A Cohesive Surface Separation Potential

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This paper presents a form of the cohesive surface separation potential, which can produce potential curves by varying a single dimensionless parameter. Results show that a partial modification of Xu and Needleman's (1994) cohesive surface separation potential makes it possible to present the other potential curves as a special case as long as the normal separation is concerned. The proposed potential may describe interfacial debonding-crack initiation and growth-character of materials and, through numerical simulation, provide an insight for the effect of different cohesive surface separation potentials on the interfacial debonding.

Key Words: Debonding, Separation Potential, Normal Separation, Traction-Separation Law

Nomenclature

u_n	: Normal displacement component
u_t	: Tangential displacement component
T_n	: Normal component of the interfacial traction
T_t	: Tangential component of the interfacial traction
α	: Ratio of shear to normal stiffness of the interface
δ_n	: Normal characteristic length
δ_t	: Tangential characteristic length
σ_{max}	: Normal cohesive surface strength
τ_{max}	: Tangential cohesive surface strength
$\phi_{sep}(=\phi_n)$	: Work of normal separation ($u_t\equiv 0$)

1. Introduction

The use of a cohesive debonding model in failure analysis is attractive since they can be easily accommodated in the numerical analysis code. The initiation, growth, and arrest of a crack are the outcome of interaction between the imposed loading and the cohesive debonding model on the interface. There are two approaches. One is to impose the potential (Rose et al., 1981; Needleman, 1987; Needleman, 1990; Xu and Needleman, 1994).; the traction-seperation law for interfacial debonding can be derived from the potentials. The Other is to impose a traction-seperation law itself (Tvergaard and Hutchinson, 1992; Varias, 1998); energy can be obtained by integrating the relationship.

The traction-seperation law-constitutive eqation-for the interfacial debonding is as follows. With increasing interfacial separation, the traction across the interface first reaches a maximum, then deceases, and eventually vanishes as permitting a complete separation to occur. It should be noted that the cohesive debonding model mentioned here differs from the conventional ones (Dugdale, 1960; Barenblatt, 1962) in that a full account has been taken for the finite geometry changes and that in the decohesion constitutive relation is specified along the entire interface-no

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cohesive zone size needs to be determined by the analysis.

The interface constitutive response can be characterized by a cohesive surface separation potential. The potentials developed to date may be classified into two forms: the polynomial and exponential form. The polynomial form was suggested by Needleman (1990), as following the development of the exponential form (1987). In addition Xu and Needleman (1994) suggested a potential of an exponential form to fit the atomic separation analysis proposed by Rose et al. (1981).

This study presents a modified form of the cohesive surface separation potential that can generate the potential curves (Needleman, 1987; Needleman, 1990; Xu and Needleman, 1994) by varying a single dimensionless parameter introduced in the equation. The cohesive surface separation potential and corresponding traction curves produced by the modified form are compared with those previously suggested.

2. Cohesive Surface Separation Potentials

The following sections review some potential forms and describe the modified form. Attention is centered on two-dimensional analysis with neglecting plastic deformation at the interface.

2.1 A review of the potentials

2.1.1 The polynomial form

Needleman (1987) constructed a polynomial form of the interface potential that has a feature that the traction vanishes at a finite separation so that there is a well defined decohesion point. The polynomial potential for $u_n \leq \delta$ is given by

$$\phi(u_n, u_t) = \frac{27}{4}\sigma_{\max}\delta\left\{\frac{1}{2}\left(\frac{u_n}{\delta}\right)^2\left[1-\frac{4}{3}\left(\frac{u_n}{\delta}\right)+\frac{1}{2}\left(\frac{u_n}{\delta}\right)^2\right] + \frac{1}{2}\alpha\left(\frac{u_t}{\delta}\right)^2\left[1-2\left(\frac{u_n}{\delta}\right)+\left(\frac{u_n}{\delta}\right)^2\right]\right\} \quad (1)$$

where $\sigma_{\max}$ is the maximum traction carried by the interface undergoing purely normal separation ($u_t \equiv 0$), δ ths characteristic length and α the ratio of shear to normal stiffness of the interface.

