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NUMERICAL APPROXIMATION OF VEHICLE JOINT STIFFNESS BY USING RESPONSE SURFACE METHOD

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ABSTRACT–Joint stiffness can affect the vibration characteristics of car body structures. Therefore, it should be included in vehicle system model. In this paper, a numerical approximation of joint stiffness is presented considering joint flexibility of thin walled beam-jointed structures. Using the proposed method, it is possible to optimize joint structures considering the change of section shapes in vehicle structures. The numerical approximation of joint stiffness is derived using the response surface method in terms of beam section properties. The study shows that joint stiffnesses can be effectively determined in designing vehicle structures.

KEY WORDS : Joint stiffness, Optimal design, Response surface method, Section property, Vehicle structure

1. INTRODUCTION

In the early design stage of a vehicle structure, the vibration modes less than 50 Hz are considered. Therefore, it is common practice to use simplified finite element models rather than detailed models since the simplified models substantially reduce computational cost and facilitate any design change (Kim *et al.*, 1995). In general, the thin walled beam type structures such as pillars and rockers are modeled using beam elements, the roof and floor panels are modeled using shell elements, and, to consider the local compliance, the joint structures are modeled using spring elements, of which spring constants represent the flexibilities of the joint structures (Chang, 1974).

In general, the static and dynamic behaviors of the vehicle system are mainly influenced by the stiffnesses of the joint structures (Chang, 1974; Kamal *et al.*, 1982; Lee, 1996). Therefore, we have properly to design the joint structures to obtain the improved vibration characteristics of the vehicle. In general the stiffness of the joint structure can be obtained using an experiment or a finite element analysis.

During the optimal design for the vehicle structure, the section properties of the beam elements adjacent to the joint are changed. But the stiffnesses of the spring element consist of the joint are unchanged. Therefore, reliable vibration analysis and optimal design for the vehicle structure cannot be performed.

In this paper a numerical approximation method of joint stiffness by using response surface model is presented considering the joint flexibility of thin walled beam-jointed structures. Using the proposed method, the optimal design of the vehicle joint stiffness considering the change of the section shapes is performed.

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2. PROCESS OF APPROXIMATE FORMULATION

Figure 1 shows the process of approximate formulation proposed in this study.

As shown in this figure, first, the joint stiffnesses are calculated using detailed shell finite element model. Also the section properties of each section consist of the detailed shell joint model are calculated using the section analysis program (Lee, 1995). The joint stiffnesses of the simplified beam model are calculated by applying the obtained section properties to the equation of simplified beam joint stiffness. And the joint stiffness percent changes are calculated using the joint stiffness of detailed shell model and simplified beam model. The joint stiffness percent change is defined as the correction factor, and the equation of simplified beam joint stiffness modified using the correction factors. Finally, the equations of approximate joint stiffness are formulated using the least square method, response surface method, and the optimal design method (Haftka *et al.*, 1991).

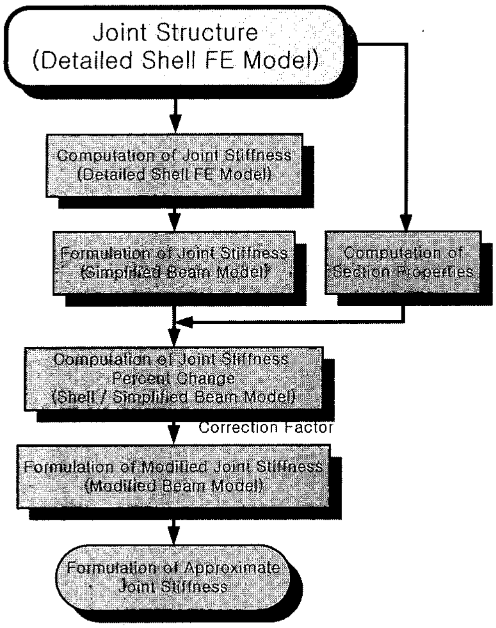


Figure 1. Process of approximate formulation.

