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A STUDY ON NONSTATIONARY RANDOM VIBRATION OF A VEHICLE IN TIME AND FREQUENCY DOMAINS

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ABSTRACT–A time domain method for solving nonstationary random vibration caused by vehicle acceleration is first proposed in which a time changing model is established for representing nonstationary excitation of a rough road. Furthermore a novel frequency domain method called the transient power spectral density with spatial frequency (TPSD) is presented to obtain a response of vehicle system in frequency domain. This method has been proved to be valid by comparing numerical results with the exact solution.

KEY WORDS : Nonstationary excitation model of rough road, Vehicle acceleration, Transient power spectral density with spatial frequency

1. INTRODUCTION

Random vibration of a vehicle caused by the excitation of rough road has a major influence on vehicle performances such as ride performance, dynamic load of parts and etc. Therefore a lot of research works have been done on this field. Most of research works were based on linear or nonlinear models with the constant running speed. The excitation of rough road was given in terms of PSD (power spectral density) or time course. Some of research works utilized the parameter optimization to improve ride characteristics (Dokainish and Elmadany, 1980; Hac, 1985). In recent years, studies on active and semi-active suspension have become popular (Hady and Crolla, 1989). However, because of the cost of active and semi-active suspensions, they have not been widely put into practice yet. All that mentioned above show that most of the research works supposed the vehicle vibration to be a stationary random process in both passive and active suspensions. In fact, while a car is traveling at variable speeds such as starting, accelerating, braking and so on, the vibration excited by the rough road should be nonstationary random vibration in time domain. Since this problem is very close to actual situation, random vibration of a vehicle is a very important topic, and it is also significant in engineering. However on account of the restriction of the theory of nonstationary random vibration, there are only a few of approaches on the topic being developed. In this paper, two methods for working out this problem are newly proposed.

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2. RESEARCH BACKGROUND

In general case, the function of road roughness is regarded as a stationary random process in space domain, while a car is moving with the constant velocity. It is also a stationary random process in time domain. However, road roughness is a non-stationary random process in time domain while a vehicle is traveling at variable speeds. Therefore, the vibration caused by rough road surface should be considered as a non-stationary random process. A state-space approach was presented to analyze the response of a vehicle traveling on homogenous rough road (Hammond and Harrison, 1981). In that, the dynamics were modeled by liner ordinary differential equations in time domain while the excitation process was modeled by a differential equation in spatial domain. The variance of the response was obtained by using so-called covariance equivalent modeling. Based on this work, an improved method with more computational efficiency was proposed by using complex modal analysis (Hwang and Kim, 2000). There was another method for solving this problem (Nigam and Yaday, 1974; Fang *et al.*, 1997), in which differential equations with variable coefficient were established first in space domain, and then, the time changing covariance was computed. However, the amount computation of this method is too large. This paper mainly investigates a new time domain method by using the nonstationary excitation model of a rough road. Moreover a new frequency method so-called transient spatial PSD (TPSD) is proposed to solve this problem.

3. MODELING

A new method of solving non-stationary vibration of a vehicle is mainly investigated, which can also be used in various kinds of vehicle models. As an example, a vehicle model with 5 degrees of freedom is established. The vehicle model is shown in Figure 1.

Where,

Z_s – vertical displacement of seat,

Z_b – vertical displacement of vehicle body at center of gravity,

Z_p – pitch angular displacement of vehicle body,

Z_f – bouncing displacement of unsprung mass of front suspension,

M_s – mass of human body and seat,

M_b – mass of vehicle body,

M_p – moment of inertia of vehicle body with respect to axis,

M_f, M_r – unsprung masses of front and rear suspensions, respectively,

K_s – stiffness coefficient of seat,

K_f, K_r – stiffness coefficients of front and rear suspensions, respectively,

K_{tf}, K_{tr} – stiffness coefficients of front and rear tires, respectively,

C_s – damping coefficient of seat,

C_f, C_r – damping coefficients of front and rear suspensions, respectively,

q_f, q_r – displacements of road input at front and rear tires, respectively,

l_1 – distance from the center of gravity to seat,

l_2, l_3 – distances from the center of gravity to front and rear tires, respectively,

L – wheelbase.

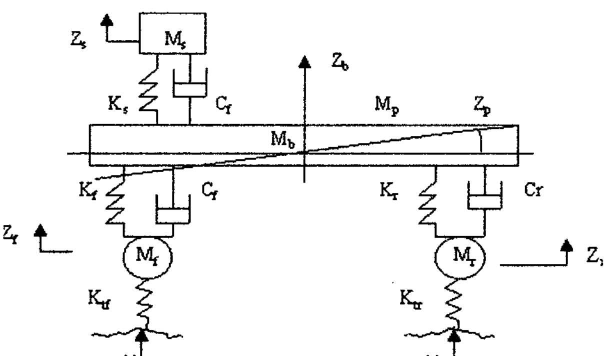

Figure 1. Vehicle model with 5 degrees of freedom.

In this model, damping of the tires is neglected since it is so small. The nonlinear damping and stiffness of the suspension can also be considered if necessary.

