내부튜브가 있는 골조 튜브 구조물의 전단응력에 대한 수치해석

Numerical Analysis of Shear Stresses in Framed Tube Structures with Internal Tube(s)

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요 지

내부튜브를 가진 튜브 구조물의 보에 있는 전단응력을 평가하기 위하여 수치적인 해석기법이 제안되었다. 이 기법은 휨과 전단변형을 고려한 튜브 보 개념 위에서 각각의 튜브를 연속 보로서 모델링 한다. 전단응력에 대한 수치해석은 패널에 작용하는 응력에 대한 탄성이론과 수학적인 유도를 기초로 하고 있다. 내부와 외부 튜브에 있는 변형곡선에 대한 표현식을 가정함으로써, 그 튜브에 있는 전단응력은 작용하중과 단면 2차 모멘트 그리고 구조물의 기하학적인 표현으로 구성된 선형 함수로서 표현된다. 전단지체와 축 응력을 다룬 이전의 연구[1,2]가 튜브내에 존재하는 전단응력을 수치적으로 해석하기 위하여 보완 발전되었다. 제안된 방법의 정확성과 유용성이 3개의 튜브 구조물의 해석을 통하여 증명되었다.

핵심용어 : 튜브 구조, 연속보 해석, 변형분포, 전단지체, 미지 변형 함수

Abstract

A simple numerical modelling technique is proposed for estimating the shear stress distribution in beams of framed tube structures with multiple internal tubes. The structures are analysed using a continuum approach in which each tube is individually modelled by a tube beam that accounts for the flexural and shear deformations, as well as the shear lag effects. The numerical analysis of shear stress is based on the mathematical analogy in conjunction with the elastic theory. By simplifying assumptions regarding the form of strain distributions in external and internal tubes, the shear stress distributions are expressed in terms of a series of linear functions of the second moments of area of the structures and the corresponding geometric and material properties, as well as the applied loads. Previous studies[1,2] for axial stresses and shear lag phenomenon are further developed for the numerical analysis of shear stresses in the tubes. The simplicity and accuracy of the proposed method are demonstrated through the solutions of three numerical examples.

Keywords : *tube structures, continuum beam analogy, strain distribution, shear lag, unknown strain function*

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1. Introduction

Modern highrise buildings of the framed-tube system exhibit a considerable degree of shear-lag with consequential loss of cantilever efficiency. Despite this drawback, the framed-tube structures are accepted as an economical system capable of maximising the structural efficiency for highrise buildings over a wide range of building heights. In particular, the framed-tube structures with multiple internal tubes, or tube(s)-in-tube structures, are widely used due to their high stiffness in resisting lateral loads. In addition, this type of structure shows a reduced shear-lag due to the existence of the internal tubes, the columns of which also participate more effectively in resisting lateral forces.

Existing methods[3)~5)] of modelling a framed tube structure as equivalent orthotropic plate panels with shear and bending rigidities are not sufficient to capture the true behaviour of the framed tube structure with multiple internal tubes. These models for approximate analysis not only ignore the contribution to lateral stiffness provided by the internal tubes but also neglect the negative shear lag effects and the tube-tube interaction in the tubes. In addition, the existence of the tube-tube interaction along with the negative shear lags in the tubes makes it difficult to estimate the structural performance and the accurate analysis of such system. As a result, they are inadequate in capturing the true shear behaviour as well as the true bending behaviour of such structures.

The proposed method, which is intended to be used as a tool for preliminary design purposes, can be applied for the shear stress analysis of framed-tube structures with single and multiple internal tubes as well as those without internal tubes.

A series of framed tube structure with multiple internal tubes subjected to lateral loading is analysed to verify the simplicity and accuracy of the proposed method. Three 40-storey reinforced concrete framed structures(of tube-in-tube, 2tubes-in-tube and 3tubes-in-tube construction) are analysed using the proposed method and a 3-D frame analysis program.[6)] The results are compared to estimate the shear stress distribution in beams of each tube. The shear stress distribution, affected by the tube-tube interaction, explains well the overall shear stress distributions in the structures.

2. Analysis Method

2.1 Structural Modelling

The analytical method of the discrete tube structure with multiple internal tubes was proposed previously by Lee.[1)] The simplicity and accuracy of the proposed method was verified through the comparison of deflection and stress distributions. The tubes-in-tube structure is modelled as an assemblage of equivalent multiple tubes of uniform thicknesses so that the framed tubes can be analysed as continuous structures.