3. COMPUTATION OF THE JOINT STIFFNESS OF DETAILED SHELL MODEL

To compute the joint stiffness of the vehicle system, a joint structure is modeled using detailed shell elements. Figure 2 shows the center pillar-roof rail joint finite element model used in this study. To calculate the joint stiffness of the model, the tip moment method (Yim *et al.*, 1995) is utilized. That is, unit moments are applied at the tip of the center pillar with both ends of the roof rail fixed for the center pillar-roof rail joint, and the corresponding rotation angles are calculated. From the determined rotation angles the joint stiffnesses can be computed using the Equation (1).

$$K_x=\frac{M_x}{\Theta_x},\ K_y=\frac{M_y}{\Theta_y},\ K_z=\frac{M_z}{\Theta_z} \tag{1}$$

In Equation (1), M_x is a unit moment with respect to the x-axis; M_y is a unit moment with respect to the y-axis; M_z is a unit moment with respect to the z-axis. And, Θ_x, Θ_y, and, Θ_z denote, respectively, the rotation angles calculated at the load point.

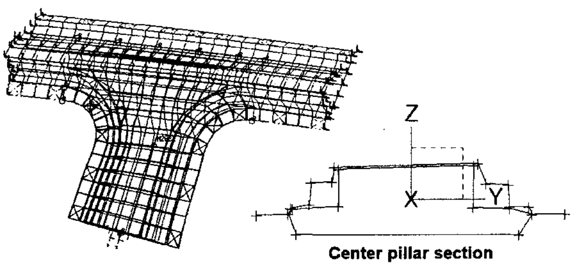


Figure 2. Center pillar-roof rail joint model.

4. JOINT STIFFNESS OF SIMPLIFIED BEAM MODEL

4.1. Right Angle T-type Joint Model

Figure 3(a) shows the simplified beam joint model with a right angle T-type. The joint can be regarded as a coupled system of two beam structures: the cantilever and the fixed-end beam. Therefore, when unit moments are applied at tip of the center pillar with both ends of the roof rail fixed, the total rotation angles of the tip can be obtained with adding the rotation angles of each member. The equations of simplified beam joint stiffness can be expressed as follows.

$$K_x=\frac{M_{x1}}{\Theta_{x1}+\Theta_{x2}}=\frac{8EGJ_1I_{x2}}{GL_2J_1+8EL_1I_{x2}} \tag{2}$$

$$K_y=\frac{M_{y1}}{\Theta_{y1}+\Theta_{y2}}=\frac{2EGI_{y1}J_2}{EL_2I_{y1}+2GL_1J_2} \tag{3}$$

$$K_z=\frac{M_{z1}}{\Theta_{z1}+\Theta_{z2}}=\frac{8EI_{z1}I_{z2}}{L_2I_{z1}+8L_1I_{z2}} \tag{4}$$

In Equation (2), M_{x1} is the unit moment applying with respect to the x-axis at the tip of the member 1; Θ_{x1} and Θ_{x2} are respectively the rotation angle with respect to the x-axis of the member 1 and 2; E is a modulus of elasticity; G is a modulus of elasticity in shear; J_1 is the polar moment of inertia with respect to the x-axis of the member 1; I_{x2} is the moment of inertia with respect to the x-axis of the member 2; L_1 is the length of the member 1.

In Equation (3), M_{y1} is the unit moment applying with respect to the y-axis at the tip of the member 1; Θ_{y1} and Θ_{y2} are respectively the rotation angle with respect to the y-axis of the member 1 and 2; I_{y1} is the moment of inertia with respect to the y-axis of the member 1; J_2 is the polar moment of inertia with respect to the y-axis of the member 2.

In Equation (4), M_{z1} is the unit moment applying with respect to the z-axis at the tip of the member 1; Θ_{z1} and Θ_{z2} are respectively the rotation angle with respect to the z-axis of the member 1 and 2; I_{z1} and I_{z2} are respectively the moment of inertia with respect to the z-axis of the member 1 and 2.

4.2. Oblique T-type Joint Model

In the previous section, we concentrate on the joint whose members meet at the right angle. In most actual situations, however, joint members do not meet at the right angle. The purpose of this section is to propose an equation of simplified beam joint stiffness for such oblique joints.

As shown in Figure 3(b), the member 1 is inclined to the member 2 by the angle α. In order not to complicate the involved analysis, we choose a simplified oblique T-type joint with its symmetry plane on $z = 0$. When a moment with respect to the x-axis is applied at the tip of the member 1, the tip rotates not only in the x-axis, but also in the y-axis. Due to this coupling, we should modify the equations of right angle T-type joint stiffness.