The differential equations for the vehicle model can be described as follows:

$$[M]\ddot{Z}+[C]\dot{Z}+[K]Z=[F]Q \tag{1}$$

Where,

Z – output vector,

$Z=[z_s\ z_b\ z_p\ z_f\ z_r]^T$

$[M]$ – mass matrix,

$$[M]=\begin{bmatrix} M_s & 0 & 0 & 0 & 0 \\ 0 & M_b & 0 & 0 & 0 \\ 0 & 0 & M_p & 0 & 0 \\ 0 & 0 & 0 & M_f & 0 \\ 0 & 0 & 0 & 0 & M_r \end{bmatrix}$$

$[C]$ – damping matrix,

$$[M]=\begin{bmatrix} C_s & -C_s & C_s l_1 & 0 & 0 \\ -C_s & C_s+C_f+C_r & -C_s l_1-C_f l_2+C_r l_3 & -C_f & -C_r \\ C_s l_1 & -C_s l_1-C_f l_2+C_r l_3 & C_s l_1^2+C_f l_2^2+C_r l_3^2 & C_f l_2 & -C_r l_3 \\ 0 & -C_f & C_f l_2 & C_f & 0 \\ 0 & -C_r & -C_r l_3 & 0 & C_r \end{bmatrix}$$

$[K]$ – stiffness matrix,

$$[K]=\begin{bmatrix} K_s & -K_s & l_1 K_s & -K_f & 0 \\ -K_s & K_s+K_f+K_r & -K_s l_1-K_f l_2+K_r l_3 & K_f l_2 & -K_r \\ l_1 K_s & -K_s l_1-K_f l_2+K_r l_3 & K_s l_1^2+K_f l_2^2+K_r l_3^2 & K_f+K l_2 & -K_r l_3 \\ 0 & -K_f & K_f l_2 & K_f+K_{tf} & 0 \\ 0 & K_r & -K_r l_3 & 0 & K_r+K_{tr} \end{bmatrix}$$

$[F]$ – excitation force matrix,

$$[F]=\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ K_{tf} & 0 \\ 0 & K_{tr} \end{bmatrix}$$

$[Q]$ – excitation vector,

$[Q] = [q_f\ q_r]^T$

When a car is running at the variable speed, functions of $q_f(t)$ and $q_r(t)$ are non-stationary in time domain, but it is noted that they are stationary in the spatial domain.

4. TIME DOMAIN METHOD

4.1. State Equations

To obtain the results of Equation (1) in the time domain, state vectors are utilized as follows:

$$[x_1\ x_2\ x_3\ x_4\ x_5]^T = [z_s\ z_b\ z_p\ z_f\ z_r] \quad (1\text{-a})$$

$$[x_6\ x_7\ x_8\ x_9\ x_{10}]^T = [\dot{z}_s\ \dot{z}_b\ \dot{z}_p\ \dot{z}_f\ \dot{z}_r] \quad (1\text{-b})$$

Hence

$$[\dot{x}_1\ \dot{x}_2\ \dot{x}_3\ \dot{x}_4\ \dot{x}_5]^T=[x_6\ x_7\ x_8\ x_9\ x_{10}] \quad (1\text{-c})$$

$$[\dot{x}_6\ \dot{x}_7\ \dot{x}_8\ \dot{x}_9\ \dot{x}_{10}]^T=[\ddot{z}_s\ \ddot{z}_b\ \ddot{z}_p\ \ddot{z}_f\ \ddot{z}_r] \quad (1\text{-d})$$

Substituting these state vectors into Equation (1), then the following state equation can be obtained:

$$\dot{X}=AX+BU \quad (2)$$

where,
X – state variables

$$X=[x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7\ x_8\ x_9\ x_{10}]^T$$

A – state coefficient matrix

$$A=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -K_s/M_s & K_s/M_s & -K_s l_1/M_s & 0 & 0 & -C_s/M_s & C_s/M_s & -C_s l_1/M_s & 0 & 0 \\ K_s/M_b & a_{72} & a_{73} & K_f/M_b & K_r/M_b & -C_s/M_b & a_{77} & a_{78} & C_f/M_b & C_r/M_b \\ -K_s l_1/M_p & a_{82} & a_{83} & -K_f l_2/M_p & K_r l_3/M_f & -l_1 C_s/M_p & a_{87} & a_{88} & -l_2 C_f/M_p & l_3 C_r/M_p \\ 0 & K_f/M_f & -K_f l_2/M_f & a_{94} & 0 & 0 & C_f/M_f & -C_f l_2/M_f & -C_f/M_f & 0 \\ 0 & K_r/M_r & l_3 K_r/M_r & 0 & a_{105} & 0 & C_r/M_r & C_r l_3/M_r & 0 & C_r/M_r \end{bmatrix}$$