The discrete framed tube structure with multiple internal tubes(2 in this case) composed of equivalent multiple tubes is shown in Fig. 1. All framed tubes under consideration consist of an assemblage of equivalent orthotropic plates of uniform thickness in vertical planes. The high in-plane stiffness of the floor slabs restricts the relative lateral displacements between external tube and internal tubes, and it may therefore be assumed negligible at each floor level. The analogy method proposed herein has the following characteristics: (1) The modified Reissner's function1) is adopted for the independent distribution of vertical displacement in the flange frame panels, thereby taking into consideration the net shear lag; (2) the effect of positive and negative shear lag in the external and internal tubes is considered in assessing the overall shear behaviour of the tubular structure; (3) lateral stiffness provided by the internal tubes is taken into the analysis; (4) additional bending stresses, developed by he tube-tube interaction, are also included in the numerical analysis of the structure.

It is assumed that the stress of each member in the structure can be expressed in terms of a family of linear functions of its second moment of area, member property and geometry of the structure. As the procedure used in the proposed method is an extension of the continuum beam analogy, shape functions are the same as those adopted in previous studies[1),2)] to describe the variation of displacements in flange and web frame panels of each tube. The shape functions can be varied with change of the number of bays and storeys.

The following assumption is added to the previous assumptions to simplify the modelling concept and the expressions in the analysis for idealisation of the structure: The tubes-in-tube structure has two horizontal axes of symmetry (x and y), passing through the vertical axis (z)(see Fig. 1). It is further assumed that the strain distributions in the flange frame panels of internal tubes are symmetrical about the vertical central axis(z). As a result, the strain distributions in the flange frame panels of internal tubes are symmetrically equal in magnitude. Assumptions analogous to this additional assumption can be applied to all the framed tube structure with different number of internal tubes.

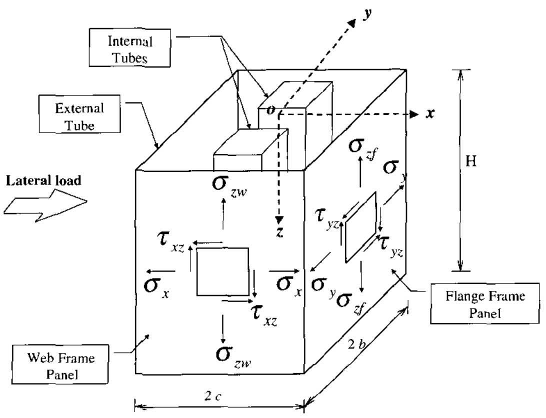


Fig. 1 Equivalent Tube Structure with Multiple(Two) Internal Tubes

The equilibrium equations for the flange frame panel shown in Fig. 1 are

$$\frac{\partial \sigma_y}{\partial y}+\frac{\partial \tau_{yz}}{\partial z}=0 \text{ and } \frac{\partial \sigma_{zf}}{\partial z}+\frac{\partial \tau_{yz}}{\partial y}=0 \tag{1}$$

The corresponding equilibrium equations for the web frame panel shown in Fig. 1 are obtained as

$$\frac{\partial \sigma_x}{\partial x}+\frac{\partial \tau_{xz}}{\partial z}=0 \text{ and } \frac{\partial \sigma_{zw}}{\partial z}+\frac{\partial \tau_{xz}}{\partial x}=0 \tag{2}$$

Thus, the stress-strain relations of the orthotropic plate elements are expressed as

$$\sigma_y = E_y\varepsilon_y + E_{yz}\varepsilon_z;\ \sigma_{zf} = E_{zf}\varepsilon_{zf} + E_{yz}\varepsilon_y;\ \tau_{yz} = G_{yz}\cdot\gamma_{yz}$$
$$\sigma_x = E_x\varepsilon_x + E_{xz}\varepsilon_{zw};\ \sigma_{zw} = E_{zw}\varepsilon_{zw} + E_{xz}\varepsilon_x;\ \tau_{xz} = G_{xz}\cdot\gamma_{xz} \tag{3}$$

where E_y and E_{yz} are, respectively, the horizontal and cross elasticity moduli in the flange frame panel; E_x and E_{xz} are, respectively, the horizontal and cross elasticity moduli in the web frame panel; E_{zf} and E_{zw} are the vertical elasticity moduli in the flange and web frame panels, respectively; and γ_{yz} and γ_{xz} are the shear moduli in the flange and web frame panels, respectively. In Eq. (3), the elasticity moduli, E_{yz} and E_{xz}, are assumed to be negligible in nature.

