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Dynamic Analysis of a Moving Vehicle on Flexible beam Structure (Ⅱ) : Application

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ABSTRACT

Recently, mechanical systems such as a high-speed vehicles and railway trains moving on flexible beam structures have become a very important issue to consider. Using the general approach proposed in the first part of this paper, it is possible to predict motion of the constrained mechanical system and the elastic structure, with various kinds of foundation supporting conditions. Combined differential-algebraic equation of motion derived from both multibody dynamics theory and finite element method can be analyzed numerically using a generalized coordinate partitioning algorithm. To verify the validity of this approach, results from the simply supported elastic beam subjected to a moving load are compared with the exact solution from a reference. Finally, parametric study is conducted for a moving vehicle model on a simply supported 3-span bridge.

Keywords : Flexible beam structure, Combined differential-algebraic equation, Multibody dynamics, Finite element method, Bernoulli-Euler beam, 3-span bridge

1. Combined system equation of motion

Ignoring shear strain and rotary inertia, if a cross-section of a beam is small compared to its length, then the beam can be assumed to be a Bernoulli-Euler beam.

A linear and non-uniform Bernoulli-Euler beam with an arbitrary cross-section is shown in Fig. 1. Horizontal and vertical motions are assumed to be independent of each other.

If a constrained multibody system is moving with initial velocities on a flexible beam, the equations of motion for the constrained multibody system and the elastic beam can be obtained as explained next. Continuous vertical and horizontal elastic foundations can be modeled by adding terms based on elastic foundations to the stiffness and damping matrices in the equations of motion for the beam. Discrete horizontal, vertical and rotational foundations can be modeled by adding the proper terms to the force vector in the equations of motion for the beam. Also, nonlinear elastic Hertzian contact between the moving system and the elastic structure can be considered by adding the proper force in the force vector. [1]

$$[M_{aa}]\{\ddot{v}_a\}+[C]^*\{\dot{v}_a\}+[K]^*\{v_a\}=\{P(q,\dot{q},v_a,\dot{v}_a,t)\}^* \quad (1)$$

$$\begin{bmatrix} N & \Phi_q^T \\ \Phi_q & 0 \end{bmatrix}\begin{Bmatrix} \ddot{q} \\ \lambda \end{Bmatrix}=\begin{Bmatrix} g(q,\dot{q},v_a,\dot{v}_a,t) \\ \gamma(q,\dot{q},t) \end{Bmatrix} \quad (2)$$

$$\{\Phi(q,t)\}=\{0\} \quad (3)$$

where, $[C]^* = [C_{VC}]+[C_{LC}]$
$[K]^* = [K_{aa}]+[K_{VC}]+[K_{LC}]$
$\{P\}^* = \{P_a\}+\{F_{VD}\}+\{F_{LD}\}+\{F_{NR}\}$

$[M_{aa}]$ and $\{v_a\}$ represent the mass matrix and nodal coordinate vector, respectively. Constraint relationships are considered in the equation. [N] is the diagonal mass matrix of the moving system and $[\Phi_q]$ is the Jacobian matrix of the constraint equations, $[\lambda]$ is the Lagrange multiplier vector and $\{\gamma\}$ is the right hand side of the acceleration equation of the constraint equations. $[K_{VC}]$

and [C_{VC}] are stiffness and damping matrices due to continuous and longitudinal elastic foundations, {F_{VD}} and {F_{LD}} are force terms due to discrete, vertical and longitudinal elastic foundations and {F_{NR}} is the force term due to discrete and rotational elastic foundations.

The generalized force vector {g} in the DAE (Differential Algebraic Equation) of the constrained multibody system and the nodal force vector $\{P\}^*$ of the ordinary differential equation of the elastic beam are determined based in part depending on the space and time variables for the two systems. Thus, equations of motion for the two systems can be combined into one system matrix equation as in Eq. (4).

$$\begin{bmatrix} M_{aa} & 0 & 0 \\ 0 & N & \Phi_q^T \\ 0 & \Phi_q & 0 \end{bmatrix} \begin{Bmatrix} \ddot{v}_a \\ \ddot{q} \\ \lambda \end{Bmatrix} = \begin{Bmatrix} Q \\ g \\ \gamma \end{Bmatrix} \tag{4}$$

where, $\{Q\} = \{P\}^* - [C]^*\{\dot{v}_a\} - [K]^*\{v_a\}$. The two systems have to be solved simultaneously using Eq. (4). Generalized coordinates which are obtained by integrating Eq. (4) must satisfy the nonlinear constraint relationship of Eq. (3).

