Efficient Calculation of a Step Discontinuity for Shielded-Microstrip using Vector Finite Element (VFEM) and Mode Matching Method

Young-Tae Kim, Jun-Seok Park and Hyeong-Seok Kim

Abstract - In this paper, we proposed a procedure to analyze a shielded-microstrip step discontinuity using the mode matching method (MMM) combined with the vector finite element method (VFEM), which is used to find the equivalent waveguide-model for a microstrip. In order to calculate the effective-widths and –dielectric permittivity of the equivalent waveguide-model corresponding shielded-microstrip, the propagation constant and characteristic impedance are calculated from the VFEM. MMM is then applied to find the scattering parameter in the planar waveguide. This technique makes it possible to take advantage of the high accuracy of the VFEM as well as the high efficiency of the MMM.

Keywords: shielded-microstrip, vector finite element method, mode matching method

1. Introduction

The calculation of the scattering at the step discontinuities for the planar transmission line becomes increasingly important as more precise design procedure are required for monolithic microwave and millimeter-wave integrated circuits. Therefore, it is important to develop an efficient and accurate calculation procedure to compute the characteristics of step discontinuities. Microstrip discontinuity problems have been treated during the past years by several authors. Several comprehensive reviews on microstrip discontinuity problems are also presented in several articles [1]–[4]. The unshielded asymmetric microstrip step discontinuity was studied in Ref. [5] where they employed an equivalent waveguide model based on a spectral domain approach using hybrid mode analysis [5]–[6]. This method is well suited to handle geometries where modal expansions of electromagnetic fields can be derived analytically. However, this method has a limitation in handling a shielded structure with complex and irregular shape.

In this paper, we present a technique to analyze a shielded microstrip step discontinuity based on VFEM [7] and MMM [3]. In order to achieve an equivalent waveguide model of a microstrip discontinuity, the propagation constant and characteristic impedance are calculated from the VFEM in the shielded microstrip cross section.

2D electromagnetic full analysis was used to calculate the shielded microstrip. In this paper, the propagation constant and characteristic impedance of a cross section are calculated in the analysis domain when the operating frequency is specified. This method could improve the accuracy of the extracted equivalence parameter values for the equivalent waveguide-model compared with existing methods. Furthermore, in order to save computing time, MMM is used for deriving scattering parameters in the analysis domain. The analytical approach used in computing the mode properties allows the development of an efficient MMM procedure.

Manuscript received: June 12, 2002 accepted: Oct. 25, 2002

Young-Tae Kim and Jun-Seok Park are with the Division of Information Technology Engineering, Soonchunhyang Univerity, Asan, 336-745, Korea (lyann@sch.ac.kr)

Hyeong-Seok Kim is with the School of Electrical and Electronic Engineering, Chungang Univerity, Seoul, 156-756, Korea (kimcaf2@cau.ac.kr)

2. The equivalent waveguide for the shielded microstrip

For shielded-microstrip analysis, the propagating modes can be divided into TE or TM modes, which can be solved separately. For the TE mode $E_z = 0$, the transverse electric field vector E_t satisfies the vector wave equation.

$$\nabla_t \times \left(\frac{1}{\mu_r} \nabla_t \times E_t \right) - k_o^2 \varepsilon_r E_t = 0 \quad (1)$$

where ε_r and μ_r are the permeability and permittivity of the materials in the microstrip, respectively.