2.2 Vertical displacements in the flange and web frame panels

Figs. 2 and 3 show, respectively, the variations of displacement distributions in the flange and web frame panels. The structure behaves diffe-

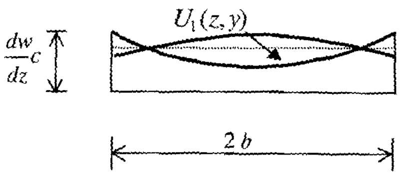


Fig. 2 Distribution of vertical displacement in the web frame panel

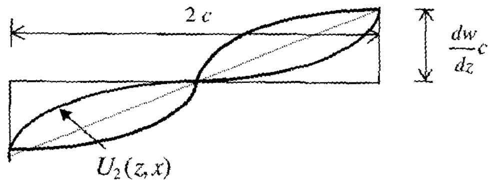


Fig. 3 Distribution of vertical displacement in the flange frame panel

rently from that predicted by the primary bending theory, in that the distribution of stresses in the flange frame panels is not uniform, and that in the web frame panels is nonlinear. This phenomenon is referred to as shear-lag. The shear lag due to tube-tube interaction and the flexibility of the spandrel beam causes the variation of axial stress distributions along the height of the structure. From Fig. 2, which shows the distribution of axial stresses across the flange frame, it can be seen that when the degree of the shear lag varies along the height, the distribution of the axial stresses in the flange frame changes concave or convex. Similarly, from Fig. 3, which shows the distribution of axial stress across the web frame, it can be also seen that when the degree of the shear lag varies, the axial stresses near the centre of the web significantly lag down or up in the linear distribution.

For the displacement distributions in the flange frames of framed-tube structures, the displacement distributions across the width of the flanges were approximated as second-order-polynomial curves by Coull and Bose,[3] as well as Kwan.[4] For the shear wall structures, the displacement distributions were assumed as second-order-polynomial curves by Coull and Abu El Magd[7]; and as fourth-order-polynomial curves by Kwan.[8] The shape functions adopted herein are the modified Reissner's functions.[1] Essentially, the modification involves up-grading the displacement functions from second-order polynomials to third-order polynomials. The third-order-polynomial curve, whose equation is given by

$$U_1(z,y)=c\left[\frac{dw}{dz}+\left(1-\left(\frac{y}{b}\right)^3\right)u_1(z)\right] \tag{4}$$

is shown in Fig. 2.

Regarding the axial displacement distributions in the web frame panels, most researchers neglected the shear lag in the webs and assumed a linear distribution of axial displacement across the width of the web except Coull and Bose[3] and Kwan,[4] who assumed that the variations of the axial displacements in the webs were expressed as third-order-polynomial functions. Recently, Kwan[8] also developed the displacement functions from third-order polynomials to fifth-order polynomials. However, the functions do not take into account the negative shear-lag as shown in Fig. 3. The shape function adopted herein is a third-order-polynomial curve, whose equation is given by

$$U_2(z,x)=\left[\frac{dw}{dz}x+\left(\frac{x}{c}-\left(\frac{x}{c}\right)^3\right)u_2(z)\right] \tag{5}$$

In Eqs. (4) and (5), $u_1(z)$ and $u_2(z)$ are shear lag coefficients of the flange and web frames, respectively, due to the shear deformation, and the expressions can be found elsewhere(Lee 2001).

A pilot study of the proposed displacement functions[1),2)] indicates that they are adequate to cover the important characteristics of the shear-lag phenomenon in assessing the global behaviour of the framed-tube structures with multiple internal tube. Consequently, the proposed numerical method explains well the deflection and stress distributions, affected by shear lag phenomenon, in such structures.

The methodology of modelling the anticipated distribution of shear strain as affected by the shear-lag in the web and flange panels is presented in the following sections.

