EXCHANGE RINGS SATISFYING
STABLE RANGE CONDITIONS

HUANYIN CHEN AND MIAOSEN CHEN

ABSTRACT. In this paper, we establish necessary and sufficient conditions for an exchange ring $R$ to satisfy the $n$-stable range condition. It is shown that an exchange ring $R$ satisfies the $n$-stable range condition if and only if for any regular $a \in R^n$, there exists a unimodular $u \in R^n$ such that $au \in R$ is a group member, and if and only if whenever $a \preceq_{n} b$ with $a \in R, b \in M_n(R)$, there exist $u \in R^n, v \in R^n$ such that $a = ubv$ with $uv = 1$. As an application, we observe that exchange rings satisfying the $n$-stable range condition can be characterized by Drazin inverses. These also give nontrivial generalizations of [7, Theorem 10], [13, Theorem 10], [15, Theorem] and [16, Theorem 2A].

1. Introduction

Let $R$ be an associative ring with identity. $R$ is said to satisfy the $n$-stable range condition if for every split epimorphism $M = (\oplus_{i=1}^{n} R) \oplus H \rightarrow R$ there is a splitting $\psi : R \rightarrow M$ such that $M = \psi(R) \oplus L \oplus H$ with $L \subseteq \oplus_{i=1}^{n} R$. It is well known that, if $A$ is a right $R$-module and $\text{End}_R A$ satisfies the $n$-stable range condition, then $A^{n+1} \oplus B \cong A \oplus C$ implies $A^n \oplus B \cong C$ for any right $R$-modules $B$ and $C$. The $n$-stable range condition plays an important role in algebraic $K$-theory. For example, if $R$ satisfies the $n$-stable range condition then the central extension $1 \rightarrow K_2(m, R) \rightarrow \text{St}(m, R) \rightarrow E(m, R) \rightarrow 1$ is a universal central extension for all $m \geq \max(n+3, 5)$ and $K_2(m, R) \rightarrow K_2(m+1, R)$ are surjective for all $m \geq n + 2$. Furthermore, one sees that $K_{2,m}(R) \cong K_2(R)$ provided that $m \geq n + 2$. Also we have $K_1(R) \cong U(R)/W(R)$ provided that $R$ satisfies the $1$-stable range condition. Many authors have studied rings satisfying the $n$-stable range condition (see [1], [3-6] and [10-14]).

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Recall that $R$ is an exchange ring if, for every right $R$-module $A$ and any two decompositions $A = M \oplus N = \bigoplus_{i \in I} A_i$, where $M_R \cong R$ and the index set $I$ is finite, there exist submodules $A'_i \subseteq A_i$ such that $A = M \oplus (\bigoplus_{i \in I} A'_i)$. The class of exchange rings is very large. It includes all regular rings, all $\pi$-regular rings, all strongly $\pi$-regular rings, all semiperfect rings, all left or right continuous rings, all clean rings, all unit $C^*$-algebras of real rank zero and all right semi-artinian rings (cf. [2-4], [9-12] and [14] and [17]).

In this paper, we establish necessary and sufficient conditions for an exchange ring $R$ to satisfy the $n$-stable range condition. It is shown that an exchange ring $R$ satisfies the $n$-stable range condition if and only if for any regular $a \in R^n$, there exists a unimodular $u \in R$ such that $au \in R$ is a group member. In addition, we introduce $n$-pseudo similarity for exchange rings. Using this new concept, we show that an exchange ring $R$ satisfies the $n$-stable range condition is equivalent to whenever $a^n b$ with $a \in R, b \in M_n(R)$, there exist $u \in R^n, v \in R$ such that $a = uvw$ with $uv = 1$. As an application, we observe that exchange rings satisfying the $n$-stable range condition can be characterized by Drazin inverses. These also give nontrivial generalizations of [7, Theorem 10], [13, Theorem 10], [15, Theorem] and [16, Theorem 2A].