Equation (5) expresses the moments with respect to the x and y axes transmitted to the member 2 when the unit moment with respect to the x-axis acts on the tip of the center pillar. Equations (6) and (7), respectively, express the rotation angle and the joint stiffness with respect to the x-axis.

$$M_{x2,x}=M_{x1}\cos\alpha,\ M_{y2,x}=M_{x1}\sin\alpha \tag{5}$$

$$\Theta_x=\frac{M_{x1}L_1}{GJ_1}+\frac{M_{x2,x}L_2}{8EI_{x2}}\cos\alpha+\frac{M_{y2,x}L_2}{2GJ_2}\sin\alpha \tag{6}$$

$$K_x=\frac{1}{\dfrac{L_1}{GJ_1}+\dfrac{L_2}{8EI_{x2}}\cos^2\alpha+\dfrac{L_2}{2GJ_2}\sin^2\alpha} \tag{7}$$

Equation (8) expresses the moments with respect to the x and y axes transmitted to the member 2 when the unit moment with respect to the y-axis acts on the tip of the center pillar. Equations (9) and (10), respectively, express the rotation angle and the joint stiffness with respect to the y-axis.

$$M_{x2,y}=M_{y1}\sin\alpha,\ M_{y2,y}=M_{y1}\cos\alpha \tag{8}$$

$$\Theta_y=\frac{M_{y1}L_1}{EI_{y1}}+\frac{M_{x2,y}L_2}{8EI_{x2}}\sin\alpha+\frac{M_{y2,y}L_2}{2GJ_2}\cos\alpha \tag{9}$$

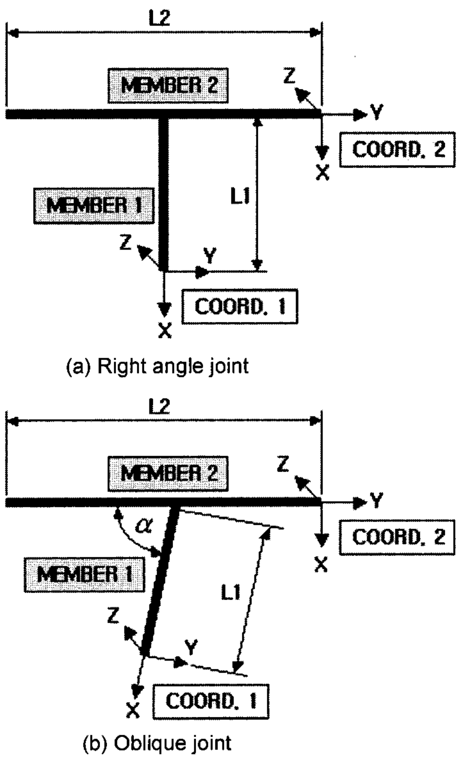
$$K_y=\frac{1}{\dfrac{L_1}{EI_{y1}}+\dfrac{L_2}{8EI_{x2}}\sin^2\alpha+\dfrac{L_2}{2GJ_2}\cos^2\alpha} \tag{10}$$

5. FORMULATION OF APPROXIMATE JOINT STIFFNESS

There are many differences between the simplified beam model and the detailed shell model for the joint stiffness of a vehicle structure, because a local deformation and distortion are not occurred for the simplified beam model when a load acts on the joint structure, but occurred for the detailed shell model. Therefore, the joint stiffness percent changes are used to the numerical approximation process of the vehicle joint stiffness as correction factors.

5.1. Computation of Correction Factor

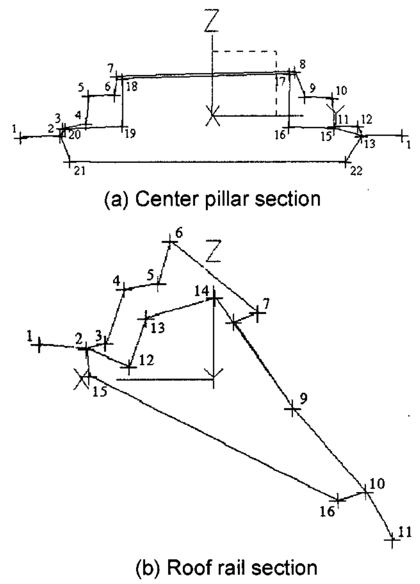
Figure 4 shows each section consist of the center pillar-roof rail joint. The joint stiffnesses of the detailed shell model with the initial sections are calculated using the NASTRAN finite element analysis. Also the joint stiffnesses of the simplified beam model with the initial section properties are calculated using Equations (4), (7), and (10), respectively. And the joint stiffness percent changes



(a) Right angle joint

(b) Oblique joint

Figure 3. T-type joint models.



(a) Center pillar section

(b) Roof rail section

Figure 4. Center pillar-roof rail joint sections.