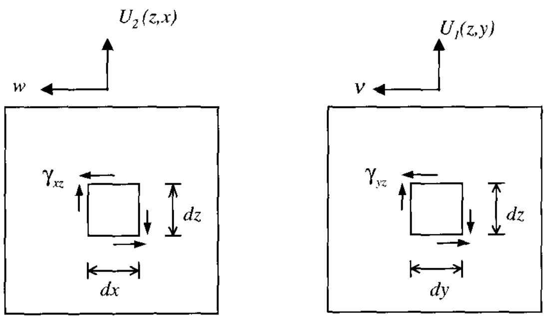
2.3 Shear strains

Fig. 4 shows the shear strains in the external web and flange frame panels. The shear strain expressions are given as

$$\gamma_{xz}=\frac{\partial w}{\partial z}+\frac{\partial U_2(z,x)}{\partial x} \quad \text{for web frame panel}$$

and (6)

$$\gamma_{yz}=\frac{\partial U_1(z,y)}{\partial y}+\frac{\partial(\nu)}{\partial z} \quad \text{for flange frame panel}$$



(a) Web frame panel (b) Flange frame panel

Fig. 4 Shear strain diagrams in the external web and flange frame panels

where

$$\frac{\partial w}{\partial z}=\theta$$

$$\frac{\partial U_2(z,x)}{\partial x}=\left[\frac{dw}{dz}+\frac{1}{c}\left(1-3\left(\frac{x}{c}\right)^2\right)u_z(z)\right]$$

$$\frac{\partial U_1(z,y)}{\partial y}=-\frac{3c}{b}\left(\frac{y}{b}\right)^2 u_1(z)$$

Note that the displacement normal to the lateral load, ν, can be neglected, θ is the rotation of the plane section joining the four corners of the external tube, and $u_1(z)$ and $u_z(z)$ are the undetermined functions, including shear-lag coefficients, of the external flange and web, respectively

Likewise, the shear strains in the internal tube can be obtained as

$$\gamma_{xzi}=\frac{\partial w}{\partial z}+\frac{\partial U_{i2}(z,x)}{\partial x}$$

and (7)

$$\gamma_{yzi}=\frac{\partial U_{i1}(z,y)}{\partial y}+\frac{\partial \nu_i}{\partial z}$$

where; $\frac{\partial U_{i2}(z,x)}{\partial x}=\left[\frac{dw}{dz}+\frac{1}{c_i}\left(1-3\left(\frac{x}{c_i}\right)^2\right)u_{i2}(z)\right]$; $\frac{\partial U_{i1}(z,y)}{\partial y}=-\frac{3c_i}{y}\left(\frac{y}{b_i}\right)^3 u_{i1}$ and the displacement of the internal flange frame panel, ν_i, is also neglected.

2.4 Undetermined strain functions

Fig. 5 shows a typical cross section of the equivalent tubes-in-tube structure. The second moment of the entire tubes-in-tube system with regard to the y-axis is

$$I=I_e+I_i \tag{8}$$

where I_e and I_i are the second moments of area of the external and multiple internal tubes, respectively, and

$$I_e=\frac{4}{3}tc^2(3b+c)+4A_c c^2$$

$$I_i=\frac{4}{3}Nt_i c_i^2(3b_i+c_i)+4NA_{ci}c_i^2$$

in which N is number of internal tubes; t and t_i are respectively the equivalent orthotropic panel thicknesses in the external and internal tubes; and A_c and A_{ci} are the cross-sectional areas of the corner column in the external and internal tubes, respectively.

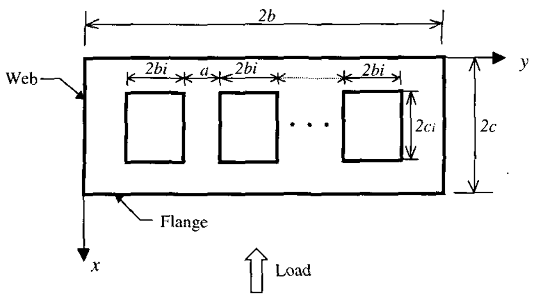


Fig. 5 Typical cross section of equivalent tubes-in-tube structure

The equilibrium equation of the overall moment at any height is

$$M(z)=2\int_{-b}^{b}E\varepsilon_{zf}tc\,dy+2\int_{-c}^{c}E\varepsilon_{zw}tx\,dx+4A_c c^2E\varepsilon_c$$
$$+\sum_{m=1}^{n}2m\int_{-bi}^{bi}E\varepsilon_{zif}\,t_i\,c_i\,dy$$
$$+2m\int_{-ci}^{ci}E\varepsilon_{ziw}\,t_i\,x\,dx+4mA_{ci}\,c_i^2\,E\varepsilon_{ci} \quad (9)$$

where ε_c and ε_{ci} are respectively the corner column strains in the external and internal tubes; ε_{zf} and ε_{zw} are respectively the strains in the external flange and web frame panels; and ε_{zif} and ε_{ziw} are respectively the strains in the internal flange and web frame panels. The strain functions are obtained by differentiating the vertical displacement distributions with respect to z.