The representative variational functional for such a problem is given by

$$F(E) = \frac{1}{2} \iint_\Omega \left[(\nabla_t \times E) \frac{1}{\mu_r} (\nabla_t \times E)^* - k_0^2 E \varepsilon_r E^* \right] d\Omega \quad (2)$$

Assuming that the dependence of the fields in the z-direction is $e^{-jk_o z}$, the functional can be written in terms of the transverse and the longitudinal fields similar to [7];

$$F(E) = \frac{1}{2} \iint_\Omega \left[(\nabla_t \times E) \mu_r (\nabla_t \times E)^* \right.$$

$$-k_0^2(E_t\varepsilon_r E_t^* + E_z\varepsilon_r E_z^*) + (\nabla_t E_z + jk_z E_t)\mu_r(\nabla_t E_z + jk_z E_t)^* \Big] d\Omega \tag{3}$$

where ∇_t is the transverse del operator; these field components can be subsequently expanded as a summation of scalar and vector basis functions. For obtaining more accurate solutions, the second-order vector elements are applied [8].

$$e_t = k_z E_t = \sum_{i=1}^{n} N_i^e e_{ti}^e \tag{4}$$

$$e_z = -jE_Z = \sum_{i=1}^{n} N_i^e e_{zi}^e \tag{5}$$

where n denotes the number of degree of freedom in each element which is 9 as shown in Fig. 1

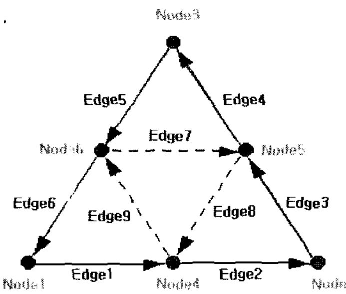


Fig .1 The second-order triangular vector shape function.

$$S_{el(tt)} = \frac{1}{\mu_r}\iint_\Delta (\nabla_t \times N)\cdot(\nabla_t \times N)ds - k_o^2\varepsilon_r \iint_\Delta (N_{tm}\cdot N_{tn})ds \tag{6}$$

$$T_{el(tt)} = \varepsilon_r \iint_\Delta (N_{tm}\cdot N_{tn}) \tag{7}$$

$$T_{el(tz)} = \frac{1}{\mu_r}\iint_\Delta (N_{tm}\cdot \nabla L_j)ds \tag{8}$$

$$T_{el(zt)} = \frac{1}{\mu_r}\iint_\Delta (\nabla L_i \cdot N_{tn})ds \tag{9}$$

$$T_{el(zz)} = \frac{1}{\mu_r}\iint_\Delta (\nabla L_i \cdot \nabla L_j)ds - k_o^2\varepsilon_r \iint_\Delta L_i L_j\, ds \tag{10}$$

These element matrices can be assembled over all the triangles in the cross section of the shielded microstrip to obtain a global eigenvalue equation.

$$\begin{bmatrix} S_{tt} & 0 \\ 0 & 0 \end{bmatrix}\cdot\begin{bmatrix} e_t \\ e_z \end{bmatrix} = (-\beta^2)\begin{bmatrix} T_{tt} & T_{tz} \\ T_{zt} & T_{zz} \end{bmatrix}\cdot\begin{bmatrix} e_t \\ e_z \end{bmatrix} \tag{11}$$

The shielded microstrip can be divided into the dielectric substrate material with a height h and a dielectric constant ε_r, and a metallic strip of the width W and the thickness t. In this model, the shielded microstrip is represented by a parallel plate waveguide of width W_{eff} and height h as shown in Fig. 2.

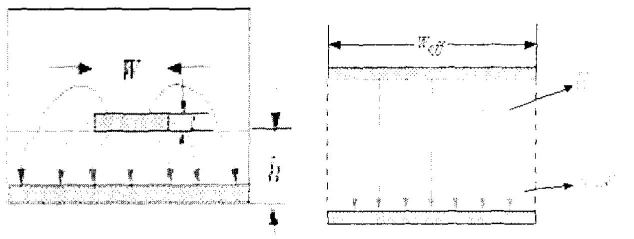


Fig. 2 The electromagnetic fields of the dominant (TEM) mode in the shielded microstrip line and equivalent waveguide model.