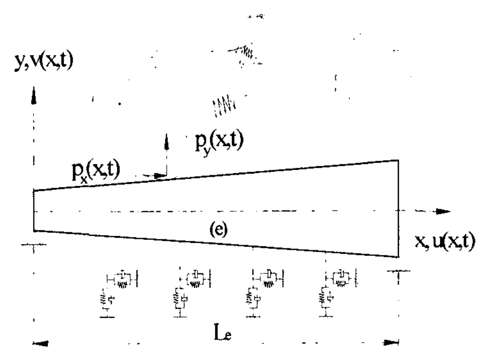


Fig. 1 Constrained multibody system moving on a flexible beam structure

Several methods which utilize the ordinary differential equation solution method can be used to solve the DAE of Eq. (4). All generalized coordinates can be solved simultaneously using the backward difference formula and the nonlinear equation solution method. This method is suitable for the stiff system but the simulation time is longer than with the other methods. [2]

The generalized coordinate partitioning method partitions the generalized coordinates into independent and dependant coordinates. Independent coordinates are obtained from the differential equations and dependant coordinates are solved using the constraint equations. [3]

The constraint violation stabilization method uses feedback control theory by adding constraint violation items to the right side of the differential equations. The exact solution can be obtained theoretically by choosing the proper gain values from the constraint violation terms. [4] Defining the correct values of the gains for this method is very important. But defining the optimal values of the gains is not an easy task because the values change depending on the type and status of the system. This method does not solve the position and velocity equations, thus, if the system is near the kinematically singular configuration or impulsive force is applied to the system, then the solution may not be reliable.

In this study, the generalized coordinate partitioning method is used to solve the combined system equations. The independent coordinates are obtained using the implicit multi-step predictor corrector integration method. To solve Eq. (4), the solution method with the following 6 steps are suggested. Figure 2 shows the flow diagram of the algorithm.

Step 1) Define the initial conditions for the moving system and the elastic beam.

Step 2) Partition the generalized coordinates of the moving system into independent and dependent coordinates. To partition the coordinates, the LU decomposition method is applied to the Jacobian matrix of the constraint. QR decomposition, singular value decomposition or the Gram-Schmidt method can be used.

Step 3) Solve Eq. (4) using the initial conditions from Step 1.

Step 4) Numerically integrate the independent coordinates using the predictor corrector algorithm. During the corrector process, partition the generalized coordinate again if the convergence rate is slow.

Step 5) Solve the nonlinear constraint equation, Eq. (3), to obtain the position values of the dependent coordinates. The Newton-Raphson method is used to solve the nonlinear equations. If the convergence rate is low then partition the generalized coordinates again. The velocity equation, which is the time derivative of Eq. (3), is used to obtain the velocities of the dependent coordinates.

Step 6) Repeat steps 3,4,5 until the time reaches t_{END}. The positions and velocities obtained in Steps 4 and 5 are used as the initial conditions in Step 3.

2. Concentrated load moving on a simply supported beam

To verify the reliability of the proposed method, the solution of the simple model is used. Timoshenko and other investigators have used the mode synthesis method to obtain the analytical solution of the vertical motion of the constant load moving on a simply supported beam with continuous contact.