Throughout this paper, all rings are associative rings with identity and all modules are right unitary modules. The symbol $M \subseteq N$ means that $M$ is isomorphic to a direct summand of a module $N$. The symbol $mR$ denotes the $M_m(R)$-bimodule $\{(x_1, \ldots, x_m)^T \mid x_1, \ldots, x_m \in R\}$, where scalar multiplications are products of matrices over the ring $R$. $R^n = \{(a_1, \ldots, a_n) \mid a_1, \ldots, a_n \in R\}$. $R = \{(a_1, \ldots, a_n)^T \mid a_1, \ldots, a_n \in R\}$. Call $x \in R^n(x \in R)$ is unimodular provided that $xy = 1(yz = 1)$ for some $y \in R(y \in R^n)$. $x \in R^n(x \in R)$ is said to be regular provided that $x = xyx$ for some $y \in R(y \in R^n)$.

2. Group members

Recall that $g \in R$ is group member of $R$ provided that there exists some $g^y \in R$ such that $g = gg^y g, g^y = g^y gg^y$ and $gg^y = g^y g$. That is, $g \in R$ has group inverse $g^y \in R$. The author proved that a regular ring $R$ is one-sided unit-regular if and only if for any regular $a \in R$, there exists some one-sided unit $u \in R$ such that $au \in R$ is a group member. By virtue of group members, we now extend [7, Theorem 10] and characterize exchange rings satisfying the $n$-stable range condition, which also extend [16, Theorem 2A].
LEMMA 2.1. Let $R$ be an exchange ring. Then the following are equivalent:

(1) $R$ satisfies the $n$-stable range condition.
(2) Whenever $a(nR) = eR$ with $a \in R^n$, $e = e^2 \in R$, there exists a unimodular $u \in nR$ such that $au = e$.
(3) Whenever $a(nR) = eR$ with $a \in R^n$, $e = e^2 \in R$, there exists a unimodular $u \in R^n$ such that $a = eu$.

Proof. (1)$\Rightarrow$(2) Given $a(nR) = eR$ with $a \in R^n$, $e = e^2 \in R$, then $ax = e$ for some $x \in R^n$. Furthermore, we have $r_1, \ldots, r_n \in R$ such that $a(1, 0, \ldots, 0)^T = e(r_1, 0, \ldots, 0)^T = e(r_2, 0, \ldots, 0)^T = \cdots = e(r_n, 0, \ldots, 0)^T = e(r_1, r_2, \ldots, r_n)$. Set $y = (r_1, r_2, \ldots, r_n)$. Then we claim that $ax = e$ and $a = ey$ for $x \in R^n$, $y \in R^n$. Since $yx + (1 - yx) = 1$, we can find a $z \in R^n$ such that $y + (1 - yx)z = u \in R^n$ is unimodular. So we have $uv = 1$ for a $v \in R^n$, and then $(y + (1 - yx)z)v = 1$. It is easy to verify that $eyz = az = e$. Therefore $e = e(y + (1 - yx)z)v = eyv = av$, as desired.

(2)$\Rightarrow$(1) Given any regular $x \in R^n$, there exists a $y \in R$ such that $x = xyx$. We easily check that $x(nR) = (xy)R$. Hence we can find a unimodular $u \in R^n$ such that $xu = xyx$. Therefore $x = xyx = xux$. It follows by [18, Theorem 9] that $R$ satisfies the $n$-stable range condition.

(1)$\Rightarrow$(3) is analogous to the consideration above.

(3)$\Rightarrow$(1) For any regular $a \in R^n$, there exists $b \in R^n$ such that $a = aba$. Set $e = ab$. Then $a(nR) = eR$ with $e = e^2 \in R$. Hence $a = eu$ for a unimodular $u \in R^n$. It follows by [12, Theorem 4.2] that $R$ satisfies the $n$-stable range condition. \qed

THEOREM 2.2. Let $R$ be an exchange ring. Then the following are equivalent:

(1) $R$ satisfies the $n$-stable range condition.
(2) Whenever $a(nR) = gR$ with $a \in R^n$ and group member $g \in R$, there exists a unimodular $u \in R$ such that $au = g$.
(3) Whenever $a(nR) = gR$ with $a \in R^n$ and group member $g \in R$, there exists a unimodular $u \in nR$ such that $a = gu$.

Proof. (2)$\Rightarrow$(1) is clear by Lemma 2.1.