where,

$a_{72}=(K_s+K_f+K_r)/M_b$

$a_{73}=(K_s l_1+K_f l_2-K_r l_3)/M_b$

$a_{77}=-(C_s+C_f+C_r)/M_b$

$a_{78}=(C_s l_1+C_f l_2-C_r l_3)/M_b$

$a_{82}=(K_s l_1+K_f l_2-K_r l_3)/M_p$

$a_{83}=-(l_1^2 K_s+l_2^2 K_f+l_3^2 K_r)/M_p$

$a_{87}=(l_1 C_s+l_2 C_f-l_3 C_r)/M_p$

$a_{88}=-(l_1 C_s+l_2 C_f-l_3 C_r)/M_p$

$a_{94}=-(K_f+K_{tf})/M_f$

$a_{105}=-(K_r+K_{tr})/M_r$

B - input coefficient matrix,

$$B=\begin{bmatrix} 0\,0\,0\,0\,0\,0\,0\,0 & K_{tf}/M_f & 0 \\ 0\,0\,0\,0\,0\,0\,0\,0 & 0 & K_{tr}/M_r \end{bmatrix}^T$$

U - input vector,

$$U=\begin{bmatrix} q_f \\ q_r \end{bmatrix}$$

4.2. Simulation of Road Roughness

In general case, the function of road roughness $q(s)$ is regarded as a stationary random process in the space domain, if a car moves with the constant speed v, on account of the relation between moving distance s and time t, it is also a stationary random process in the time domain. The PSD (power spectral density) of rough road in the space domain can be expressed as,

$$S_q(\Omega)=S_q(\Omega_0)\left(\frac{\Omega}{\Omega_0}\right)^{-2} \quad (3)$$

Where, $S_q(\Omega)$ is the spatial power spectral density, $S_q(\Omega_0)$ is the coefficient of road roughness, Ω is the spatial angular frequency, and Ω_0 is the reference spatial angular frequency $\Omega_0=1(rad/m)$. Equation (3) can also be described as a function of spatial frequency n.

$$S_q(n)=S_q(n_0)\left(\frac{n}{n_0}\right)^{-2} \quad (4)$$

where $S_q(n_0)$ is also called the coefficient of road roughness, and n is spatial frequency $(1/m)$, $n_0=1/2\pi$. The relationship between spatial angular frequency and spatial frequency can be expressed as

$$\Omega=2\pi n \quad (5)$$

According to the classification of the road, the values of $S_q(\Omega_0)$ and $S_q(n_0)$ can be obtained in some references, such as Applications of Random Vibrations (Nigam and Narayanan, 1994). While a car is running with the constant speed, the relationship between traveling speed of vehicle and spatial frequency n is expressed as,

$$f=nv \quad (6)$$

Thus, Equations (3) and (5) can be described by following equations in temporal frequency domain,

$$S_q(\omega)=S_q(\Omega)/v=S_q(\Omega_0)v/\omega^2 \quad (7)$$

$$S_q(f)=S_q(n)/v=S_q(n_0)v/f^2 \quad (8)$$

However, there is difficulty in both of the equations at $\omega=0$, $f=0$, $S_q(\omega)=\infty$ or $S_q(f)=\infty$. An improved equation for PSD of road roughness in frequency domain is shown as,

$$S_q(\omega)=S_q(\Omega_0)v/(\omega^2+\omega_0^2) \quad (9)$$

Where, ω_0 is the lowest cut-off angular frequency, $\omega_0=2\pi f_0=2\pi n_0 v$. Equation (9) can be considered as a response of a first order linear system to white noise excitation.

Based on the theory of random vibration, following relationship can be obtained,

$$S_q(\omega)=|H(\omega)|^2 S_w \quad (10)$$

where $H(\omega)$ is the transfer function, and S_w is the PSD of white noise, where normally S_w=1. From Equations (9) and (10), $H(\omega)$ is written by,

$$H(\omega)=\frac{\sqrt{S_q(\Omega_0)v}}{\omega_0+j\omega} \quad (11)$$

From Equation (11), the differential equation about road roughness can be described as,

$$\dot{q}(t)+2\pi n_0 v q(t)=\sqrt{S_q(\Omega_0)v}\,w(t) \quad (12)$$

According to Equation (12), a numerical simulation can be carried out. Figure 2 is the simulation of road roughness in time domain while a car is running with the constant speed, and Figure 3 is its PSD got by FFT method from the time domain to the frequency domain, in which the time sample length used is 10.24 seconds. To verify simulating results, the theoretical PSD of road roughness is directly carried out in frequency domain from Equation (9), the comparison between theoretical and computing values shows that the simulating result is very close to the theoretical value.

While a car is running with a variable speed, it is nonstationary random process in time domain even road roughness input is stationary random process in space domain. Therefore, Equation (12) becomes as follows,

$$\dot{q}(t)+2\pi n_0 v(t) q(t)=\sqrt{S_q(\Omega_0)v(t)}\,w(t) \quad (13)$$

where $v(t)+v_0+at$, a is the acceleration of car moving, t is accelerating time. The Figure 4 and Figure 5 represent the simulation consequences of non- stationary function of rough road and its PSD, respectively. From Figure 4, it is concluded that with increasing the velocity of the car, the amplitude of road roughness increases, accordingly.