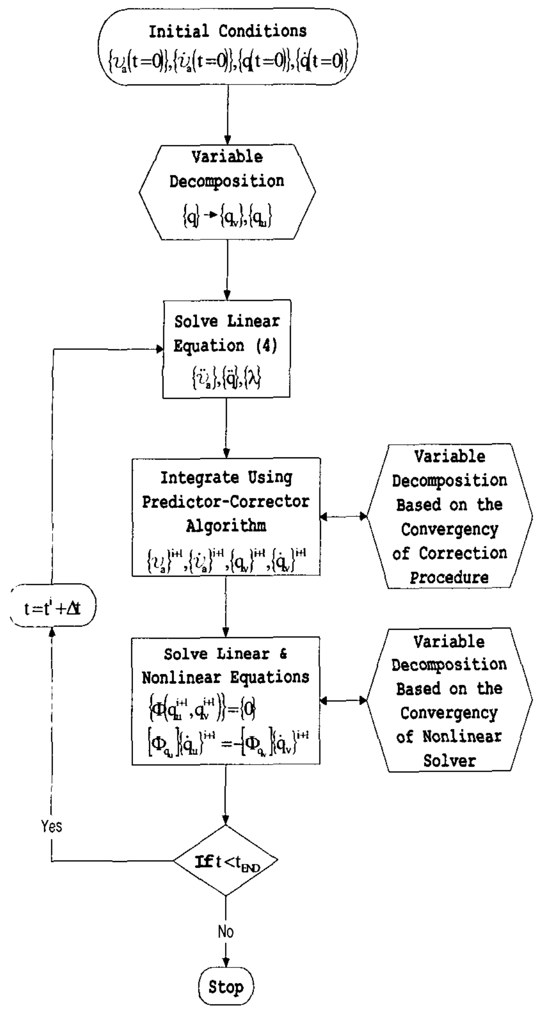


Fig. 2 Numerical analysis procedure

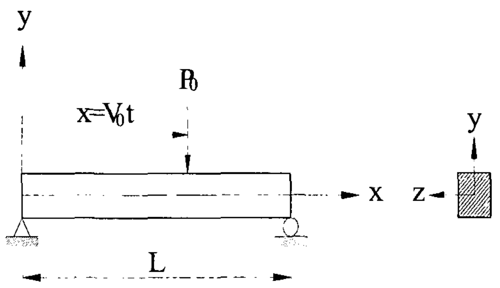


Fig. 3 Simply supported beam subjected to a moving concentrated load

To compare the results from the elastic beam, the DAF (Dynamic Amplification Factor) and IF (Impact Factor) of the simply supported beam are defined as follows:

$$D.A.F. = \frac{\textit{dynamic deflection at the beam center}}{\max\textit{imum static deflection at the beam center}} \tag{5}$$

$$I.F. = \frac{\max\textit{imum dynamic deflection at the beam center}}{\max\textit{imum static deflection at the beam center}} \tag{6}$$

Table 1 Material properties and other conditions of the moving load model

Symbol	Description	Value
EI	flexural rigidity (N.m^2)	4.03×10^{6}
M_B	mass of elastic beam (kg/m)	50.47
A	cross section area (m^2)	6.4×10^{-3}
L	span length (m)	6.25×10^{-1}
V_0	moving speed (km/h)	37
P_0	moving load (kgf)	349

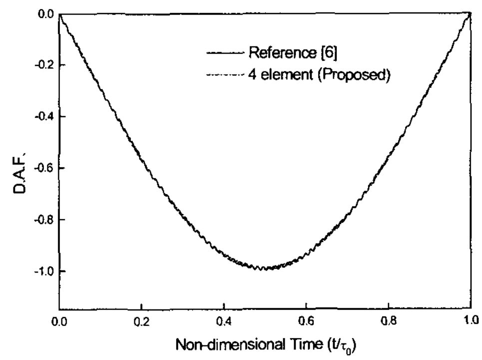


Fig. 4 Dynamic Amplification Factor of the moving load model

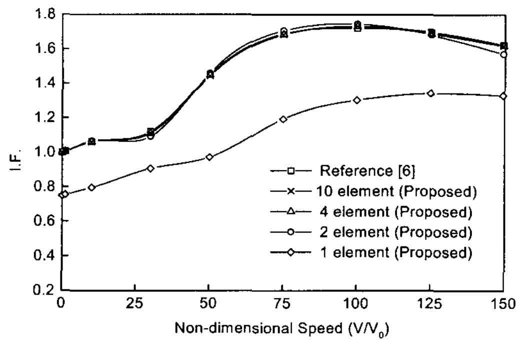


Fig. 5 Velocity effect of the Impact Factor according to element increment

The DAF of the proposed method with 4 beam elements and the results from the reference [6] are compared in Fig. 4. The results using the proposed method are very close to those of the reference [6]. τ_0 is the time required for the load moving from the left end to the right end.