(1)$\Rightarrow$(2) Suppose that $a(nR) = gR$ with $a \in R^n$ and group member $g \in R$. Because $g \in R$ has a group inverse $g^\#$ in $R$, we see that $g = g(g^\# + 1 - g^\#)g$g. Clearly, $(g^\# + 1 - g^\#)g^{-1} = g + 1 - g^\#g \in U(R)$. Set $g(g^\# + 1 - g^\#)g = e$. Then $a(nR) = eR$. In view of Lemma 2.1, we can find a unimodular $u \in nR$ such that $au = e = g(g^\# + 1 - g^\#)g$. Hence
One easily checks that \( u(\overline{g} + 1 - \overline{g}g)^{-1} \in R \) is unimodular, as required.

(3)\( \Rightarrow \) (1) Since every idempotent has group inverse, the implication is obvious by Lemma 2.1.

(1)\( \Rightarrow \) (3) Suppose that \( a^nR = gR \) with \( a \in R^n \) and group member \( g \in R \). Similarly to the consideration above, we have \( a^nR = eR \) with \( e = g(\overline{g} + 1 - \overline{g}g) \). Using Lemma 2.1 again, we can find a unimodular \( u \in R \) such that \( a = eu = g(\overline{g} + 1 - \overline{g}g)u \). Clearly, \( (\overline{g} + 1 - \overline{g}g)u \in R^n \) is also unimodular. Therefore we complete the proof.

In general, \( R \) satisfies the \( n \)-stable range condition is not equivalent to for any regular \( a \in R^n \), there exists a unimodular \( u \in R \) such that \( au \) is a group member if \( R \) is not an exchange ring. For example, \( \mathbb{Z} \) satisfies the condition (2) above for \( n = 1 \), while it has no stable range one. But we note that the class of exchange rings satisfying the finite stable ranges is very large. We refer the reader to [3], [11-12] and [18].

In order to have more examples of exchange rings satisfying the \( n \)-stable range condition, we raise the question of when a trivial extension is an exchange ring satisfying the \( n \)-stable range condition.

**Proposition 2.3.** Let \( R \) be a ring, \( M \) an \( R-R \)-bimodule. If \( R \) is an exchange ring satisfying the \( n \)-stable range condition, then so is the trivial extension \( R \rtimes M \).

**Proof.** We easily check that \( U(R \rtimes M) = \{(u, m) | u \in U(R), m \in M \} \). Hence \( J(R \rtimes M) = \{(r, m) | r \in J(R), m \in M \} \). Therefore \( R \rtimes M/J(R \rtimes M) \cong R/J(R) \). Since \( R \) is an exchange ring satisfies the \( n \)-stable range condition, so is \( R \rtimes M/J(R \rtimes M) \). Hence \( R \rtimes M \) satisfies the \( n \)-stable range condition. Clearly, \( J(R \rtimes M) \) is an exchange ring and idempotents in \( R \rtimes M/J(R \rtimes M) \) can be lifted modulo \( J(R \rtimes M) \). In view of [2, Theorem 2.2], we claim that \( R \rtimes M \) is an exchange ring, as asserted.

**3. \( n \)-Pseudo similarity**

So as to get more characterizations of exchange rings satisfying the \( n \)-stable range condition, we now introduce the concept of the \( n \)-pseudo similarity over exchange rings.

Let \( a \in R, b \in M_n(R) \). Then we say that \( a \) is \( n \)-pseudo similar to \( b \) in \( R \) provided that there exist \( x \in R \), \( y, z \in R^n \) such that \( xay = b, zbx = a \).
and \(xyx = xzx = x\). We denote it by \(a \simeq_R b\). Clearly, pseudo similarity on a ring \(R\) is equivalent to 1-pseudo similarity. It is well known that \(R\) satisfies cancellation if and only if pseudo similarity coincides with pseudo similarity (see [15, Theorem]). Unfortunately, Guralnick and Lanski's technique cannot be applied to exchange rings satisfying the \(n\)-stable range condition. We need to develop some new methods.

**Lemma 3.1.** Let \(e = e^2 \in R\) and \(f = f^2 \in M_n(R)\). Then

1. \(eR \simeq f(nR)\) if and only if there exist \(a \in R^n\) and \(b \in R^n\) such that \(e = ab\), \(a = eaf\) and \(b = fbe\).

2. \(eR \cong f(nR)\) if and only if there exist \(a \in R^n\) and \(b \in R^n\) such that \(e = ab\), \(f = ba\), \(a = eaf\) and \(b = fbe\).

