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Students' Cognitive Style and Mathematical Word Problem Solving

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Students approach mathematical problem solving in fundamentally different ways, particularly problems requiring conceptual understanding and complicated strategies such as mathematical word problems.

The main objective of this study is to compare students' performance with different cognitive styles (Field-dependent vs. Field-independent) on mathematics problem solving, particularly, in word problems. A sample of 180 school girls (13-years-old) were tested on the Witkin's cognitive style (Group Embedded Figures Test) and two mathematics exams. Results obtained support the hypothesis that students with Field-independent cognitive style achieved much better results than Field-dependent ones in word problems. The implications of these results on teaching and setting problems emphasizes that word problems and cognitive predictor variables (Field-dependent/Field-independent) could be challenging and rather distinctive factors on the part of school learners.

INTRODUCTION

Cognitive style is composed of characteristics in individuals that influence how they respond and function in different situations. Cognitive style relates to basic differences in the individuals' expectations of life, their relationship to other persons,and the way in which they seek solutions to problems (Saracho 1998). Cognitive style is an individual approach to organizing and representing information (Riding & Al-Sanabani 1998).

In recent years the study of cognitive styles has become a broad stream in cognitive psychology and math's education. Individuals display their own personal cognitive styles and, that is, broad attributes that become evident in an individual's response to various situations (Anastasi 1996).

According to Messick (1976), cognitive styles are information processing habits representing the learner's typical mode of perceiving, thinking, problem solving, and remembering.

A large body of researches suggest that students with different cognitive styles approach processing of information and problem solving in different ways. (Alamolhodaei 2001; Johnstone & Al-Naeme 1991; Kempa 1979; Kogan 1976; Messick 1976; Witkin et al. 1977; Witkin & Goodenough 1981).

Styles regardless of their types, are different from ability which some believe to be a characteristic of intelligence. Whereas ability refers more to the content of cognition, cognitive styles help one predict how information is being processed by each individual (Messick 1976 & Witkin et al. 1977; Witkin & Goodenough 1981). One cognitive style may lead to more effective learning in one discipline but may be detrimental in another discipline. A student with an analytic cognitive style may succeed in a situation requiring analytical skills whereas a student with a global cognitive style may fail in the same situation (Saracho 1998). A widely used dimension of cognitive style in education is field-dependence/field-independence (FD/FI) which specifies an individual's mode of perceiving, thinking, problem solving and remembering.

FD/FI COGNITIVE STYLE

Field-dependency/field-independency defines the individual's dependency in a perceptual field when analysing a structure or a form which is part of that field. Field-independent (FI) individuals can abstract an item from the surrounding field and work out problems that have a vital component out of the content.

In contrast, field-dependent (FD) learners rely on the organization of the field as a whole, they retain a global perspective as they work through problems, while FI learners are analytical (Goodenough 1976; Witkin 1976; Witkin, Moore, Goodenough & Cox 1977; Witkin et al. 1976; 1977; Witkin & Goodenough 1981; Saracho 1998).

Dickstein (1968) and Ohnmacht (1966) have shown the differences between FI and FD learners in the strategies they employ during concept attainment and conceptualizing the environment. The same results were obtained by Goodenough (1976).

Based on (Johnstone & Al-Naeme 1991), FD students have difficulty in separating "signal" from "noise", relevant from irrelevant; what is important from the surrounding and confusing information. This learner tends to give equal attention to both "signal" and "noise" and hence tends to overload with information.

In contrast, the FI learner can get to the nub of a situation and ignore distracting information. Therefore, such a learner is not in danger of overloading his ability to process information.

Witkin's studies have consistently demonstrated cognitive styles to be independent of intelligence and thus field-dependency/field-independency appears to be more related to the "how" than to the "how much" of cognitive functioning. The feature of bipolarity of cognitive styles rejected the issue of good or bad in connection with both field-dependent/field-independent learners (Witkin 1978; Witkin et al. 1977; Witkin & Goodenough 1981;Clarizio, Craig & Mehrens 1987).

According to Laughlin (1966) FI students solved the problems in fewer trials and shorter time in comparison with, FD ones. The cognitive restructuring aspect of field-dependence/field-independence has been found to relate to problem solving ability (e.g., Witkin et al. 1977; Threadgill-Sowder & Sowder 1982; Threadgill-Sowder et al. 1985). This means that students scoring high on the (FD/FI) test are generally more successful problem solvers that those who score low on the same task.