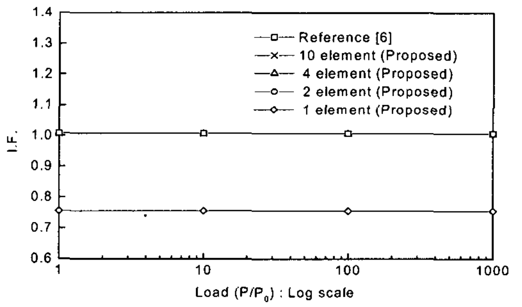


Fig. 6 Load effect of the Impact Factor according to element increment

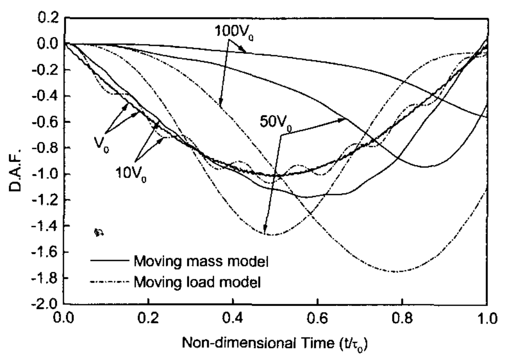


Fig. 7 Dynamic Amplification Factor according to moving model

The vertical movement of the elastic beam is affected by the velocity and magnitude of the moving load. To investigate the effect of the velocity, the IF of a 349kgf moving load is compared with the results from the reference [6], at various velocities. Figure 5 shows the comparison graphs. Except for the result with one beam element, the results are very close to the those of the reference [6].

The IF of the elastic beam increases nonlinearly with increasing velocity, but after a certain velocity, it decreases, even with further increases in velocity. The critical speed is the velocity where the IF curve changes its slope. The critical speed can be defined using the length of the span and the first bending frequency along the span.

The accuracy of the proposed method will depend on the number of the finite elements used to model the elastic beam. The result will be more accurate with an increased number of finite elements, but the simulation time will be increased too. Thus, the number of finite elements used should be optimized in accordance with the required accuracy of the result. Figure 6 shows the IF of the moving load with various magnitudes at a constant speed of 37km/h. The IF is constant with various magnitudes of the load as expected from the assumption of the linear Bernoulli-Euler beam. Figures 5 and 6 indicate that the result is accurate if the number of finite elements is more than four.

Another assumption in the previous simulation is that the moving load maintains contact with the beam. But to model a high speed vehicle or railway train moving on the bridge accurately, the inertia effect for the moving system needs to be considered in the model. To achieve that, the contact between the moving system and the beam is assumed to be discrete.

Figure 7 shows the DAF curves of the moving load model and the moving mass model with the material properties in Table 2. The results of the two models are similar when the speed is low but the results are quite different when the speed is increased. When the speed is low, the curves of the two models follow the static deflection of the beam, but at the increased speed, the dynamic effect needs to be considered. The moving load model cannot consider the dynamic effect of the moving system. Thus when the speed of the moving system is high, the proposed method has to be used.

3. Vehicle model moving on a 3-span bridge

The multibody model of a vehicle moves along on a 3 span bridge as shown in Fig. 8. The design parameters of the vehicle are changed to study the dynamic behavior of the vehicle while it is accelerating or decelerating on the bridge. Also, the supporting conditions of the bridge are changed to see the dynamic behavior of the vehicle and the bridge.

The vehicle is modeled with one sprung mass and two unsprung masses. Springs and dampers are defined for the wheels and suspensions of the vehicle. Two links are defined between the sprung mass and two unsprung masses to restrain the longitudinal motion of the vehicle. Discrete Hertzian contact between the unsprung masses and elastic beams makes the vehicle to vibrate on the bridge. Four finite beam elements are used for each span, and two end points A and D are constrained to have only rotational degree of freedom. Only vertical displacements are constrained for the supports B and C. Properties for the elastic beam and material properties for the Hertzian contact are shown in Table 1 and Table 2. Other necessary data for the model is in Table 3.

Table 2 Contact properties of the moving mass model

Symbol	Description	Value
E	Young's modulus of the moving mass (GPa)	206.0
	Young's modulus of the beam (GPa)	205.6
v	Poisson ratio of the moving mass	0.29
	Poisson ratio of the beam	0.30