Substituting the strain distributions into Eq. (9) and integrating with respect to z yields

$$u'_2(z)=-\frac{15b}{2}u'_1(z) \text{ for external tubes} \quad (10a)$$

and

$$u'_{i2}(z)=-\frac{45b_i}{8}u'_{i1}(z) \text{ for multiple internal tubes} \quad (10b)$$

As a result of Eq. (10), the vertical strain distributions can then be simply expressed in terms of the only two undetermined strain functions, $u_1'(z)$ and $u_{i1}'(z)$, including the effects of shear lag. The two unknown functions can also be obtained on the basis of the potential energy principle in conjunction with the variational approach and those are given in previous study.[1)]

τ_{xz} can be derived from the second expression of Eq. (2). Or,

$$\frac{\partial\tau_{xz}}{\partial x}=-\frac{\partial\sigma_{zw}}{\partial z} \quad (11)$$

where the expression of σ_{zw} is available elsewhere (Lee 2001).

Integrating Eq. (11) with respect to x and considering the boundary conditions[1)] yield the shear stress expression as

$$\tau_{xz}=\alpha x^2\frac{dM(z)}{dz}+\beta x^2\left[2-\left(\frac{x}{c}\right)^2\right]u''(z)$$
$$+\gamma p(H)+\Psi\frac{dM(z)}{dz} \quad (12)$$

where α, β, γ and Ψ are the constants to be determined according to the geometric and structural properties conditions; $P(H)$ is the external shear force in the lowest level of the whole tube(s)-in-tube system; I is the second moment of area of the structure; and $M(z)$ is the total bending moment of the structure induced by the applied load.

The second expression of Eq. (6) is the shear strain equation for the flange frame panel of the external tube. Multiplying it by the shear modulus(G) yields the shear stress as

$$\tau_{yz}=G\left[-\frac{3c}{b}\left(\frac{y}{b}\right)^2u_1(z)\right] \quad (13)$$

The shear forces in the beams at positions x_i and y_i are

$$S_{zw}=t\int_{z_i-\frac{h}{2}}^{z_i+\frac{h}{2}}\tau_{xz}\,dz \text{ for the web frame} \quad (14a)$$

and

$$S_{zf}=t_i\int_{z_i-\frac{h}{2}}^{z_i+\frac{h}{2}}\tau_{yz}\,dz \text{ for the flange frame} \quad (14b)$$

where $h/2$ is the mid-height of column measured from the centreline of floor; and t and t_i are the thicknesses[4] of the orthotropic plate in the external and internal tubes respectively.

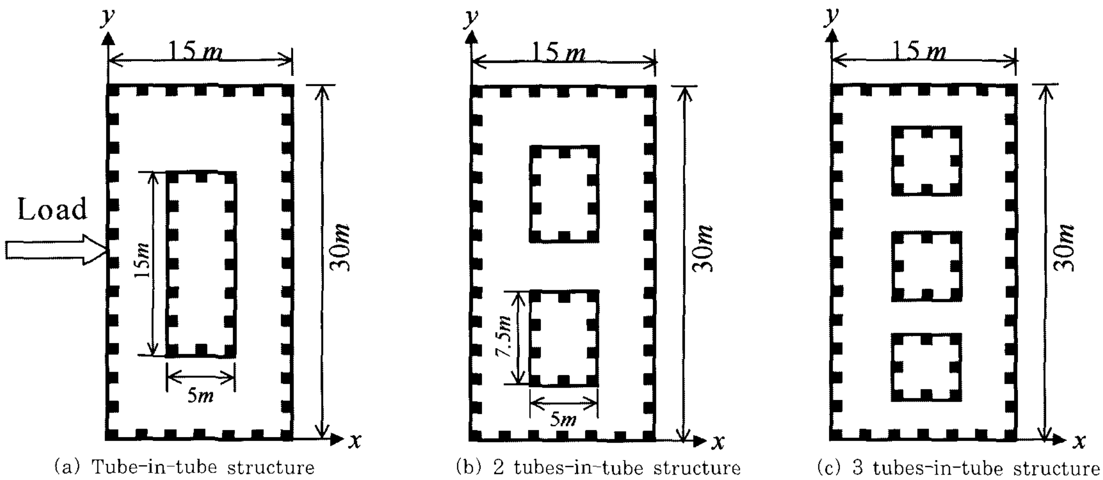
A manner analogous to those for the shear force distribution in the external tube can also be applied to the internal tube.