Several researches have demonstrated the importance of field-dependency/field-independency in science education and mathematical problem solving (Witkin 1976; Witkin et al. 1977; Witkin & Goodenough 1981; Johnstone & Al-Naeme 1991; 1995; Alamolhodaei 1996; Ekbia & Alamolhodaei 2000).

FD/FI COGNITIVE STYLE AND MATHEMATICAL PERFORMANCE

The role of FD/FI style in mathematical performance has been emphasized by various studies. For example:

1. Bieri, Bradbaurn & Galinsky (1958) reported significant correlations between FI cognitive style and mathematical ability among college students.
2. Witkin et al. (1962) noted that work in mathematics seems to demand the analytic operations required for success in FI test.
3. Similarly, Spitler (1970) found that mathematical performance is significantly related to FD/FI cognitive style. FI students behaved more efficiently than FD ones in a maths class.
4. According to a number of researches, FI learners were more successful in math's achievement and problem solving than FD learners (for example; Witkin et al. 1977; Roberge & Flexr 1983; Dugger 1984; Van & Malcom 1988; Talbi 1990).
5. Mrosla (1983) noted that low achieving students in high school algebra were mainly FD in their learning styles.
6. Alamolhodaei (1996) has shown that the FD/FI dimensions of cognitive style do affect students' performance in calculus education. It was found that FI learners tend to get higher results than FD ones in calculus problem solving at university level.
7. Srivastava (1997) suggested that students achieving high marks in mathematics differ significantly in their cognitive styles. Students achieving high marks in

math are more FI than those achieving low marks.

8. Ekbia & Alamolhodaei (2000) reported that school boys with FI cognitive style were more successful than FD ones in the math's word problems.

THE PRESENT STUDY

In brief, the main aim of the current study was to identify students' difficulties associated with the mathematics word problems.

The focus of this research was to provide a profile of learners' performance with different cognitive styles (FD/FI) in tackling word problems. Thus, the main question addressed in this study is "How different would the achievement of students with (FD/FI) learning style be in such math's task?"

It seems to the researcher, as a main hypothesis, that the field-independent (FI) students would be expected to show higher achievement than the field-dependent (FD) ones in the math's word problems. In fact, such questions tend to make the task rather distinctive and challenging on the part of students.

MATHEMATICAL WORD PROBLEMS

Students express great difficulties in handling word problems (Clement, Lochhead & Monk 1981; Carpenter & Moser 1982; Gagne 1983; Orton 1992; Takahashi 2000).

A word problem or verbal problem is a question which requires the application of math's in order to achieve a solution, but in which the required procedure has first to be extracted from within sentences (Orton 1992). According to Carpenter and Moser (1982), verbal problems are difficult for students of all ages. They need to master operations before solving the problem, specifically set out to embed the math's within a real life context. Again, as Gagne` (1983) suggested, in the process of mathematical word problem solving, the learner should be able to translate the concrete to the abstract, and the abstract to the concrete. In fact, translating verbally described situations into mathematical expressions is the main step that would be expected to achieve the solution.

In addition, previously learned knowledge, necessary rules, skills and concepts have to be combined in a new way by students.

The order of information, the relation between known and unknown and the transition from known to unknown all influence understanding of a word problem on younger learners (Laborde 1990; Orton 1992).

METHOD

Sample

The sample comprised 180 students from 3 guidance schools in a city in the Khorasan province in the north east of Iran. They were 13-year-old school girls having almost the same social and educational background.

Research instruments and procedures

Cognitive styles Measure: The independent variables were cognitive style and the position of a learner on each of the learning style dimensions (FD and FI) was determined using the Group Embedded Figures Test (GEFT) (Oltman, Raskin & Witkin 1971).

On the test subjects are required to disembed a simple figure in each complex figure.

There are 8 simple and 18 complex figures which make up the GEFT. Each of the simple figures is embedded in several different complex ones.

The students' cognitive styles were determined according to a criterion used by Scardamalia (1977), Case (1974) and Case & Golberson (1974). Students who had a score less than $\frac{1}{4}$ standard deviation (SD) below the mean were classified as field-dependent (FD) (i. e. FD < Mean – $\frac{1}{4}$ SD).