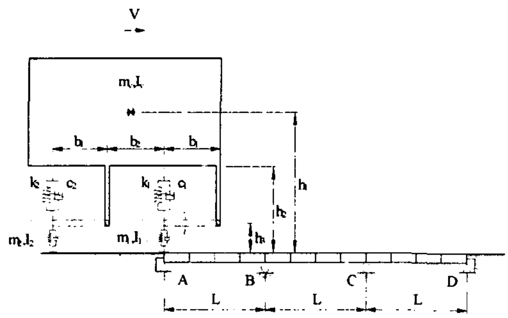


Fig. 8 Vehicle moving on a 3-span bridge

Table 3 Vehicle data

Symbol	Value
m_v (kg)	500
I_v (kg.m^2)	70
m_1, m_2 (kg)	350
I_1, I_2 (kg.m^2)	10
k_1, k_2 (N/m)	2.0×10^6
c_1, c_2 (N·s/m)	2.0×10^6
L (m)	8
h_1 (m)	0.22
h_2 (m)	0.2
h_3 (m)	0.1
b_1 (m)	0.15
b_2 (m)	0.45

3.1 Moving with constant velocity

The dynamic behavior of the vehicle and the vertical motion of the elastic beam are studied with the vehicle moving at a constant speed of 50 km/h on the 3-span bridge. Vertical deflections at the center of the moving load model and moving vehicle model are compared in Fig. 9. The vertical deflection of the vehicle model is larger than that of the moving load model, as shown in Fig. 9.

The wheelbase of the vehicle is lengthened to show the dynamic behavior of the vehicle. The vertical deflection of the longer wheelbase is smaller while the vehicle is on the bridge, but after the vehicle passes over the bridge, the residual strain becomes greater than with the smaller wheelbase, as shown in Fig. 10. The same contact force is transmitted to the vehicle, so the pitch and bounce motion behave similarly to the vertical motion of the beam.

Next, continuous vertical foundations are used to support the 3-span bridge. The vertical motion is compared with that of the initial model. The stiffness and damping coefficients per unit length are K_c=10kN/m^2 and C_c=5kN·s/m^2 respectively. The continuous elastic support can be considered in the system equations of motion by adding stiffness and damping matrices due to the supporting conditions. The continuous support reduces the vertical motion of the vehicle, as shown in Fig. 11.

Finally, dynamic response for the discrete elastic foundation is studied. This supporting condition can be considered in the system equations of motion by adding force terms due to the discrete supporting conditions. The vertical motion of the vehicle is also reduced by this support condition, as shown in Fig. 12.

3.2 Moving with deceleration

The full-lock braking condition is applied to the front unsprung mass. The initial velocity of the vehicle is 50 km/h. The friction coefficient between the wheel and the bridge is assumed to be μ_0=0.85. The motions of the vehicle and bridge are studied. The vehicle comes to a full stop after 3.4 seconds. Figures 13(a) and 13(b) show the vertical and longitudinal deflection of the bridge. Figure 13(c) and 13(d) compare the bounce and longitudinal motion of the vehicle to the result with the model which assumed the bridge to be a rigid body.

The longitudinal stiffness of the bridge is greater than the vertical stiffness, so the braking of the vehicle will excite the high frequency longitudinal motion of the bridge. The same effect will be observed in the case of acceleration. Thus, to study the dynamic behavior of the vehicle accurately, the bridge needs to be modeled as an elastic structure.

Even though the reliability of the presented method is shown in chapter 2, the results of the moving vehicle model need to be verified with the experimental results in the future.

4. Conclusions

This paper presents combined system equation of motion of a vehicle moving on an elastic structure. Various supporting foundations are considered in the equations of motion. The vehicle and the structure are assumed to have discrete Hertzian contact. The numerical procedure for the proposed equations is also presented. To validate the proposed method, the result of the moving mass on a simply supported beam is compared with the exact solution of the moving load model.

Moving velocity, moving load, and the number of finite elements are changed to study the results. Accurate results can be obtained if more than four finite elements are used for the elastic structure. The results with the moving load model and the proposed method are similar at a low speed, but the results are quite different for the high speed case, due to the inertia effect of the moving vehicle.

Finally, a multibody model of a vehicle moving on a 3-span bridge is studied to show the applicability of the proposed method. The dynamic behavior of the bridge and vehicle can be studied for many design parameters and different vehicle operating conditions.