Note that the shear stress of each structural member can be expressed in terms of a series of linear functions by its second moment of area, the corresponding geometric and material properties and the applied loads. The shear stress functions are able to describe the variation of stress distributions in the flange and web frame panels of each tube. Further, the shear stresses are the functions of z, i.e. the coordinate along the height of the structure.

3. Comparison of Results

A series of framed tube structure with multiple internal tubes subjected to lateral loading is analysed to verify the simplicity and accuracy of the proposed method in the shear stress distributions. Three 40-story tube(s)-in-tube structures consisting of horizontal beams and vertical columns are analysed by a 3-D frame analysis,[5] and the structures idealized as a tube assemblage of the vertical equivalent frame panels are also analysed using the proposed method. The results are compared to estimate the shear stress distribution in each tube.

Each building has a 3.0m story height, 2.5m centre-to-centre column spacing and a uniformly distributed lateral load along the entire height of the structure. The cross-sectional area of all the columns and beams in the external tube of the example structures is taken to be 0.64m^2, and Young's modulus E and shear modulus G are equal to 2.06×10^{10}N/m^2 and 0.824×10^{10}N/m, respectively. The second moment of area of the internal tube of each the example structures is taken to be 90m^4. In order to consider the critical case of the structures, a uniformly distributed lateral load of 88.24KN/m is assumed to be applied to long side frame panel(flange frame panel) parallel to y-axis(see Fig. 6). The data of the example structures is summarised in Table 1.



(a) Tube-in-tube structure (b) 2 tubes-in-tube structure (c) 3 tubes-in-tube structure

Fig. 6 Plans of the three tube(s)-in-tube structures

Table 1 Data for Example Structures

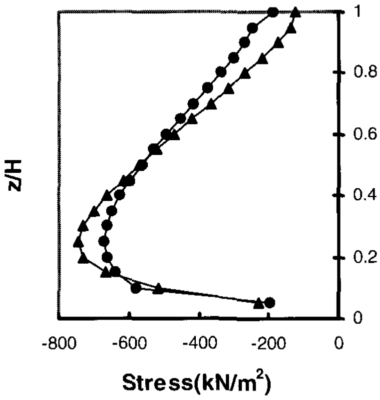
	Tube structure with one internal tube	Tube structure with two internal tubes	Tube structure with three internal tubes
Column & Beam Size of External Tube	80cm×80cm		
Column & Beam Size of Internal tube	91cm×91cm	80cm×80cm	72cm×72cm
External Tube Size	30m×15m(12bays by 6bays)		
Internal Tube Size	15m×5m (6bays by 2bays)	2 - 7.5m×5m (2-3bays by 2bays)	3 - 5m×5m (3-2bays by 2bays)
Lateral Load	88.24kN/m		
Total Story Height	120m		

Fig. 7 shows the shear stresses and forces in the beams of the three tube(s)-in-tube structures. It is observed that the predicted beam shear stresses along the structural height and the forces along the flange and web panels agree fairly well with the 3-D frame analysis results. Note that the maximum shear stress occurs in the web frame panel. The centre beam shear stress distribution in the web frame panel obtained by the proposed method is more linear, whereas the 3-D frame analysis program shows nonlinear distribution of the shear stress(see Fig. 7). It is further found that the maximum shear stress due to the proposed method occurs in the first floor, and that due to the 3-D frame analysis program occurs in the sixth floor. In order to find out the maximum errors between the predicted results and the 3-D frame analysis results, the maximum beam shear stresses in the first and the sixth storeys of the three structures, and those of the entire system of each structure, are compared in Table 2.

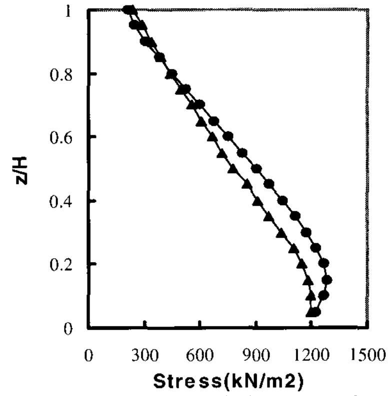
In any comparison of the deflection and the member stresses in a structure, the errors between the results by a preliminary analysis and these by all commercial program analysis are generally expected to be within about 16 %3), 4). Thus the proposed method yields good results for the shear stresses for all three tube(s)-in-tube structures as shown in Fig. 7. This implies that the proposed method is capable of satisfactorily predicting the structural behaviour in the shear stresses as well as the bending stresses verified in previously study(1).