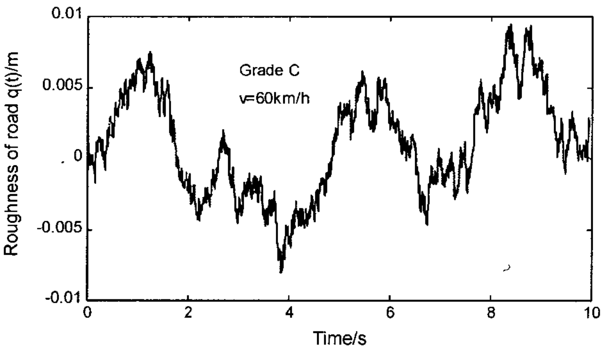

Figure 2. Time course of road roughness.

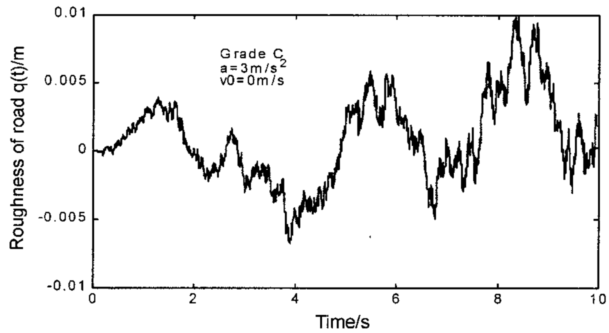

Figure 4. Nonstationary road roughness.

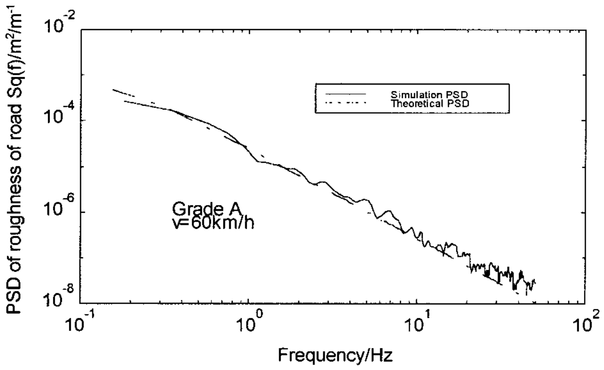

Figure 3. PSD of road roughness.

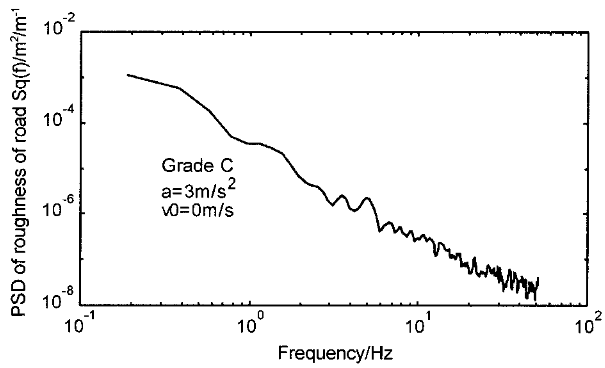

Figure 5. PSD of nonstationary road roughness.

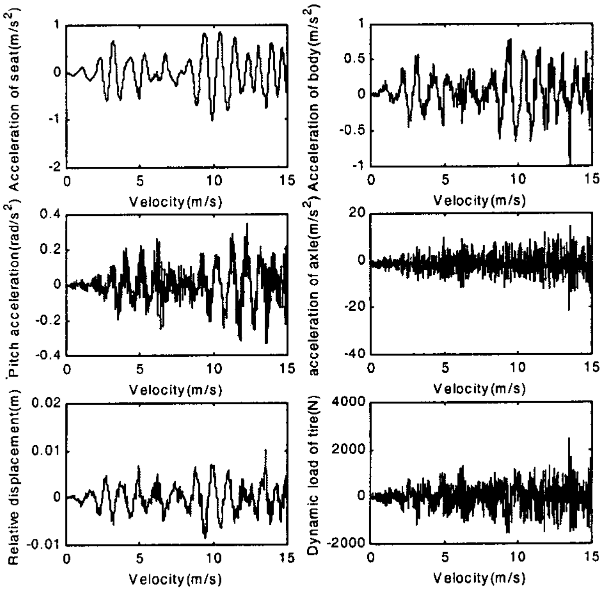

Figure 6. Nonstationary responses (a = 1.5 m/s^2).