Neglecting energy dissipation associated with the separation process, the normal and tangential components of the interfacial traction are obtained from Eq. (1) through $T_n = \partial\phi/\partial u_n$ and $T_t = \partial\phi/\partial u_t$:

$$T_n = \frac{-27}{4}\sigma_{\max}\left\{\left(\frac{u_n}{\delta}\right)\left[1-2\left(\frac{u_n}{\delta}\right)+\left(\frac{u_n}{\delta}\right)^2\right] + \alpha\left(\frac{u_t}{\delta}\right)^2\left[\left(\frac{u_n}{\delta}\right)-1\right]\right\} \quad (2)$$

$$T_t = \frac{-27}{4}\sigma_{\max}\left\{\alpha\left(\frac{u_t}{\delta}\right)\left[1-2\left(\frac{u_n}{\delta}\right)+\left(\frac{u_n}{\delta}\right)^2\right]\right\} \quad (3)$$

for $u_n \leq \delta$ and $T_n \equiv T_t = 0$ when for $u_n > \delta$.

2.1.2 The exponential form

Needleman (1990) also proposed an exponential form of the potential, based on the polynomial form given by Eq. (1). Using the same dependence of the shear as in Eq. (1) and $\phi \rightarrow \phi_{sep}$ as $u_n \rightarrow \infty$ where ϕ_{sep} is the work of normal separation ($u_t \equiv 0$), the exponential potential is

$$\phi(u_n, u_t) = \frac{9}{16}\sigma_{\max}\delta\left\{1-\left[1+z\left(\frac{u_n}{\delta}\right)-\frac{1}{2}\alpha z^2\left(\frac{u_t}{\delta}\right)^2\right]\exp\left[-z\left(\frac{u_n}{\delta}\right)\right]\right\} \quad (4)$$

where $z = 16e/9$. Using Eq. (4), the interfacial traction is obtained as

$$T_n = -\sigma_{\max} e\left\{z\left(\frac{u_n}{\delta}\right)-\frac{1}{2}\alpha z^2\left(\frac{u_t}{\delta}\right)^2\right\}\exp\left[-z\left(\frac{u_n}{\delta}\right)\right] \quad (5)$$

$$T_t = -\sigma_{\max} e\left\{\alpha z\left(\frac{u_t}{\delta}\right)\right\}\exp\left[-z\left(\frac{u_n}{\delta}\right)\right] \quad (6)$$

The parameters characterizing the exponential potential are chosen so that ϕ_{sep} is $9/16\sigma_{\max}\delta$ and the maximum absolute value of T_n for a normal separation only is $\sigma_{\max}$. It should be noted that Eq. (1) through Eq. (6) have no distinction between the normal and tangential characteristic lengths.

On the other hand, Rose et al. (1981) earlier suggested an exponential form of the interfacial potential based on the atomic analysis of a normal interfacial separation; this equation was later modified by Xu and Needleman (1994) to include a tangential separation. The exponential potential can be written in the form

$$\phi(u_n, u_t) = \phi_n + \phi_n \exp\left(-\frac{u_n}{\delta_n}\right)\left\{-\left(1+\frac{u_n}{\delta_n}\right)\exp\left(-\frac{u_t^2}{\delta_t^2}\right)\right\} \quad (7)$$

The normal and the tangential component of the interfacial traction are then obtained as

$$T_n = -\frac{\phi_n}{\delta_n}\exp\left(\frac{u_n}{\delta_n}\right)\left\{\left(\frac{u_n}{\delta_n}\right)\exp\left(-\frac{u_t^2}{\delta_t^2}\right)\right\} \quad (8)$$

$$T_t = \frac{\phi_n}{\delta_n}\frac{2\delta_n}{\delta_t}\frac{u_t}{\delta_t}\left(1+\frac{u_n}{\delta_n}\right)\exp\left(-\frac{u_n}{\delta_n}\right)\exp\left(-\frac{u_t^2}{\delta_t^2}\right) \quad (9)$$

ϕ_n $(=e\sigma_{max}\delta_n)$ is the work of a normal separation; the respective δ_n and δ_t are normal and tangential characteristic length; σ_{max} and τ_{max} are the normal and tangential cohesive surface strength, respectively. The dependence of the tangential traction component on the relative shear displacement was assumed to be periodic.