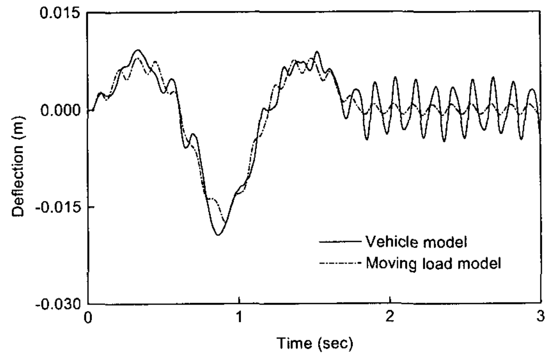
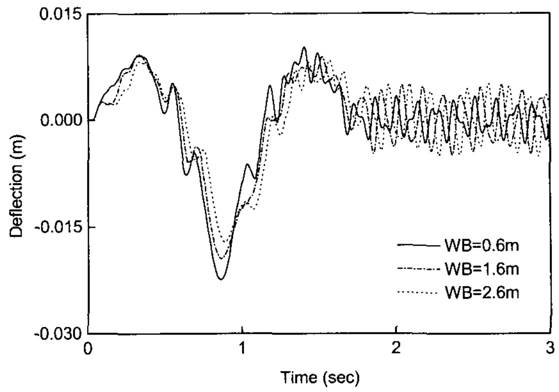
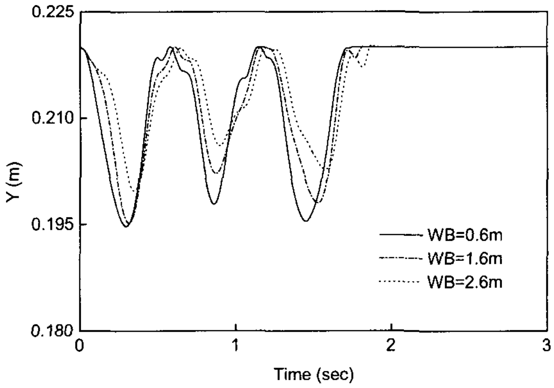