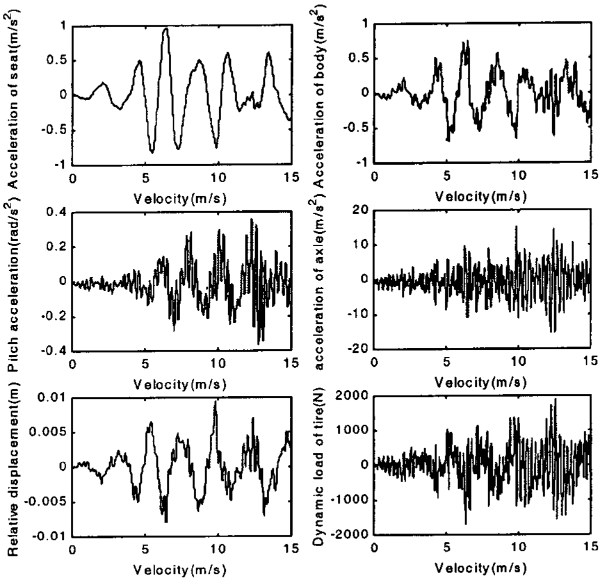

Figure 7. Nonstationary responses (a = 3 m/s^2).

4.3. Simulation of Nonstationary Responses

Inputting the values of road roughness above into Equation (2), the nonstationary responses of the vehicle system can be obtained. The model parameters representing 5 degrees of freedom are shown in Table 1. Figure 6 represents the responses of vehicle system to nonstationary excitation, i.e., the responses of the vehicle running with acceleration a = 1.5 m/s. It can be seen that the accelerations of seat, vehicle body, relative displacement and pitch acceleration have one periodicity per second in low frequency range. The accelerations of the axle and the dynamic load of the tire do not show evident periodicity with the velocity increasing. Figure 7 is simulation results of the vehicle responses with acceleration a = 3 m/s^2. From Figure 7, the periodicity numbers of the accelerations of the seat, vehicle body, relative displacement and the pitch acceleration decrease from one to half per second compared with that in Figure 6. The accelerations of the axle and the dynamic load show a little change. Namely, it takes less time for a car to reach same velocity with a big acceleration than that with a small acceleration. It is better to accelerate a car with bigger acceleration. Further analysis indicates that the response periodicity comes from the road excitation in low frequency scope. Some peaks of responses are lower than that in low acceleration case. Table 2 is comparison

Table 1. Parameters of vehicle model with 5 degrees of freedom.

Symbol	M_s (kg)	M_b (kg)	M_p (kg.m^2)	M_f (kg)	M_r (kg)
Value	70	2100	3500	140	210
Symbol	K_s (N/m)	K_f (N/m)	K_r (N/m)	K_{tf} (N/m)	K_{tr} (N/m)
Value	12200	74000	120000	520000	520000
Symbol	C_s (Ns/m)	C_f (Ns/m)	C_r (Ns/m)		
Value	550	1800	1200		

Table 2. Comparison results of RMS values (T = 10.24 s).

Condition	Seat (m/s^2)	Vehicle body (m/s^2)	Pitch (rad/m/s^2)	Axle (m/s^2)	Relative displacement (m)	Dynamic load (N)
A=1.5 m/s^2	0.366	0.269	0.100	4.190	0.0028	484.10
A=3.0 m/s^2	0.367	0.268	0.111	4.166	0.0030	504.68

results of RMS (roots mean square) values, in which the time sample length T=10.24 seconds. From Table 2, it is concluded that while a car speed up to same velocity with different acceleration, the average RMS values of various responses change a little.

5. METHOD IN SPATIAL DOMAIN

The most efficient method for solving stationary random response to a linear system is the transfer function. However, it is not suitable for solving nonstationary random response. Therefore, a new method in frequency domain needs to be proposed.

5.1 Transfer Function with Time Frequency

Taking Fourier transform to both sides of Equation (1), following equation is obtained,

$$(-\omega^2[M]+j\omega[C]+[K])Z_i(\omega)=[F_0]Q_f(\omega) \tag{14}$$

in which,

$$[F_0]=\begin{bmatrix} 0 & 0 & 0 & 0 & e^{-j\omega\tau}K_{tr} \\ 0 & 0 & 0 & K_{tf} & 0 \end{bmatrix}^T$$

where τ is time delay between the inputs of front and rear tires, $\tau=L/v$. Hence, the transfer functions in time frequency domain can be obtained,

$$H(\omega)_i=\frac{Z_i(\omega)}{Q_f(\omega)}=[A]^{-1}[F_0] \tag{15}$$

where $[A]=-\omega^2[M]+j\omega[C]+[K]$

The transfer functions of the suspension relative displacement and the tire dynamic load are expressed as follows, respectively.

$$H_{rd}(\omega)=\frac{Z_b(\omega)-l_2Z_p(\omega)-Z_f(\omega)}{Q_f(\omega)} \tag{16}$$

$$H_{tl}(\omega)=\frac{Z_f(\omega)-Q_f(\omega)}{Q_f(\omega)}K_{tf} \tag{17}$$

It is noted that Equations. (15) (16) and (17) cannot be directly used to calculate statistical characteristics of responses in time frequency domain since the input and response are all nonstationary random process. Therefore, it is necessary to seek a transfer function with space frequency for computing nonstationary responses in spatial domain.