4. Conclusion

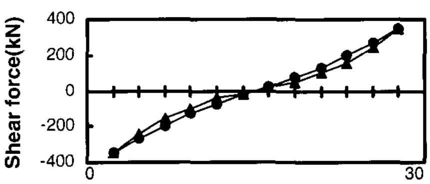
A simple mathematical model is proposed for the approximate shear stress analysis of the framed tube structure with multiple internal tubes. The numerical analysis of shear stress is based on the mathematical analogy in conjunction with the elastic theory. The net shear lag effects and the lateral stiffness of the internal tube are taken into consideration to estimate the shear stresses in the tubes-in-tube structures. In comparison with the 3-D frame analysis program, the only other approach available for the tubes-in-tube system, the proposed method provides similarly accurate results in predicting the shear stress distributions in the tubes of the framed-tube structures. In view of its simplicity, it is worth mentioning that the proposed method requires minimal data preparation effort, and for analysis, the personal computer running time is absolutely negligible when compared with the 3-D frame analysis program. Thus the proposed method is considered to be a suitable design tool to evaluate the shear stress distri-



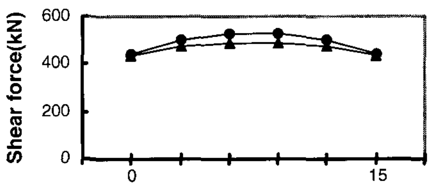
Corner beam of flange frame panel



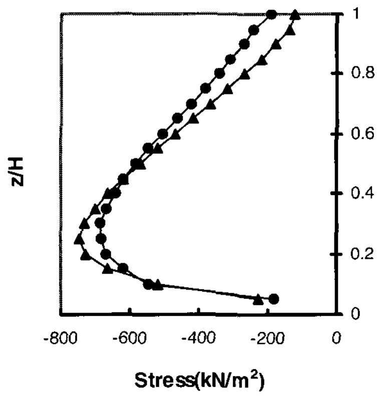
Centre beam of web frame panel



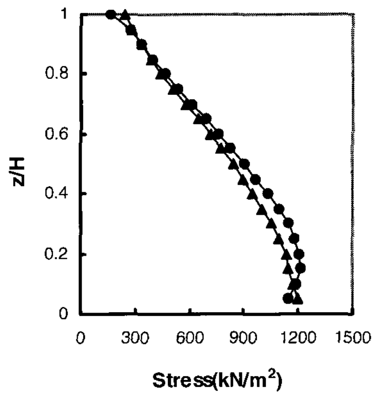
Flange frame panel at 21st floor



Web frame panel at 21st floor

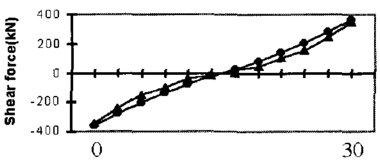


Centre beam of web frame panel

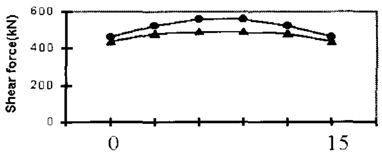


Corner beam of flange frame panel

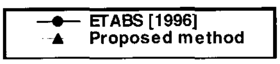
(a) Tube-in-tube structure



Flange frame panel at 21st floor

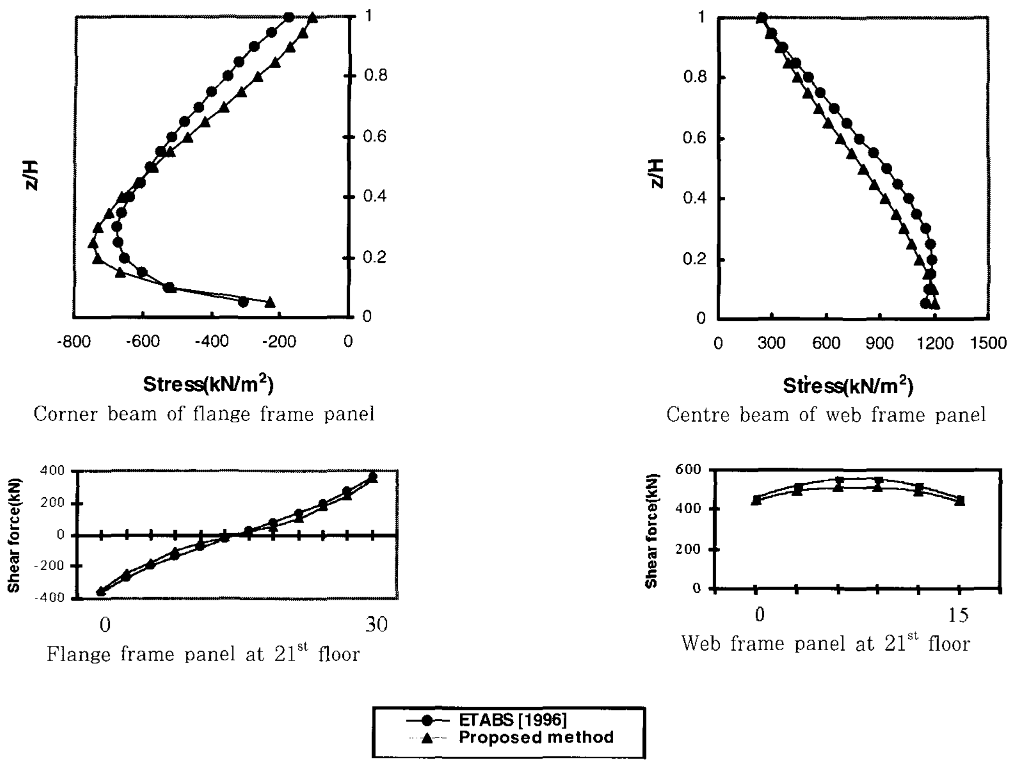


Flange frame panel at 21st floor



(b) 2 tubes-in-tube structure

Fig. 7 Shear stresses and forces in beams of the three tube(s)-in-tube structures



(c) 3 tubes-in-tube structure

Fig. 7. Shear stresses and forces in beams of the three tube(s)-in-tube structures(continue)

Table 2 Comparison of the maximum beam shear stresses in the web frame panels of the three tube(s) -in-tube structures

	Maximum shear			stress(kN/m^2)			
