

Visual Programming을 활용한 Fractal 집합의 작성

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On Constructing Fractal Sets Using Visual Programming Language

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요 약 이 논문에서는 맨델브로트집합의 개념을 n 차 복소 다항식 $z^n+c(c \in \mathbb{C}, n \geq 2)$ 에 확장하여 n 차 분기집합 및 줄리아 집합을 정의하고, 이 집합의 대칭성, 유계성 및 연결성 등에 관하여 이론적으로 연구하였다. 그 연구결과를 이용하여 n 차 분기 집합 및 줄리아 집합을 효율적으로 작성하는 알고리즘을 고안하고, C++ 컴퓨터 언어를 사용하여 마이크로소프트사의 윈도우 운영체제하에서 사용자가 마우스를 조작하여 n 차 분기집합 및 줄리아 집합을 구성할 수 있도록 소프트웨어 MANJUL을 개발하는 것이 본 논문의 목적이다. MANJUL 소프트웨어의 중요한 특징으로서 GUI(graphical user interfaces) 환경에서 단순한 마우스 조작을 통하여 n 차 분기집합 및 줄리아 집합을 작성하고 그 일부분을 확대함은 물론, n 차 분기집합 성분의 주기 등을 계산 및 저장함으로써, 이 집합들의 다양한 이론적 성질과 기하학적 구조를 시각적으로 확인할 수 있도록 하였다.

Abstract In this paper, we present a mathematical theory and algorithm constructing some fractal sets. Among such fractal sets, the degree- n bifurcation set as well as the Julia sets is defined by extending the concept of the Mandelbrot set to the complex polynomial $z^n+c(c \in \mathbb{C}, n \geq 2)$. Some properties of the degree- n bifurcation set and the Julia sets have been theoretically investigated including the symmetry, periodicity, boundedness, and connectedness. An efficient algorithm constructing both the degree- n bifurcation set and the Julia sets is proposed using theoretical results. The mouse-operated software called "MANJUL" has been developed for the effective construction of the degree- n bifurcation set and the Julia sets in graphic environments with C++ programming language under the windows operating system. Simple mouse operations can construct and magnify the degree- n bifurcation set as well as the Julia sets. They not only compute the component period but also save the images of the degree- n bifurcation set and the Julia sets to visually confirm various properties and the geometrical structure of the sets. A demonstration has verified the useful versatility of MANJUL.

Key Words : fractal set, visual programming, mandelbrot set, degree- n bifurcation set, Julia set

1. Introduction and Preliminary Studies

Definition 1 Let $P_c(z) = z^n + c$ for an integer $n \geq 2$, with $c, z \in \mathbb{C}$. Then the degree- n bifurcation set is defined to be the set

$$M = \{c \in \mathbb{C} \mid \lim_{k \rightarrow \infty} P_c^k(0) \neq \infty\}.$$

If $n = 2$, then M is called the Mandelbrot set [4-8].

Definition 2 The sets $P_m = \left\{ (r, \phi_m) : \phi_m = \frac{m\pi}{n-1}, r \geq 0 \right\}$

(for $m = 1, 2, \dots, 2n-2$) are called the rays of symmetry. The set P_1 is called the principal ray of sym-

metry and denoted by P . The set $S = \{(r, \theta) : 0 < \theta \leq \pi/(n-1), r \geq 0\}$ is called the principal sector.

The following theorem confirms the geometric symmetry of the degree- n bifurcation set with respect to rays of symmetry in the complex plane.

Theorem 1 M is symmetric in the c -parameter plane about P_m for all $m \in \{1, 2, \dots, 2n-2\}$.

Definition 3 The attracting period- k component [9] is defined to be the set $M'_k = \{c \in \mathbb{C} \mid \text{there exists } z_0$

such that $P_c^k(z_0) = z_0, \left| \frac{d}{dz} P_c^k(z) \right|_{z=z_0} < 1\}$. The

set M'_k is called the component with period- k attrac-

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tors.