5.2. Transient Transfer Function with Spatial Frequency (TTFWSF)

The function of rough road in time domain can be expressed as,

$$q(t)=he^{-j\omega t} \tag{18}$$

Where, h is the amplitude of road roughness. In space domain, it can be described by,

$$q(s)=he^{-j\Omega s} \tag{19}$$

where s is the running distance of a car. Regardless of what parameter introduced, the height of road roughness is equal, *i.e.*,

$$\omega t=\Omega s \tag{20}$$

while the car running with the constant velocity,

$$s=vt \tag{21}$$

Substituting Equation (21) into Equation (20), then,

$$\omega=\Omega v \quad \text{i.e.,} \quad f=nv \tag{22}$$

Equation (22) is widely used in the stationary random vibration of vehicle. And while a car is travelling with the constant acceleration,

$$s=v_0t+at^2/2 \tag{23}$$

From Equations (20) and (23), following equation is obtained,

$$\omega dt=\Omega ds \tag{24}$$

Hence,

$$\omega=\Omega ds/dt=2\pi n(v_0+at)=2\pi n\sqrt{2as+v_o^2} \tag{25}$$

Equation (25) is a kernel equation in which the relation between time frequency and space frequency is given. Substituting Equation (25) into Equation (15), the transient transfer function with space frequency $H_i(\Omega,s)$ can be obtained, also $H_i(\Omega,s)$ is just numerical consequence. Since the results of Equation (15) is just obtained by numerical method, the transient transfer function with space frequency is expressed as,

$$H_i(\Omega,s)=\frac{Z_i(\Omega,s)}{Q_f(\Omega)} \tag{26}$$

5.3. Transient PSD of Responses with Space Frequency (TPSDWSF)

According to Equation (26), the transient spatial PSD of response displacement can be expressed as,

$$S_{zi}(\Omega,s)=|H_i(\Omega,s)|^2S_q(\Omega) \tag{27}$$

Thus, the transient spatial PSD of response acceleration can be calculated as,

$$S_{\ddot{z}i}(\Omega,s)=\Omega^4(2as+v_0^2)^2|H_i(\Omega,s)|^2S_q(\Omega) \tag{28}$$

Eq. (28) can also be written as,

$$S_{\ddot{z}i}(n,s)=(2\pi n)^4(2as+v_0^2)^2|H_i(n,s)|^2S_q(n) \tag{29}$$

From Equations (16), (17) and (25), the transient spatial PSD of the relative displacement of suspension and the dynamic load of tire can be obtained,

$$S_{rd}(n,s)=|H_{rd}(n,s)|^2S_q(n) \tag{30}$$

Table 3. Parameters of vehicle model with two degrees of freedom.

Parameter Name	Sprung Mass (kg)	Unsprung Mass (kg)	Stiffness of Suspension (N/m)	Stiffness of Tire (N/m)	Damping of Suspension (Ns/m)
Numerical Value	1189.2	19.1	57831.6	900000	6455.9

$$S_{tl}(n,s)=|H_{tl}(n,s)|^2 S_q(n) \quad (31)$$

5.4. Simulation and Analysis of Responses

To prove the validity of the proposed method, a comparison result with the exact method (Hwang and Kim, 2000) is first carried out. The vehicle model parameters representing two degrees of freedom are shown in Table 3, Figure 8 is the comparison of covariance of the sprung mass displacement, and it shows that the simulation result of this paper is very close to the exact result. Therefore, this method can be applied to five degrees of freedom vehicle model shown in Figure 1. Figure 9 is the transient PSD of the vehicle responses at s=150m and a=1.5 m/s^2, in which the conclusion can be got that the regularity of response is similar to that of the stationary situation in frequency domain. The bounce and the pitch of the vehicle body focus on the resonant frequency of sprung mass. The transient PSD of the tire load has two peaks at both the vehicle body resonant

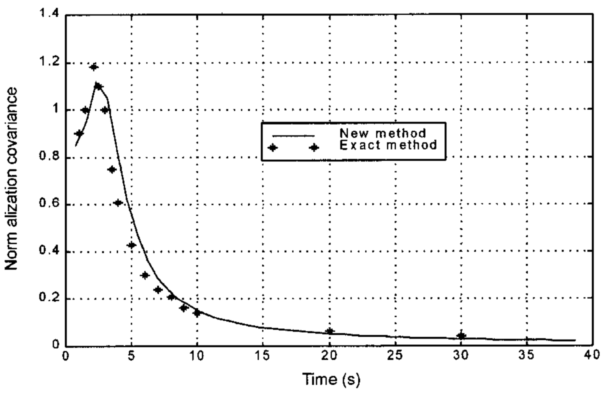

Figure 8. Comparison of covariance of sprung mass displacement.