The effective dielectric constant ε_{eff} and the effective width W_{eff} can be found from the propagation constant β and the characteristic impedance Z of the microstrip modeled by the waveguide.

$$\varepsilon_{eff} = (\beta / k_o)^2 \tag{12}$$

$$Z = (120\pi / \sqrt{\varepsilon_{eff}})(h / w_{eff}) \tag{13}$$

where β and Z can be calculated beforehand from structural parameters by the VFEM. To verify the above method, it is compared with the wheeler formula [9], [10] from (14) to (17).

$$w_{eff} = \left\{\frac{w}{h} + \frac{2}{\pi}\ln\left[2\pi e\left(\frac{w}{2h} + 0.92\right)\right]\right\}h, \quad e = 2.718 \tag{14}$$

$$\varepsilon_{eff}(f) = \varepsilon_r \cdot \varepsilon_{eff(0)}\left[\frac{(f/f_w)^2 + 1}{\sqrt{\varepsilon_{eff(0)}}(f/f_w)^2 + \sqrt{\varepsilon_r}}\right]^2 \tag{15}$$

$$f_w = 3.5 + \frac{16.2\varepsilon_r^{0.25}}{1 + 0.12(w/h)\varepsilon_r^{0.35}} \tag{16}$$

$$F(w,h,\varepsilon_r) = \left[1 + \frac{12}{(w/h)^{1.14}}\right]^{-\frac{1}{2}} + 1.2\times10^{-4}(\varepsilon_r - 6)\left(\frac{w}{h} - 40\right) \tag{17}$$

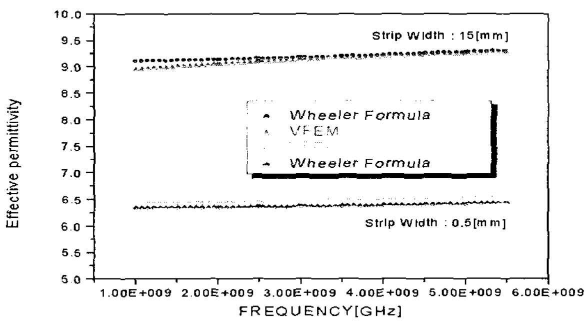


Fig. 3 Comparison between the calculated effective permittivities using the wheeler formula and VFEM for the waveguide model.

As can be seen from Fig. 3 and 4, there is good agreement between the wheeler formula results and the VFEM values of the effective width and dielectric constant.

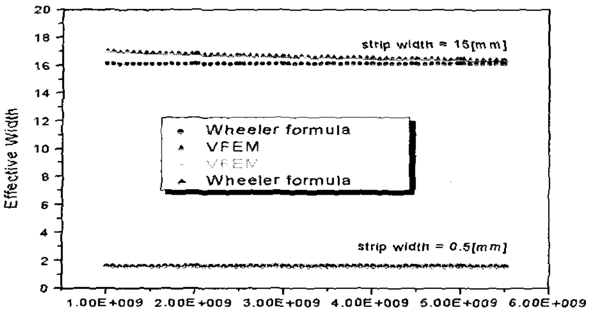


Fig. 4 Comparison between the calculated effective widths using the wheeler formula and VFEM for the waveguide model.

3. Sparameter calculation using MMM

The planar waveguide model of a step discontinuity is shown in Fig.5. Dimensions a and b are the width of the planar waveguides corresponding to the microstrip lines "a" and "b", respectively. The cross-sectional areas of these guides are denoted by $A^a(=a\times h)$ and A^b $(=b\times h)$.

In this case, only TEM and TE_{m0} modes can be excited. The transverse fields, which are expressed by each mode, in the planar waveguide model can be expressed in terms of orthogonal series expansions as follow

$$E_t = \sum_{m=0}^{\infty} U_{m0}\{\hat{z}\times\nabla_t\psi_{m0}(x,y)\} \tag{18}$$

$$H_t = \sum_{m=0}^{\infty} I_{m0}\{-\nabla_t\psi_{m0}(x,y)\} \tag{19}$$

where U_{m0} and I_{m0} are the expansion coefficients and ψ_{m0} is the scalar potential. The unit vector along the z -axis is denoted by $\hat{z}$, and m=0 corresponds to the dominant mode (TEM). The symbol ∇_t designates the transverse component of the gradient.