To create the category of field-independence, students had to score at least $\frac{1}{4}$ SD above the mean for the sample population (i.e. FI > Mean + $\frac{1}{4}$ SD) and between (Mean ± $\frac{1}{4}$ SD) were those who may be located between the above two styles (FD & FI) who were labelled as field-intermediate (FInt) learners. Table 1 shows the number of students in each of 3 dimensions (FD/FInt/FI) in this sample.

Table 1. The distribution of cognitive styles over the sample

Group	FD	FInt	FI
Total N = 180	N = 77 42.8%	N = 30 16.7%	N = 73 40.6%

Reliability for the (GEFT) has been estimated to be 0.84 (Oltman et al. 1971).

Mathematics Exams: The effectiveness of these cognitive styles were investigated by the students' performance in mathematical problem solving. Thus 2 math's exams were designed. The first one was a mathematical word problem (MWP) with 10 questions and the second one was an ordinary school class exam (CE) with 18 questions. The comparison between the two exam results was remarkable in this study. Here are two typical questions of (MWP) in this research:

1. The distance between cities A and B is 200 km, therefore it will take 5 hours to go from A to B on 40 km/h speed. How long will it take to reach city B on 50 km/h?
2. To produce 1 kg of an orange colour, red and yellow colours should be mixed on 3:2 proportion respectively. To make 100 kg orange colour, how much red and yellow ones are needed?

Data Analysis

The analysis of the data was carried out by considering the mean score as the dependent variable and by using parametric tests for the data for which the normality hypothesis was accepted (data from class exam) and non-parametric tests for the data for which the normality hypothesis rejected (data from mathematical word problem).

RESULTS

Normality assumption of the MWP data is strongly rejected (P-value = .0003) and is strongly accepted for CE data (P-value = .2). Thus a multivariate analysis based on normality assumption was not carried out (a transformation that would bring the MWP data to normality would take the CE data out of normality).

To maximize the effect of cognitive styles, only the results of FD group was compared to the FI style and the intermediate group was ignored.

Therefore, to test the hypothesis of this studies the performance of the students with cognitive styles (FI vs. FD) in both math's exams (MWP) and (CE) were investigated.

The mean scores and standard deviations (SD) in CE related to (FI/FD) group in the sample are set out in Table 2. In addition, Figure 1 exhibits the superiority of the students with different styles, based on their mean scores in CE.

According to t-test for independent samples of FI and FD on mean scores of the class exam (CE), a significant difference was found between two groups of styles at ($P < .0001$).

Table 2. Mean scores and SD in (CE) exam

Group	CE	
	Mean	SD
FI (n = 72)	14.68	3.61
FD (n = 76)	10.36	3.66

Moreover, based on Mann-Whitney non-parametric test, a significant difference was found in the MWP performance between two styles FI and FD. The results are given at

(P < .00001). Figure 2 exhibits the superiority of the FI group as compared to FD ones, based on their mean ranks on the MWP.

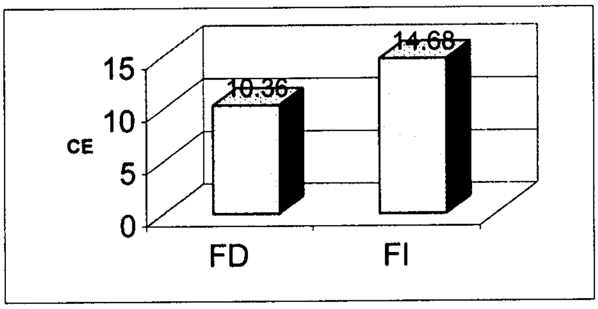

Figure 1. The students' achievements with different styles in the (CE) exam

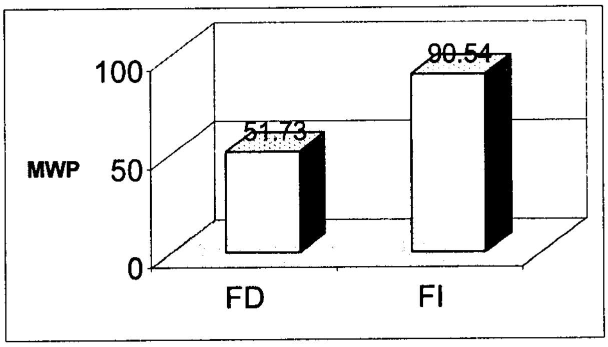

Figure 2. The students' achievements with different styles in the (MWP) exam

In addition, considering large number of pairs (69), a t-test for paired samples was used to show the performance of students with FI style in both exams CE and MWP. A significant difference was found between mean scores in each exam at (P <.0001).