**Proof.** (1) Suppose that \(eR \simeq f(nR)\). We have a projection \(p : f(nR) \rightarrow eR\). Let \(\{\eta_1, \ldots, \eta_l\}\) be the standard basis of \(nR\). Then we have \(x \in nR\) such that \(e = p(fx)\), so

\[
e = e^2 = ep(fx)e
= e(p(\sum_{j=1}^{n} f\eta_j\eta_j^T fx))e
= (e(\sum_{j=1}^{n} p(f\eta_j\eta_j^T)f)(fx)e).
\]

Let \(a = e(\sum_{j=1}^{n} p(f\eta_j\eta_j^T)f)\) and \(b = fxe\). Then \(a = eaf\), \(b = fbe\), and \(e = ab\), as desired.

Conversely, suppose \(e = ab\) with \(a \in R^n\) and \(b \in R^n\) such that \(a = eaf\) and \(b = fbe\). We construct an \(R\)-homomorphism \(\psi : f(nR) \rightarrow eR\) by \(\psi(fx) = afx\) for \(x \in R\). For any \(er \in eR\) with \(r \in R\), there exists \(x = br \in nR\) such that \(\psi(fx) = afx = afbr = abr = er\). So \(\psi\) is an epimorphism. Because \(eR\) is a projective right \(R\)-module, we know that the exact sequence \(0 \rightarrow Ker\psi \rightarrow f(nR) \rightarrow eR \rightarrow 0\) is split. Hence, \(eR \cong Ker\psi \cong f(nR)\). Therefore, \(eR \simeq f(nR)\).

(2) Analogously the consideration above, we easily complete the proof. \(\Box\)

**Lemma 3.2.** Let \(R\) be a ring with \(a \in R, b \in M_n(R)\). Then the following are equivalent:

1. \(a \simeq_R b\).

2. There exist \(x \in R^n, y \in R^n\) such that \(a = ybx, b = xay, x = xyx\) and \(y = yxy\).

**Proof.** (2)\(\Rightarrow\)(1) is trivial.

(1)\(\Rightarrow\)(2) Since \(a \sim_R b\), there are \(x \in R^n, y, z \in R^n\) such that \(xay = b, zbx = a\) and \(xyx = xzx = x\). By replacing \(y\) with \(yxy\) and \(z\) with
We can assume $y = yxy$ and $z = zxz$. We directly check that $xazx = xzbzxy = zbx = xay = b, zybx = zyxayx = xay = zbx = a, zxy = zyxzxy$ and $x = xzxy$, as required.

In [15], Guralnick and Lanski observed that the definition of pseudo-similarity in [15] differs from that introduced by Hartwig and Putcha (Linear Algebra Appl., 39, 1981, 125–132). In fact, we have proved that the two kind of concepts coincide with each other in the lemma above. Now we characterize exchange rings satisfying the $n$-stable range condition by virtue of $n$-pseudo similarity. We also note that if $R$ is an exchange ring satisfying the $n$-stable range condition then so is $M_n(R)$. Thus $n$-pseudo similarity also give new relations for matrices over exchange rings.

**Theorem 3.3.** Let $R$ be an exchange ring. Then the following are equivalent:

1. $R$ satisfies the $n$-stable range condition.
2. For any $a \in R, b \in M_n(R)$, $a \simeq_n b$ implies that there exist $u \in R^n, v \in R^n$ such that $a = uv$ with $uv = 1$.

**Proof.** (1)$\Rightarrow$ (2) Assume that $a \simeq_n b$. By Lemma 3.2, we have $x \in R^n$ such that $a = ybx, b = xay, x = xyx$ and $y = yxy$. Since $R$ satisfies the $n$-stable range condition, by [18, Theorem 9], we have a unimodular $s \in R^n$ such that $x = xsx$. Assume that $st = 1$. Set $w = x + (I_n - xs)t(1 - sx)$. Then we verify that $x = wsw$ with $sw = 1$. Let $u = (1 - yx - sx)s(I_n - xy - xs), v = (1 - xy - xs)t(1 - yx - sx)$. One easily checks that $(I_n - xy - xs)^2 = I_n, (1 - yx - sx)^2 = 1$ and $uv = 1$. Furthermore, we show that $uvb = (1 - yx - sx)s(I_n - xy - xs)b(I_n - xy - xs)t(1 - yx - sx)w = yxsbx = ybx = a$, as desired.