2.2 Modified form of the potential

In the present study, a cohesive surface separation potential is proposed by modifying Xu and Needleman (1994). A dimensionless parameter γ in Eq. (7) is introduced to yield the following equation:

$$\phi(u_n, u_t) = \phi_n + \phi_n \exp\left(-\gamma\frac{u_n}{\delta_n}\right)\left\{-\left(1+\gamma\frac{u_n}{\delta_n}\right)\exp\left(-\frac{u_t^2}{\delta_t^2}\right)\right\} \quad (10)$$

The normal and the tangential component of the cohesive surface traction are obtained as

$$T_n = \gamma\frac{\phi_n}{\delta_n}\exp\left(-\gamma\frac{u_n}{\delta_n}\right)\left\{\left(\gamma\frac{u_n}{\delta_n}\right)\exp\left(\frac{u_t^2}{\delta_t^2}\right)\right\} \quad (11)$$

$$T_t = \frac{2\phi_n}{\delta_t}\frac{u_t}{\delta_t}\left\{1+\gamma\frac{u_n}{\delta_n}\right\}\exp\left(-\gamma\frac{u_n}{\delta_n}\right)\exp\left(-\frac{u_t^2}{\delta_t^2}\right) \quad (12)$$

Notice that Eqs. (11) and (12) satisfy the constraints of $T_n=0$ when $u_n/\delta_n=0$, and that $T_t=0$ when $u_t/\delta_t=0$. With $u_t=0$ in Eq. (11), the maximum normal traction occurs when the normalized relative displacement (u_n/δ_n), equals $1/\gamma$. Consequently, the work ϕ_n of a normal separation, ϕ_n (with $u_t=0$), can be expressed as

$$\phi_n = \sigma_{max}\delta_n \exp(1)/\gamma \quad (13)$$

3. Results and Discussion

Figure 1 illustrates the normalized cohesive surface separation potential (ϕ/ϕ_n), as a function of the normalized normal displacement, u_n/δ_n (with $u_t=0$), for several different values of the parameter γ. The work of a normal separation (represented by the area under the potential curve) increases with decreasing γ.

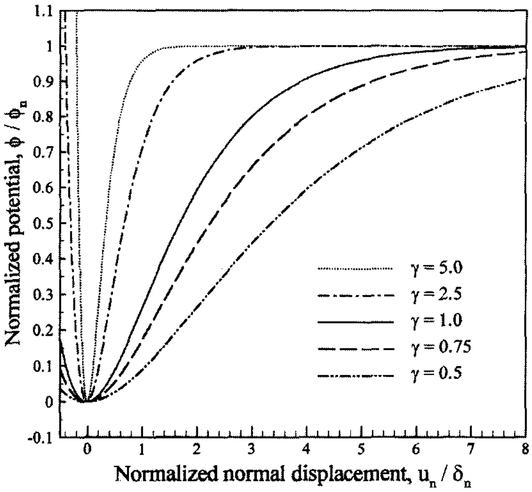

Fig. 1 Cohesive surface potential curves across the interface as a function of u_n with $u_t=0$, generated from the modified form of surface potential

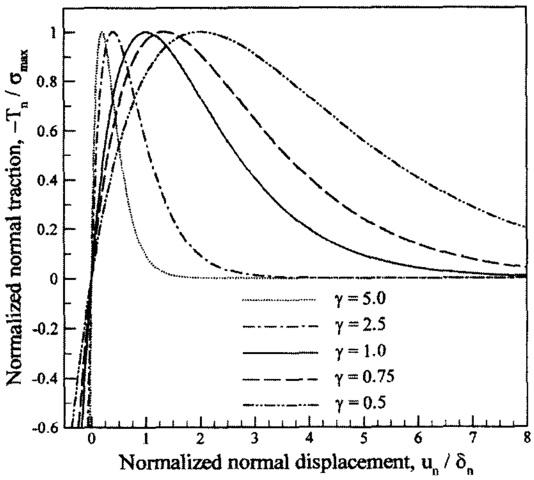

Fig. 2 Normal traction curves across the interface as a function of u_n with $u_t=0$, generated from the modified form of surface potential