Structures	Entire system			Selected stories			
	ETABS [1989]	Proposed method	Error*(%)	Storey	ETABS [1989]	Proposed method	Error*(%)
Tube-in-tube	1283.8	1181.9	7.9	1st	1225.8	1181.9	3.5
				6th	1283.8	1162.5	9.44
2 tubes-in-tube	1215.3	1191.9	1.9	1st	1150.0	1191.9	3.6
				6th	1215.3	1131.5	6.89
3 tubes-in-tube	1183.8	1200.3	1.4	1st	1150.0	1200.3	4.3
				6th	1183.8	1118.2	5.54

Note : $* = \left| \dfrac{\text{Proposed method} - \text{3D frame analysis [ETABS]}}{\text{3-D frame analysis}} \times 100\% \right|$

butions in framed tube structures, particularly at the preliminary stages where numerous analysis iterations need to be carried out.

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References

1. Lee, K. K., Loo, Y. C. and Guan, H., "Simple Analysis of Framed Tube Structure with Multiple Internal Tubes", *Journal of Structural Engineering*, ASCE, Vol. 127, No. 4, 2001, pp.450~460
2. Lee, K. K. and Lee, L. H., "Shear Lag in Framed Tube Structures with Multiple Internal Tubes", *Journal of the Computational Structural Engineering Institute of Korea*, Vol. 13, No. 3, 2000, pp.351~360
3. Coull, A. and Bose, B., "Simplified analysis of framed-tube structures", *Journal of Structural Engineering*, ASCE, Vol. 101, No. 11, 1975, pp.2223~2240
4. Kwan, A. K. H., "Simple method for approximate analysis of framed tube structures", *Journal of Structural Engineering*, ASCE, Vol. 120, No. 4, 1994, pp.1221~1239
5. Chang, P. C., "Analytical modelling of tube-in-tube structure", *Journal of Structural Engineering*, ASCE, Vol. 111, No. 6, 1985, pp. 1326~1337
6. ETABS(1996), *Three Dimensional Analysis of Building System,* Computers and Structures Inc., Berkerly, California, U.S.A.
7. Coull, A., and Abu El Magd, S. A., "Analysis of wide-flanged shearwall structures", *Reinforced concrete structures subjected to wind and earthquake forces*, ACI Spec. Publ. 63, Paper No. SP63-23, Concrete Institute, Detroit, Mich., 1980, pp.575~607
8. Kwan, A. K. H., "Shear lag in shear/core walls", *Journal of Structural Engineering*, ASCE, Vol. 122, No. 9, 1996, pp.1097~1104