According to the same test based on mean score of FD group in each math exam, a significant difference was found at (P < .0001).

Table 3 shows the results.

Table 3. Mean scores and SD in (MWP) and (CE) exams of FI/FD group

Group	MWP		CE		Number of pairs
	Mean	SD	Mean	SD	
FI	11.07	5.77	14.57	3.64	69
FD	5.38	4.66	10.22	3.62	71

Moreover, regardless of students' cognitive styles, their performance in MWP and CE was tested by t-test for paired samples. Again, a significant difference was found between mean scores in both exams at ($P < .0001$). The findings were shown in Table 4.

Table 4. Mean scores and SD in (MWP) and (CE) exams of all students

Number of pairs	MWP		CE	
	Mean	SD	Mean	SD
170	8.15	5.8	12.36	4.16

DISCUSSION

According to the students' performance in this study, the FI learners achieved signify-cantly higher results than FD learners in both math exams (MWP) and (CE). Therefore, this finding could support the hypothesis of this research. This means that being FI in thinking style may be more beneficial than being FD.

Moreover, Table 3 indicates that for students with an FI cognitive style, the mean score on the CE (Mean = 14.57) is higher than mean score on the MWP (Mean = 11.07).

The difference in means was found to be significant owing to $P < .0001$.

In addition, FD students performed higher result in the CE exam (Mean = 10.22) was compared to the MWP exam (Mean = 5.38, $P < .0001$).

On the other hand, regardless of students' cognitive styles their performance in both math exams, in Table 4, shows their higher results in CE (Mean = 12.36) as compared to MWP (Mean = 8.15). Again the difference in mean scores was found to be significant owing to $P < .0001$.

It seems from these results that the mathematical word problem exam (MWP) is a more distinctive and challenging task on the part of learners than the ordinary class exam (CE).

The results may support the previous research indications that students experience more troubles when handling the complexity of mathematical word problems. In fact, learners can show higher results in an ordinary math's exam based upon routine opera-tions, rules and skills, such as a class exam (CE) in this research.

CONCLUSIONS AND EDUCATIONAL IMPLICATIONS

The present study showed a positive relationship between cognitive styles (FI/FInt/FD), which are based upon individual differences and students' mathematical performance and could have implications for math's education. This confirms the previous studies in this area.

This research supports the findings of the previous studies that FI way of thinking may promote higher performance in math's problem solving compared with FD way.

Moreover, as was discussed, students with an FI cognitive style demonstrated higher results than FD ones in tackling the complexity of word problems, according to presented study. However, differences in performance in word problem exam were significantly higher than ordinary exam. This finding supports the previous research about male students (Ekbia & Alamolhodaei 2000).

In fact, study on the results obtained form exam sheets, along with students' mean scores may suggest that they experience more troubles in mathematical word problems than ordinary class exam.

That was due to the fact that ordinary class exam was based on algorithmic solution strategies that can be achieved by simply applying the operation currently being studied. It should be added that higher demands of thought steps required to find a solution in word problems may exhibit students difficulties than algorithmic problems. Students' mean scores in this regard confirms this assertion in the present study.

Harmon (1984) noted that perhaps the most important use of cognitive style is in alerting math's educators to the possibility that some learners may understand the mathematical concepts differently from the way the teacher does. Therefore, it is safe to suggest that, teaching styles and mathematical tasks should be planned to be adapted to the cognitive style (FI/FD) of learners, which requires that FD learners need to be assisted to overcome with mathematical word problems difficulties in conceptual and prcedural knowledege.

It is not however, a simple issue to explain students' difficulties with the complexity of mathematical word problems, so it is not easy to find safe ways of improving performance. The findings of previous studies have encouraged researchers to continue investigating in this area.

When the difficulties of word problems are being discussed to reduce the complexity of the task is to make language (verbal words) more comprehensible, thus rewording verbal problems so that the semantic relations may be made more explicit could be an important step to achieve the solution. As Johnstone and Cassels (1985) suggested, incautious use of language is a very real source of noise and misconceptions. On the other hand, FD learners have more difficulty than FI learners in separating signal from noise in a confusing field. Thus, it is reasonable to suggest that FD students could be low achieving learners in mathematical problem solving, in particular, word problems and they need to be assisted in performing suitable task and teaching methods.

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