(2)$\Rightarrow$ (1) Given $eR \cong f(nR)$ with $e = e^2 \in R, f = f^2 \in M_n(R)$, in view of Lemma 3.2, there exist $a \in R^n$ and $b \in R^n$ such that $e = ab, f = ba, e = eaf, a = ebf = ebea$. So $e \cong_n f$. Thus we can find $u \in R^n$ and $v \in R^n$ such that $e = uvf$ with $uv = 1$. It is easy to check that $1 - e = u(I_n - f)v = (1 - e)(u(I_n - f))((I_n - f)v(1 - e))$. Using Lemma 3.2 again, we conclude that $(1 - e)R \cong (I_n - f)(nR)$.

Given $R$-module decompositions $R = A_1 \oplus B_1$ and $nR = A_2 \oplus B_2$ with $A_1 \cong A_2$, there are $e = e^2 \in R$ and $f = f^2 \in M_n(R)$ such that $A_1 = eR$ and $A_2 = f(nR)$. Thus, $eR \cong f(nR)$, whence $B_1 \cong R/A_1 \cong$
\[(1 - e)R \subseteq (I_n - f)(nR) \cong nR/A_2 \cong B_2.\] According to [18, Theorem 13], we complete the proof.

Recall that an exchange ring \( R \) is strongly separative if the following condition holds: For all finitely generated projective right \( R \)-modules \( A \), \( B \) and \( C \), \( 2C \oplus A \cong C \oplus B \Rightarrow C \oplus A \cong B \) (cf. [3]).

**Corollary 3.4.** Let \( R \) be a strongly separative exchange ring, \( n \geq 2 \) a positive integer. For any \( a \in R, b \in M_n(R), a \sim_n b \), there exist \( u \in R^n, v \in n R \) such that \( a = uvb \) with \( uv = 1 \).

**Proof.** Since \( R \) is a strongly separative exchange ring, it satisfies the 2-stable range condition. So we complete the proof by Theorem 3.3. \( \square \)

**Proposition 3.5.** Let \( R \) be an exchange ring. Then the following are equivalent:

1. \( R \) satisfies the \( n \)-stable range condition.
2. For any regular \( a \in R^n \), there exists a unimodular \( u \in R^n, v \in n R \) such that \( aa^{-} = ua^{-}av \) with \( uv = 1 \).
3. For any regular \( a \in R^n \), there exists a unimodular \( u \in n R \) such that \( a^{-}au = uaa^{-} \).

**Proof.** (2)\( \Rightarrow \) (1) Assume that \( eR \cong f(nR) \) with \( e = e^2 \in R, f = f^2 \in M_n(R) \). By Lemma 3.1, we have \( a \in R^n \) and \( b \in n R \) such that \( e = ab, f = ba, a = eaf \) and \( b = fbe \). Thus we claim that \( a = ea = aba \). So we may choose \( a^{-} = b \). Hence \( ab = u(ba)v \) and \( uv = 1 \) for some \( u \in n R, v \in n R \). Therefore \( 1 - a = u(I_n - f)v = (1 - e)u(I_n - f)v(1 - e) \). Consequently, we show that \( (1 - e)R \subseteq (I_n - f)(nR) \), as needed.

(1)\( \Rightarrow \) (2) Given any regular \( a \in R^n \), there exists \( a^{-} \in n R \) such that \( a = aa^{-}a \). We construct a map \( \psi : aa^{-}R = a(nR) \to a^{-}a(nR) \) given by \( \psi(ax) = a^{-}ax \) for any \( x \in n R \). Obviously, we claim that \( \psi : aa^{-}R \cong a^{-}a(nR) \). According to Lemma 3.1 and Lemma 3.2, we see that \( aa^{-} \sim a^{-}a \). Hence the result follows by Theorem 3.3.

(1)\( \Rightarrow \) (3) is analogous to the discussion above. \( \square \)

Recall that \( a \in R \) is said to be strongly \( \pi \)-regular provided that there exist \( n \geq 1 \) and \( x \in R \) such that \( a^n = a^{n+1}x, ax = xa \) and \( x = xax \). By an argument of P. Ara, we know that every strongly \( \pi \)-regular element of an exchange ring is unit-regular. Clearly, the solution \( x \in R \) is unique, and we say that \( x \) is the Drazin inverse \( a^d \) of \( a \). As an application of Theorem 3.3, we now give a new necessary and sufficient condition under
which an exchange ring $R$ satisfies the $n$-stable range condition, which also shows that the $n$-stable range condition over exchange rings can be determined by Drazin inverses.