At the discontinuity plane ($z=0$), the following boundary and interface conditions must be satisfied.

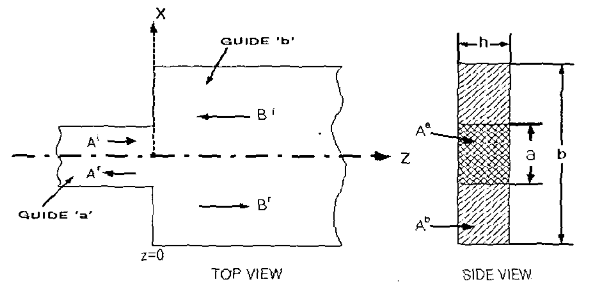


Fig. 5 Top and side views of a step in width discontinuity

$$H_t^b = 0 \quad \text{in region } (A^b - A^a) \tag{20}$$

$$H_t^a = H_t^b \quad \text{in region } A^a \tag{21}$$

$$E_t^a = E_t^b \quad \text{in region } A^a \tag{22}$$

Multiplying (22) by $(\hat{z}\times\nabla_t\Psi_{M0}^a)$ for various values of M (=0,1,2...) and integrating over the aperture A^a leads to the equation.

$$U_{M0}^a = \iint_{A^a} E_t^b \cdot \left(\hat{z}\times\nabla_t\Psi_{M0}^a\right)dA \tag{23}$$

Similarly the following expression is found for the expansion coefficients I^b:

$$I_{P0}^b = \iint_{A^a} H_t^b \cdot \left(-\nabla_t\Psi_{P0}^b\right)dA \tag{24}$$

Substituting the expansion of E_t^b and H_t^b from (18) and (19), respectively, U_{M0}^a and I_{P0}^b are given by

$$U_{M0}^a = \sum_{p=0}^{\infty} U_{p0}^b K_{(M0)(p0)} \tag{25}$$

$$I_{P0}^b = \sum_{m=0}^{\infty} I_{m0}^a K_{(m0)(P0)} \tag{26}$$

where M, m and P, p correspond to the models in the guide "a" and "b" respectively, $K's$ are called the coupling integrals. The coupling integrals are given by

$$K = K_{(M0)(p0)} = \iint_{A^a} (\hat{z}\times\nabla_t\psi_{p0}^b)(\hat{z}\times\nabla_t\psi_{M0}^a)dA \tag{27}$$

$$K^T = K_{(m0)(P0)} = \iint_{A^a} (\nabla_t\psi_{m0}^a)\cdot(\nabla_t\psi_{P0}^b)dA \tag{28}$$

The expansion coefficients U and I can be replaced by the normal mode coefficients A^i, A^r, B^i, and B^r. The resulting equations are given by

$$\sqrt{Z^a}(A^i + A^r) = \sqrt{Z^b}K\ (B^i + B^r) \tag{29}$$

$$\sqrt{Y^b}(B^i + B^r) = \sqrt{Y^b}K^T(A^i - A^r) \tag{30}$$

where m , M , p , P =0,1,2... and $Z(=1/Y)$ is the wave impedance. The incident and the reflected modes may be related by S-matrix as

$$\begin{bmatrix} A^r \\ B^r \end{bmatrix} = \begin{bmatrix} S^{AA} & S^{AB} \\ S^{BA} & S^{BB} \end{bmatrix}\begin{bmatrix} A^i \\ A^i \end{bmatrix} \tag{31}$$

By knowing K, K^T, Z^a, and Z^b, one can calculate the S-parameters of the step discontinuity.