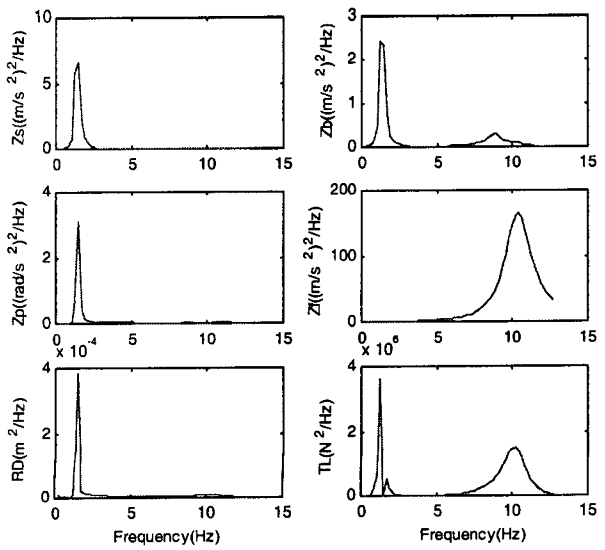

Figure 9. Transient spatial PSD of response (a=1.5 m/s^2).

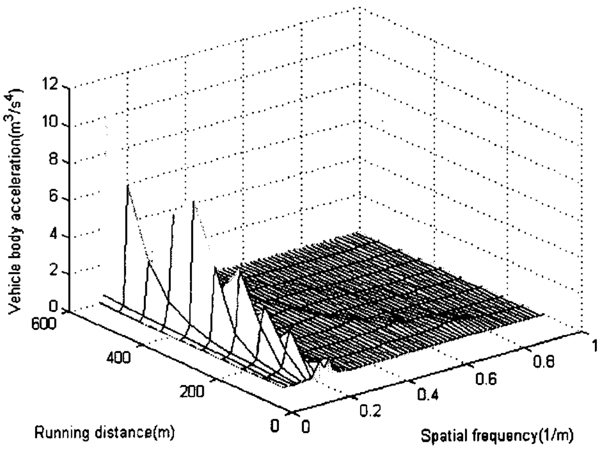

Figure 10. 3-D PSD of vehicle body acceleration.

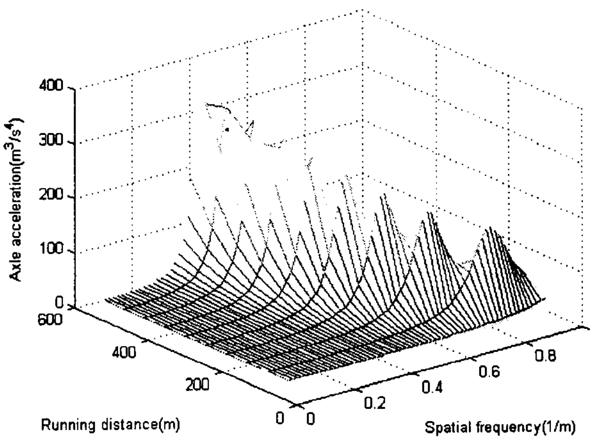

Figure 11. 3-D PSD of axle acceleration.

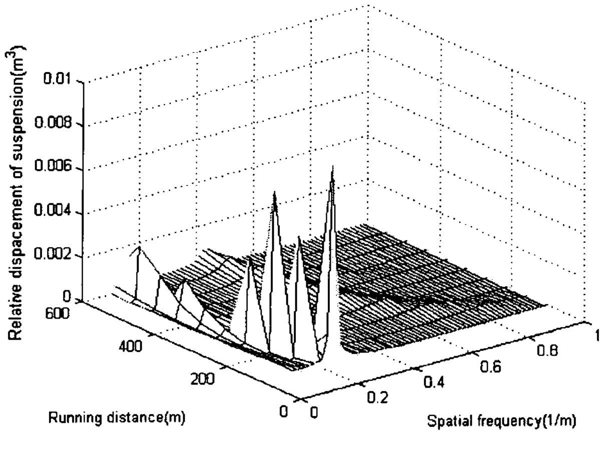

Figure 12. 3-D PSD of suspension relative displacement.

frequency and the axle resonant frequency. Figure 10 represents the three dimensional PSD of acceleration of the vehicle body. It is seen, from Figure 10, that the peak of instantaneous PSD of the vehicle body acceleration in low frequency has an increasing tendency with the running distance increasing. But in some local segments such as in the distance from 400m to 500m, the peak decreases. Figure 11 illustrates that the three-dimensional PSD of axle almost increases proportionally with increasing of the running distance. The three-dimensional PSD of the suspension relative displacement is shown in Figure 12. It is obvious that the peak in low frequency decreases with increasing of the running distance, but it does not always decrease.

Figure 13 shows that the peaks in both low frequency and high frequency increase with the running distance increasing, but the peak in low frequency does not always increase. In addition, calculation results show that the two resonant frequencies decrease with increasing of