**Theorem 3.6.** Let $R$ be an exchange ring. Then the following are equivalent:

1. $R$ satisfies the $n$-stable range condition.
2. Whenever $ab \in R, ba \in M_n(R)$ are strongly $\pi$-regular with $a \in R^n, b \in R^n$, there exists a unimodular $u \in R^n, v \in R^n$ such that $(ab)^d = u(ba)^d v$ with $uv = 1$.
3. Whenever $ab \in R, ba \in M_n(R)$ are strongly $\pi$-regular with $a \in R^n, b \in R^n$, there exists a unimodular $u \in R^n$ such that $(ab)(ab)^d = u(ba)(ba)^d v$ with $uv = 1$.

**Proof.** (1)$\Rightarrow$(2) Since $ab \in R$ are strongly $\pi$-regular, we have $k \geq 1$ such that $(ab)^k = (ab)^{k+1}(ab)^d$, $(ab)(ab)^d = (ab)^d(ab)$ and $(ab)^d = (ab)^d(ab)(ab)^d$. Likewise, we have $l \geq 1$ such that $(ba)^l = (ba)^{l+1}(ba)^d$, $(ba)(ba)^d = (ba)^d(ba)$ and $(ba)^d = (ba)^d(ba)(ba)^d$. So we verify that

\[(ab)^{k+l+1} = (ab)^{k+l+1}(ba)^d(ba)^d\]
\[(ab)(ab)^d(ba)^d = (ab)(ab)^d(ba)(ba)^d = (ab)^d(ba)^d(ba)^d = (ab)^d(ba)^d(ba)^d\]

Hence $(ab)^d = a(ba)^d(ba)^d b$. By a similar route, we also have $(ba)^d = b(ab)^d(ab)^d a$. Furthermore, we check that

\[(ba)^d = (ba)^d(ba)(ba)^d = (ba)^d bab(ab)^d(ab)^d a = (ba)^d b(ab)^d a.

Clearly, $(ba)^d(ba)(ba)^d = (ba)^d b$. Thus $(ab)^d \sim_n (ba)^d$, as required.
(2) ⇒ (1) Suppose that \( eR \cong f(R) \) with idempotents \( e \in R, f \in M_n(R) \). Then we can find some \( a \in R^n, b \in R^n \) such that \( e = ab, f = ba, a = eaf \) and \( b = fbe \). Obviously, \( e \) and \( f \) are both strongly \( \pi \)-regular, hence \( (ab)^d \cong_n (ba)^d \). Inasmuch as \( e^d = e \) and \( f^d = f \), we claim that \( e = uv \) and \( uv = 1 \) for some \( u \in R^n, v \in R^n \). Therefore \( R \) satisfies the \( n \)-stable range condition by Theorem 3.3.

(1) ⇒ (3) Similarly to the consideration above, one easily checks that
\[
(ab)^d \cong_n (ba)^d,
\]
as desired.

(3) ⇒ (1) Given \( eR \cong f(R) \) with \( e = e^2 \in R, f = f^2 \in M_n(R) \), then there exist \( a \in R^n, b \in R^n \) such that \( e = ab, f = ba, a = eaf \) and \( b = fbe \). Analogously to the consideration in (2) ⇒ (1), we obtain the result.

[Corollary 3.7] Let \( F \) be a field. For any \( a \in F^n, b \in F^n \), there exists a unimodular \( u \in F^n, v \in F^n \) such that \( (ab)^d = u(ba)^d v \) with \( uv = 1 \).

Proof. Since \( F \) is a field, we know that \( F \) and \( M_n(F) \) are both strongly \( \pi \)-regular rings. Hence \( (ab)^d \) and \( (ba)^d \) exist for any \( a \in F^n, b \in F^n \). Thus the proof is complete by Theorem 3.6.

References


DEPARTMENT OF MATHEMATICS, ZHEJIANG NORMAL UNIVERSITY, JINHUA, ZHEJIANG 321004, P. R. CHINA
E-mail: chyzxd@sparc2.hunnu.edu.cn

miao@szhptt.zj.cn