4. Numerical results

Fig. 6 shows the propagation constant β of a shielded microstrip discontinuity in Regions A and B. The substrate material is a dielectric constant of $\varepsilon_r = 9.7$ and a height of $h = 0.635[mm]$.

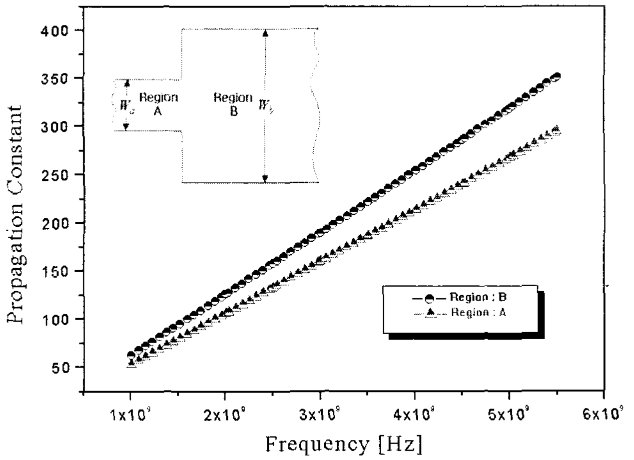


Fig. 6 Computed propagation constants of a symmetric step discontinuity where $W_a = 0.5[mm]$ and $W_b = 15[mm]$.

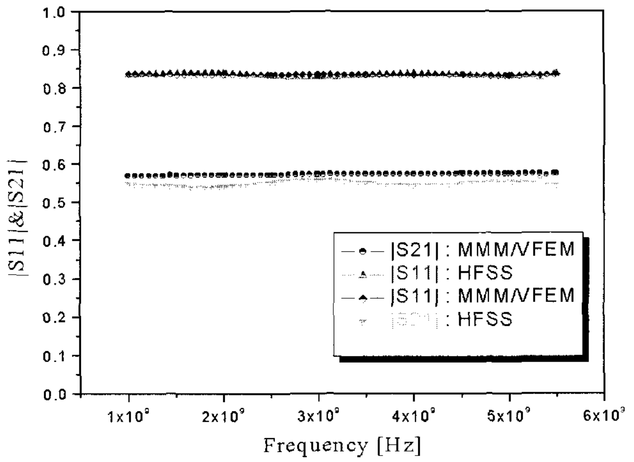


Fig. 7 Calculated S-parameters of a symmetric step discontinuity

The scattering parameter obtained in this case is plotted in Fig. 7. The analysis took about 30 seconds for this calculation with a PC. For reasonable comparison, the same structure has been analyzed with the commercial software Ansoft HFSS, which is based on a full 3-D VFEM formulation. The calculated result could confirm the efficiency and accuracy of proposed VFEM and MMM.

4. Conclusion

In this paper, we have been presented a hybrid method to analyze a step discontinuity for the shielded microstrip lines based on MMM and VFEM. For equivalent waveguide conversion in the shielded microstrip line, VFEM has been applied. MMM has been employed to calculate the S-parameters in waveguide structure with a step discontinuity. Comparisons of simulations show the validity of the theory. In principle, this method could be used to treat other types of discontinuity problems in planar lines such as strip, CPW, and so on.

References

[1] Benedek and P. Silvester, "Equivalent capacitance for microstrip gaps and steps," *IEEE Trans. Microwave Theory and Tech.*, vol. 34, pp. 213-222, 1972.

[2] A. F. Thomson and A. Gopinath, "Calculation of microstrip discontinuity inductances," *IEEE Trans. Microwave Theory and Tech.*, vol. MTT-23, pp. 177-179, 1972.

[3] G. Kompa, "S-matrix computation of microstrip discontinuities with a planar waveguide model," *Arch. Elektron, Ubertragungstechn.*, vol. 30, pp. 58-64, 1976.