Figure 2 shows the normalized normal traction T_n/σ_{max} as a function of u_n/δ_n (with $u_t=0$) for various values of the parameter γ. The maximum

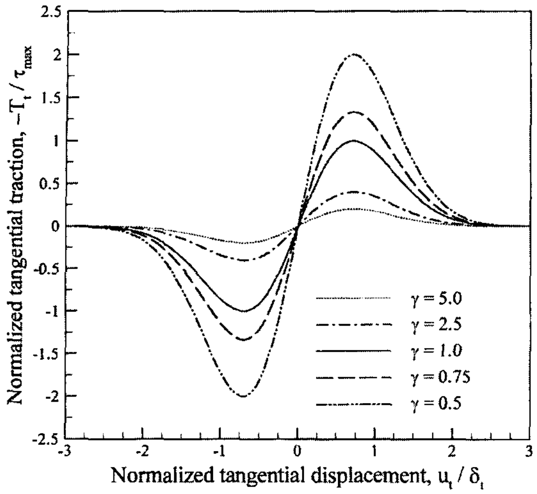

Fig. 3 Tangential traction curves across the interface as a function of u_n with $u_t=0$, generated from the modified form of surface potential

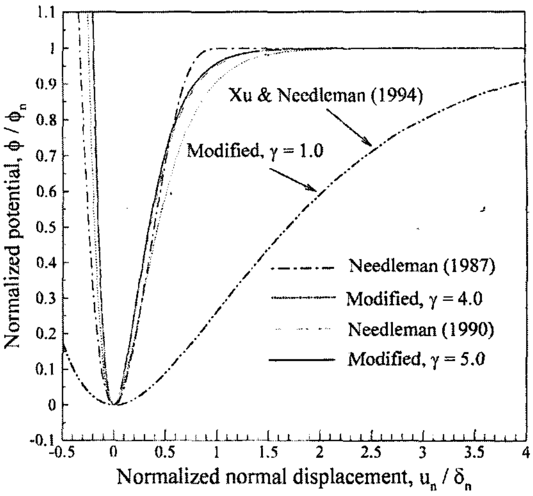

Fig. 4 Polynomial (Needleman, 1987; 1990), exponential (Xu & Needleman, 1994) and modified form of surface potential curves across the interface as a function of u_n with $u_t=0$

value of $-T_n$ is σ_{max} and occurs when $u_n/\delta_n = 1/\gamma$. Decreasing the value of the parameter γ leads to a greater level of energy required for a normal separation, by increasing the normal separation displacement un at which the maximum in the normal traction component occurs.

The variation of the tangential traction, T_t, as a function of u_t (with $u_n=0$), for various values of the parameter γ, is shown in Fig. 3. Unlike the case of normal traction, the maximum value of the normalized tangential traction depends inversely on the value of the parameter γ. However, the location of the maximum normalized tangential traction, $|-T_t/\tau_{max}|$, is independent of the parameter γ, and occurs when $|u_t| = \sqrt{2}\,\delta_t/2$. Decreasing the value of the parameter γ also leads to a higher level of energy absorption during the shear separation.

Figures 4 and 5, respectively, show the normalized cohesive surface potential curves and the normal traction across the interface, T_n, as a function of u_n (with $u_t=0$) for the polynomial, exponential and modified forms of the potential. In Fig. 5, the polynomial potential (1987) gives $T_n = \sigma_{max}$ at $u_n = \delta_n/3$, while the exponential potential (1990) gives $T_n = \sigma_{max}$ at $u_n = 9\delta_n/(16e) \approx 0.207\delta_n$. The polynomial potential leads to the separation at $u_n = \delta_n$, so that $T_n \equiv 0$ for $u_n \geq \delta_n$. The exponential potential, on the other hand, gives a continually decaying normal traction that vanishes in the limit $u_n \rightarrow \infty$. However, the work done when $u_n = \delta_n$ is about $0.95\phi_{sep}$.