Table 1. Average percentage change of joint stiffness. Unit: %

	Average percentage change (Correction factor)
C_x	68.58
C_y	12.21
C_z	40.77

are calculated using the stiffnesses of the two models. Also after a little changing the sections many times, the joint stiffnesses for the two models are calculated and the joint stiffness percent changes are obtained. Using the calculated each percent change, the average percent change is determined. The average percent change is used as a correction factor for the joint stiffness of the simplified beam model. The average percent changes of the joint stiffness are shown in Table 1.

5.2. Formulation of Approximate Joint Stiffness

As shown in Equations (11), (12), and (13), the equations of modified joint stiffness can be obtained by multiplying Equation (7) by the correction factor C_x, Equation (10) by the correction factor C_y, and Equation (4) by the correction factor C_z, respectively.

$$\hat{K}_{xa}=C_x\frac{1}{\frac{L_1}{GJ_1}+\frac{L_2}{8EI_{x2}}\cos^2\alpha+\frac{L_2}{2GJ_2}\sin^2\alpha} \tag{11}$$

$$\hat{K}_{ya}=C_y\frac{1}{\frac{L_1}{EI_{y1}}+\frac{L_2}{8EI_{x2}}\sin^2\alpha+\frac{L_2}{2GJ_2}\cos^2\alpha} \tag{12}$$

$$\hat{K}_{za}=C_z\frac{1}{\frac{L_1}{EI_{z1}}+\frac{L_2}{8EI_{z2}}} \tag{13}$$

Also, the response surface equations to correct the joint stiffnesses expressed in Eqations (11), (12), and (13) can be assumed as follows.

$$\hat{K}_{xb}=p_0(p_1A_1^2+p_2A_1J_1+p_3J_1^2+p_4A_4^2+p_5I_{x2}^2+p_6I_{x2}I_{xz2}+p_7I_{xz}^2+p_8A_2J_2+p_9J_2^2) \tag{14}$$

$$\hat{K}_{yb}=p_0(p_1A_1^2+p_2I_{y1}^2+p_3I_{y1}I_{yz1}+p_4I_{yz1}^2+p_5A_2^2+p_6I_{x2}^2+p_7I_{x2}I_{xz2}+p_8I_{xz2}^2+p_9A_2J_2+p_{10}J_2^2) \tag{15}$$

$$\hat{K}_{zb}=p_0(p_1A_1^2+p_2A_1I_{z1}+p_3I_{z1}I_{yz1}+p_4I_{z1}^2+p_5A_2^2+p_6A_2I_{z2}+p_7I_{z2}I_{xz2}+p_8I_{z2}^2) \tag{16}$$

In Equations (14)-(16), p_0, p_1, p_2, etc. denote the undetermined coefficients, I_{xz2} denotes the product of inertia with respect to the x and z axes of the member 2, and I_{yz2} denotes the product of inertia with respect to the y and z axes of the member 1. Undetermined coefficients are determined using the least square method and the optimal design method. By adding the equations of modified joint stiffness and the response surface equations, respectively, the equations of approximate joint stiffness can be obtained as follows.

$$\hat{K}_x=\hat{K}_{xa}+\hat{K}_{xb} \tag{17}$$

$$\hat{K}_y=\hat{K}_{ya}+\hat{K}_{yb} \tag{18}$$

$$\hat{K}_z=\hat{K}_{za}+\hat{K}_{zb} \tag{19}$$

Table 2. Joint stiffness of the vehicle joint with respect to the x-axis. Unit: Nmm/rad, %

		K_x	$\hat{K}_x$	Error	Percent error
Initial shape		2.4647E+10	2.5243E+10	-5.9669E+08	2.42
Center pillar	Outer panel 1	2.8274E+10	2.7842E+10	4.3245E+08	1.53
	Outer panel 2	1.9040E+10	1.9413E+10	-3.7371E+08	1.96
	Reinforce panel 1	2.2940E+10	2.2908E+10	3.2253E+07	0.14
	Reinforce panel 2	2.4019E+10	2.4437E+10	-4.1836E+08	1.74
	Reinforce panel 3	2.3906E+10	2.4166E+10	-2.5995E+08	1.09
	Inner panel 1	2.0194E+10	2.0776E+10	-5.8171E+08	2.88
	Inner panel 2	3.2930E+10	3.3112E+10	-1.8211E+08	0.55
Roof rail	Outer panel 1	2.4867E+10	2.4685E+10	1.8202E+08	0.73
	Outer panel 2	2.4732E+10	2.5894E+10	-1.1624E+09	4.70
	Reinforce panel 1	2.4658E+10	2.4971E+10	-3.1325E+08	1.27
	Reinforce panel 2	2.4524E+10	2.5527E+10	-1.0028E+09	4.09
	Reinforce panel 3	2.4577E+10	2.5654E+10	-1.0763E+09	4.38
	Inner panel 1	2.4518E+10	2.4877E+10	-3.5869E+09	1.46