[4] R. H. Jansen, "The spectral domain approach for microwave integrated circuits," *IEEE Trans. Microwave Theory and Tech.*, vol. MTT-33, pp. 1043-10565, 1985.

[5] T. S. Chu, T. Itou, and Yi Chi Shih, "Comparative study of mode matching formulations for microstrip discontinuity problems," *IEEE Trans. Microwave Theory and Tech.*, vol. MTT-33, pp. 1018-1023, 1985.

[6] T. S. Chu, T. Itou, "Generalized scattering matrix method for analysis of cascade and offset microstrip step discontinuities," *IEEE Trans. Microwave Theory and Tech.*, vol. MTT-34, pp. 280-284, Feb. 1986.

[7] J. F. Lee, D. K. Sun, and Z. J. Cendes, "Full-wave analysis of dielectric waveguide using tangential vector finite elements," *IEEE Trans. Microwave Theory and Tech.*, vol. 39, pp. 1262-1271, Aug 1991.

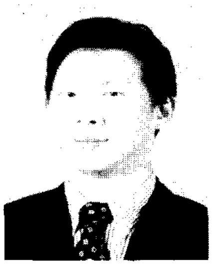
[8] A. Ahagon, T. Kashimoto, "Three-dimensional electromagnetic wave analysis using high order edge elements," *IEEE Trans. Magnetics.*, vol. 31, No.3, pp. 1753-1756, May 1995

[9] H. A. Wheeler, "Transmission line properties of parallel wide strips by conformal mapping approximation," *IEEE Trans. Microwave Theory and Tech.*, vol. MTT-12, pp. 280-289, 1964.

[10] H. A. Wheeler, "Transmission line properties of parallel strips separated by a dielectric sheet," *IEEE Trans. Microwave Theory and Tech.*, vol. MTT-13, pp. 172-185, 1965.

Young-Tae Kim was born in Pusan, Korea, on August 11, 1972. He received his B.S. and M.S. degress in electrical engineering from the Soonchunhyang University, Asan, Korea, in 1998 and 2000, respectively. He is currently working toward his Ph.D. degree at the same university since 2000. His research interests are mi- interests are microwave device analysis and design.

Jun-Seok Park was born in Seoul, Korea, on August 12, 1969. He received his B.S. and M.S. degrees in electronic engineering and his Ph.D. degree in radio frequency and monolithic microwave and integrated circuits from the Kookmin University, Seoul, Korea, in 1991, 1993 and 1996, respectively. In 1997, he joined the Department of Electrical Engineering at the University of California at Los Angeles as a Postdoctoral Researcher. In March 1998, he joined the Division of Information Technology Engineering, Soonchunhyang University, Asan, Korea, where he is currently an Assistant Professor of and Director of the Super-Module Research Laboratories. He is also currently the Technical Advisor of several RF/microwave component and system companies such as Netel Inc, Amotech Corporation, and Flambda Inc. His current research involves RF and microwave hybrid module design using low-temperature co-fired ceramic techniques and RF integrated circuit(RFIC) design such as a power amplifiers for mobile handsets based on CMOS technology. His group has developed several novel components with a DGS. His group has also worked extensively in the area of various RF and microwave components with a novel configuration and design method.



Hyeong-Seok Kim was born in Seoul, Korea, on October 9, 1962. He received his B.S., M.S. and Ph. D degrees in the department of the electrical engineering from Seoul National University, Seoul, Korea, in 1985, 1987 and 1990, respectively. From 1990 to 2002, he was with the Division of Information Technology Engineering, Soonchunhyang University, Asan, Korea. In 1997, he was a visiting professor of the Electrical Computer Science Engineering, Rensselaer Polytechnic University, Troy, New York USA. In 2002, he transferred to the School of Electrical and Electronics Engineering, Chungang University, Seoul, Korea as an Associate Professor. His current research interests include numerical analysis of electromagnetic field and waves, RF/Microwave device analysis & design as well as Electromagnetics Education