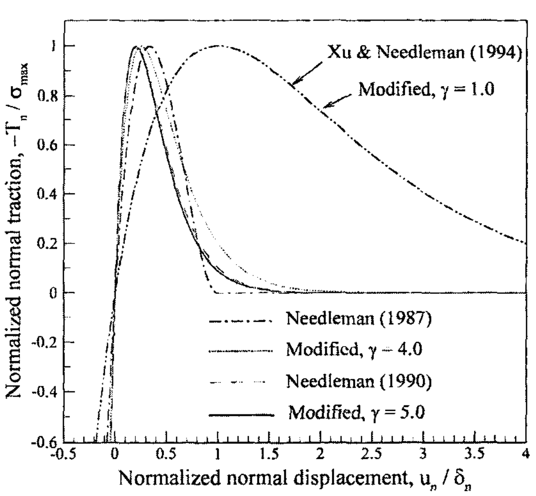

Fig. 5 Normal traction curves across the interface as a function of u_n with $u_t=0$ for the polynomial (Needleman, 1987; 1990), exponential (Xu & Needleman, 1994) and modified form of surface potential

For $\gamma=1$, the normal traction-displacement relationship reduces to that of Xu and Needleman's model (1994). For $\gamma=5$, the normal traction-displacement relationship is almost the same with that of Needleman's model (1990). But $\gamma=4$ does not yield the shape of the normal traction-displacement curve obtained from the polynomial form of the potential (1987). The normal traction-displacement produced from the polynomial form of cohesive surface separation potential decayed rapidly while that generated from modified form of it for $\gamma=4$ slowly diminished.

The tangential traction across the interface, T_t, is a monotonically increasing function of u_t (with $u_n=0$) for both polynomial and exponential form (See Eq. (3) and Eq. (6)). Therefore, the tangential traction curves were not compared.

4. Conclusions

This paper has shown that a partial modification of Xu and Needleman's cohesive surface separation potential (1994) makes it possible to treat other existing models a special cases as long as the normal separation is concerned. The modified model has a feature for a general mathematical description of the normal interfacial debonding behavior of materials and can be a tool to simulate debonding and thus fracture. Since our main interest centers on the crack growth resistance, we should carefully look at of the form of either potential or traction-seperation law on the results for simulated crack growth, which is left as a future work.

References

Barenblatt, G. I., 1962, "The Mathematical Theory of Equilibrium of Cracks in Brittle Fracture," *Advances in Applied Mechanics*, Vol. 7, pp. 56~129.

Dugdale, D. C., 1960, "Yielding of Steel Sheets Containing Slits," *J. the Mechanics and Physics of Solids*, Vol. 8, pp. 100~104.

Needleman, A., 1987, "A Continuum Model for Void Nucleation by Inclusion Debonding," *J. Applied Mechanics*, Vol. 54, pp. 525~531.

Needleman, A., 1990, "An Analysis of Decohesion Along an Imperfect Interface," *Int. J. Fracture*, Vol. 42, pp. 21~40.

Rose, J. H., Ferrante, J. and Smith, J. R., 1981, "Universal Binding Energy Curves for Metals and Bimetallic Interfacves," *Physical Reviewer letters*, Vol. 47, pp. 675~678.

Tvergaard, V. and Hutchinson, J. W., 1992, "The Relation Between Crack Growth Resistance and Fracture Process Parameters in Elastic-Plastic Solids," *J. the Mechanics and Physics of Solids*, Vol. 40, pp. 1377~1397.

Varias, A. G., 1998, "Constraint Effects During Stable Transient Crack Growth," *Computational Mechanics*, Vol. 21, pp. 316~329.

Xu, X. -P. and Needleman, A., 1994, "Numerical Simulation of Fast Crack Growth in Brittle Solids," *J. the Mechanics and Physics of Solids*, Vol. 42, pp. 1397-1434.