Table 3. Joint stiffness of the vehicle joint with respect to the *y*-axis. Unit: Nmm/rad, %

		K_x	$\hat{K}_x$	Error	Percent error
Initial shape		2.9066E+09	3.0443E+09	-1.3771E+08	4.74
Center pillar	Outer panel 1	3.7880E+09	3.4506E+09	3.3738E+08	8.91
	Outer panel 2	2.4490E+09	2.3238E+09	1.2521E+08	5.11
	Reinforce panel 1	2.7480E+09	2.6687E+09	7.9280E+07	2.88
	Reinforce panel 2	2.7982E+09	3.1060E+09	-3.0779E+08	11.00
	Reinforce panel 3	2.8291E+09	3.1403E+09	-3.1126E+08	11.00
	Inner panel 1	2.4713E+09	2.5098E+09	-3.8574E+07	1.56
	Inner panel 2	4.0346E+09	3.8327E+09	2.0190E+08	5.00
Roof rail	Outer panel 1	3.0734E+09	3.2000E+09	-1.2660E+08	4.12
	Outer panel 2	2.9374E+09	3.1767E+09	-2.3936E+08	8.15
	Reinforce panel 1	2.9136E+09	3.1333E+09	-2.1966E+08	7.54
	Reinforce panel 2	2.9018E+09	3.1273E+09	-2.2546E+08	7.77
	Reinforce panel 3	2.9109E+09	3.1240E+09	-2.1313E+08	7.32
	Inner panel 1	2.7915E+09	3.0955E+09	-3.0393E+08	10.89

Table 4. Joint stiffness of the vehicle structure with respect to the *z*-axis. Unit: Nmm/rad, %

		K_x	$\hat{K}_x$	Error	Percent error
Initial shape		1.9357E+11	1.9882E+11	-5.2537E+09	2.71
Center pillar	Outer panel 1	1.9623E+11	1.9862E+11	-2.3904E+09	1.22
	Outer panel 2	1.9415E+11	1.8988E+11	4.2655E+09	2.20
	Reinforce panel 1	1.9724E+11	1.9357E+11	3.6688E+09	1.86
	Reinforce panel 2	1.9179E+11	1.9651E+11	-4.7332E+09	2.47
	Reinforce panel 3	1.9401E+11	1.9862E+11	-4.6172E+09	2.38
	Inner panel 1	1.9038E+11	1.9854E+11	-8.1674E+09	4.29
	Inner panel 2	2.0314E+11	1.9980E+11	3.3350E+09	1.64
Roof rail	Outer panel 1	1.9436E+11	1.9859E+11	-4.2402E+09	2.18
	Outer panel 2	1.9374E+11	1.9905E+11	-5.3124E+09	2.74
	Reinforce panel 1	1.9807E+11	1.9752E+11	5.4340E+08	0.27
	Reinforce panel 2	1.9323E+11	1.9838E+11	-5.1531E+09	2.67
	Reinforce panel 3	2.0103E+11	1.9718E+11	3.8507E+09	1.92
	Inner panel 1	1.8847E+11	1.9898E+11	-1.0513E+10	5.58

Equations (20) and (21) express the objective function and the design constraints defined for the optimal design. ADS is used to perform the optimal design (Vanderplaats, 1985).

$$\text{Minimize: } F(p)=\Sigma(K_i-\hat{K}_i)^2 \tag{20}$$

$$\text{Subject to: } \left|\frac{K_i-\hat{K}_i}{K_i}\right| \le \varepsilon \tag{21}$$

In Equation (21), ε is an allowable error and set up to be 0.1. Tables (2)-(4) show the stiffness of the vehicle joint with respect to the each direction. As shown in these tables, the percent errors between the two models are less than 11%.

6. CONCLUSIONS

In this study, a numerical approximation of joint stiffness is presented for considering joint flexibility of thin walled beam-jointed structures. The approximate equation of joint stiffnesses is derived by using the response surface method, the least square method, and the optimal design method in terms of beam section properties. Using the proposed method, it is possible to optimize joint structures considering the change of section shapes in vehicle structures. The methodology presented in this study shows that joint stiffnesses can be effectively determined in designing vehicle structure.

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