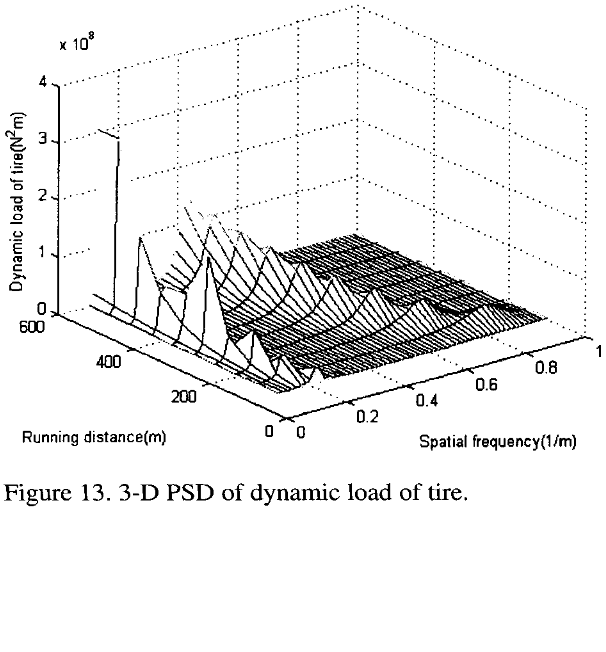

Figure 13. 3-D PSD of dynamic load of tire.

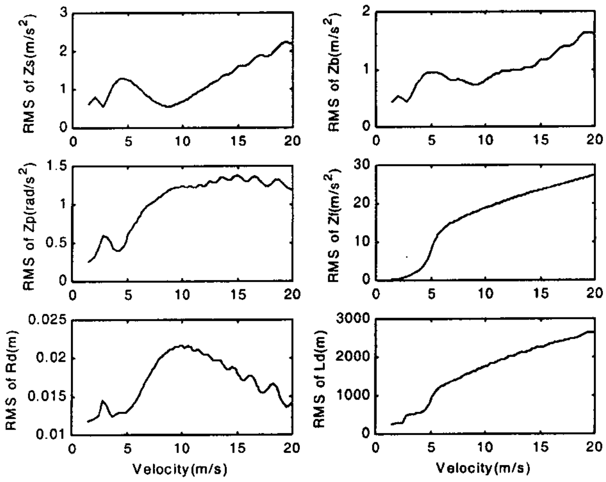

Figure 14. Response RMS versus velocity.

running distance. In Figure 14, the response RMS versus velocity is given while a car is running with a=1.5 m/s^2. When the velocity of the car is under 10 m/s, the response RMS has large change with the velocity increasing except for the axle bouncing acceleration. When the velociy of the car is over 10 m/s, the responses change stably. The RMS of the seat acceleration, the vehicle body acceleration, the axle acceleration and the dynamic load of tire increase with the velocity increasing. The RMS of relative displacement decreasing is due to the increasing of the velocity. The RMS of the pitch angular acceleration has little change.

6. CONCLUSIONS

This paper proposed two methods for solving the nonstationary random vibration of a vehicle in time and space frequency domains respectively. The nonstationary excitation model of the road roughness was modeled by using the first order time changing differential equations. A numerical simulation for the five degrees of freedom vehicle model was carried out in time domain. The computational results in time domain shows that the amplitudes of vibration caused by the vehicle accelerating with high acceleration are smaller than that with low acceleration. The conclusion can be drawn that the higher a driver accelerate his car, the better the car performance is. Based on this work, the active and semiactive suspension with the variable running speeds can be further investigated. Furthermore, a new method called transient transfer function with spatial frequency was presented in this paper by which nonstationary random vibration of vehicle was well worked out in space domain. The validity of this method was proved and a numerical example was given in frequency domain. The instantaneous PSD of the nonstationary response can be easily obtained by using the new space domain method. The two methods proposed above can also be applied to any vehicles with different parameters, even though using their nonlinear models.

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REFERENCES

Dokainish, M. A. and Elmadany, M. M. (1980). Random response of tractor-semitrailer system. *Veh. Sys. Dyn.* **9**, 87–112.

Fang, T., Sun, M. N. and Song, C. Q. (1997). Non-stationary response to the evolutionary random excitation from a same source. *Chinese Journal of Applied Mechanics.* **14, 2**, 7–13.

First Draft Proposal ISO/DP 860 for Mechanical Vibration-Road Surface Profile-Reporting Measured Data (1986).

Hac, A. (1985). Suspension optimization of a 2-DOF vehicle model using stochastic optimal control technique. *J. Sound Vib.* **100**(3), 343–357.

Haday, M. B. A. and Crolla, D. A. (1989). Theoretical analysis of active suspension performance using a four-wheel vehicle model. *Proc. Ins. Engg. J. of Automobile Engineering* (D). **203**, 125–135.

Hammond, J. K. and Harrison, R. F. (1981). Non-stationary response of vehicle on rough ground-a state space approach. *Transactions of The ASME* **103**, 245–250.

Hwang, J. H. and Kim, J. S. (2000). On the approximate solution aircraft landing gear under nonstationary random excitations. *KSME International Journal.* **14, 9**, 968–977.

Nigam, N. C. and Yadav, D. (1974). Dynamic response of accelerating vehicles to ground roughness. *Proc. Noise, Shock and Vibration Conference*, Monash University, 280–285.

Nigam, N. C. and Narayanan, S. (1994). *Applications of Random Vibrations.* Rajkamal Electric Press. 219–292.

Tompson, A. G., Davis, B. R. and Peance, C. E. M. (1980). Optimal linear active suspension with finite road preview. *SAE Paper No. 800520.*