Fig. 9 Vertical mid-span deflection of the second beam

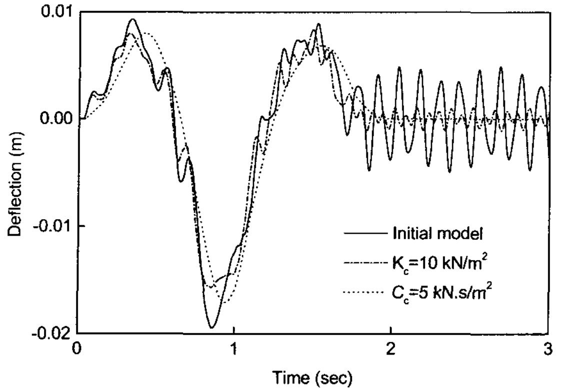


(a) Vertical mid-span deflection of the second beam

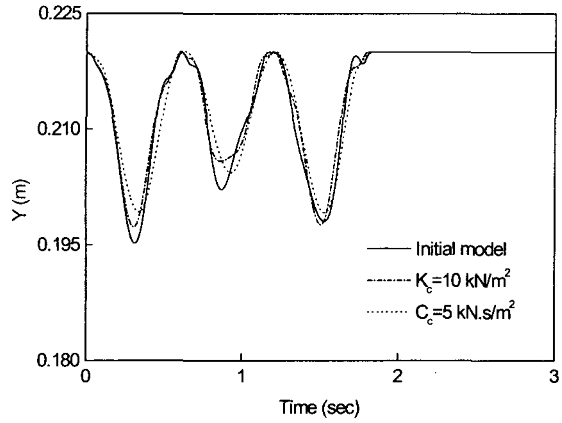


(b) Bounce motion of the vehicle

Fig. 10 Effects of the wheelbase change

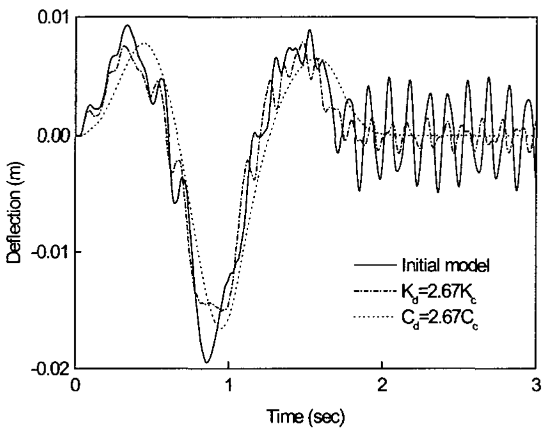


(a) Vertical mid-span deflection of the second beam

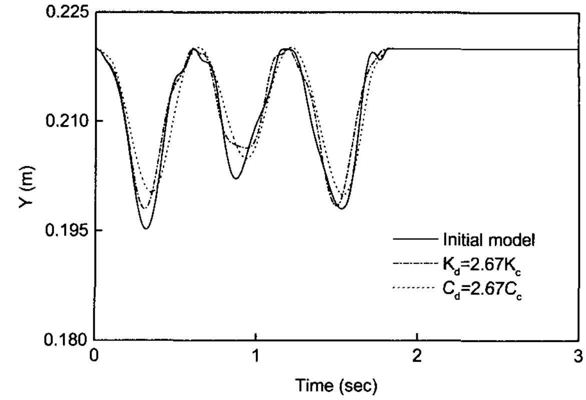


(b) Bounce motion of the vehicle

Fig. 11 Effects of continuous foundation support

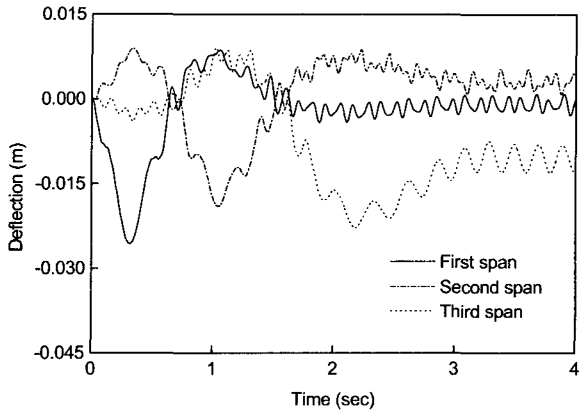


(a) Vertical mid-span deflection of the second beam

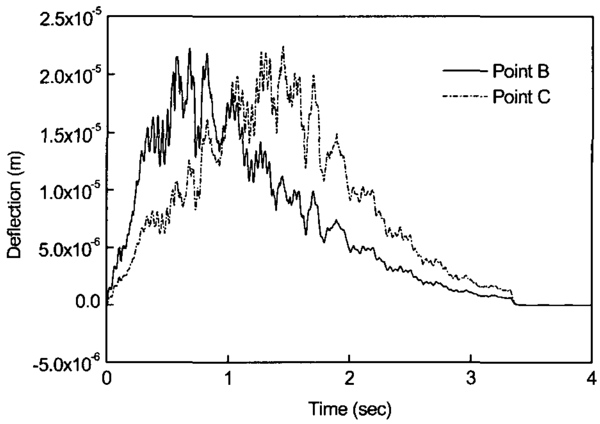


(b) Bounce motion of the vehicle

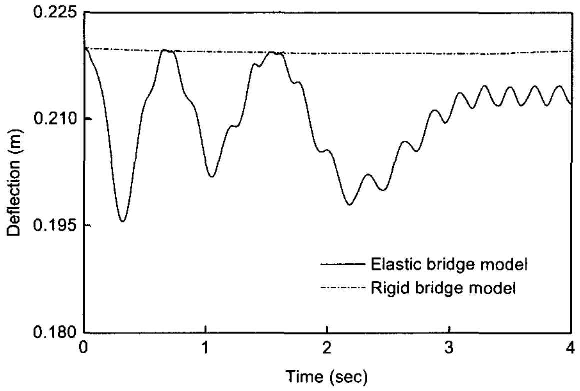
Fig. 12 Effects of discrete foundation support



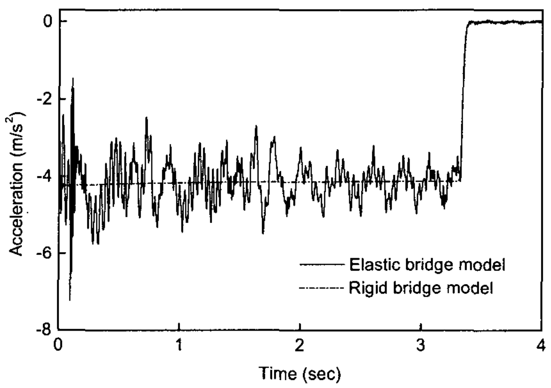
(a) Vertical mid-span deflection of the elastic bridge



(b) Longitudinal mid-span deflection of the elastic bridge



(c) Bounce motion of the vehicle



(d) Longitudinal acceleration of the vehicle

Fig. 13 Results of the vehicle model under full-lock braking condition

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