In the following theorem, the escape criterion for the critical orbit under $P_c(z)$ will be described including closedness and connectedness.

Theorem 2 Let $n \geq 2$ be an integer. Then M satisfies the following:

- (a) $M = \{c \in \mathbb{C} \mid |P_c^k(0)| \leq 2^{1/(n-1)} \text{ for all } k \geq 1\}$
 $\subset \{c \in \mathbb{C} \mid |c| \leq 2^{1/(n-1)}\}$
- (b) M is a closed simply connected set.
- (c) $M \cap \mathbb{R} = \begin{cases} [-2^{1/(n-1)}, \rho], & \text{if } n \text{ is even} \\ [-\rho, \rho], & \text{if } n \text{ is odd} \end{cases}$,

$$\text{where } \rho = \left(1 - \frac{1}{n}\right) \left(\frac{1}{n}\right)^{1/(n-1)}.$$

Definition 4 The filled-in Julia set for $P_c(z) = z^n + c$ is defined to be the set

$$K_c = \{z \in \mathbb{C} \mid \lim_{k \rightarrow \infty} P_c^k(z) \neq \infty \text{ for fixed } c \in \mathbb{C}\}.$$

Theorem 3 K_c is periodic with respect to rays $\rho_j = 2\pi j/n$ ($j = 1, 2, \dots, n$) in the z -plane.

Theorem 4 Let $n \in \mathbb{N}$ with $n \geq 2$ and c satisfy $|c| \leq 2^{1/(n-1)}$. Then the following hold:

- (a) $K_c = \{z \in \mathbb{C} : |P_c^k(z)| \leq 2^{1/(n-1)} \text{ for all } k \in \mathbb{N}\}$
 $\subset \{z \in \mathbb{C} : |z| \leq 2^{1/(n-1)}\}$
- (b) K_c is closed and connected.
- (c) If $c \in M$ or equivalently if $0 \in K_c$, then K_c is connected.

2. Algorithm for Constructing the DNBS as well as the JULIA

Listed below is a sequence of developing instructions for an algorithm constructing the DNBS and the JULIA. Many useful properties are effectively used to device an efficient user-friendly software. Numerical methods implementing the following Algorithm 2.1 can be found in [2, 12, 13].

Algorithm 2.1

Step 1. Design the appearance of MANJUL whose display structure contains various controls and graphic contents constructing the DNBS as well as the

JULIA.

Step 2. Assign global variables that can be used in all other sub-programs.

Step 3. Develop the following sub-programs that play significant roles in constructing the DNBS/JULIA.

Step 4. Design the application window that contains the windows controls executing the construction software developed in Step 3.

Step 5. Associate sub-programs developed in Step 3 with the window controls designed in Step 4 to establish MANJUL by constructing the DNBS/JULIA.

Now we describe the outline of software MANJUL developed based on Algorithm 2.1. At the end of the outline, the demonstration of the software will be given for some specified parameters.

3. The Demonstration of MANJUL

The MANJUL program will be demonstrated for the following specified parameters:

$$\begin{aligned} n &= 3, \text{ BGCO} = 221, \text{ SIZE} = 300, \\ \text{ITER} &= 216, \text{ COLINT} = 1, \text{ BNDCO} = 0 \end{aligned}$$

The demonstration procedures are given below as a sequence of operations which are numbered consecutively.

- 1) The construction of the DNBS
 - (a) Execute the program WMANJUL.exe and follow the instructions.
 - (b) Click parameter values from the Data Input and Output Menus.
 - (c) Click "GO" button to construct the DNBS and obtain the result shown in Figure 2.
- 2) Magnifying a region of the DNBS
 - (a) Select a region while dragging the mouse. This draws a dotted rectangle. The coordinates for the selected region appear in one of the Computational Result Boxes.
 - (b) Click parameter values from the Data Input and Output Menus.
 - (c) Click "MANDEL" button to obtain the magnification shown in Figure 3.

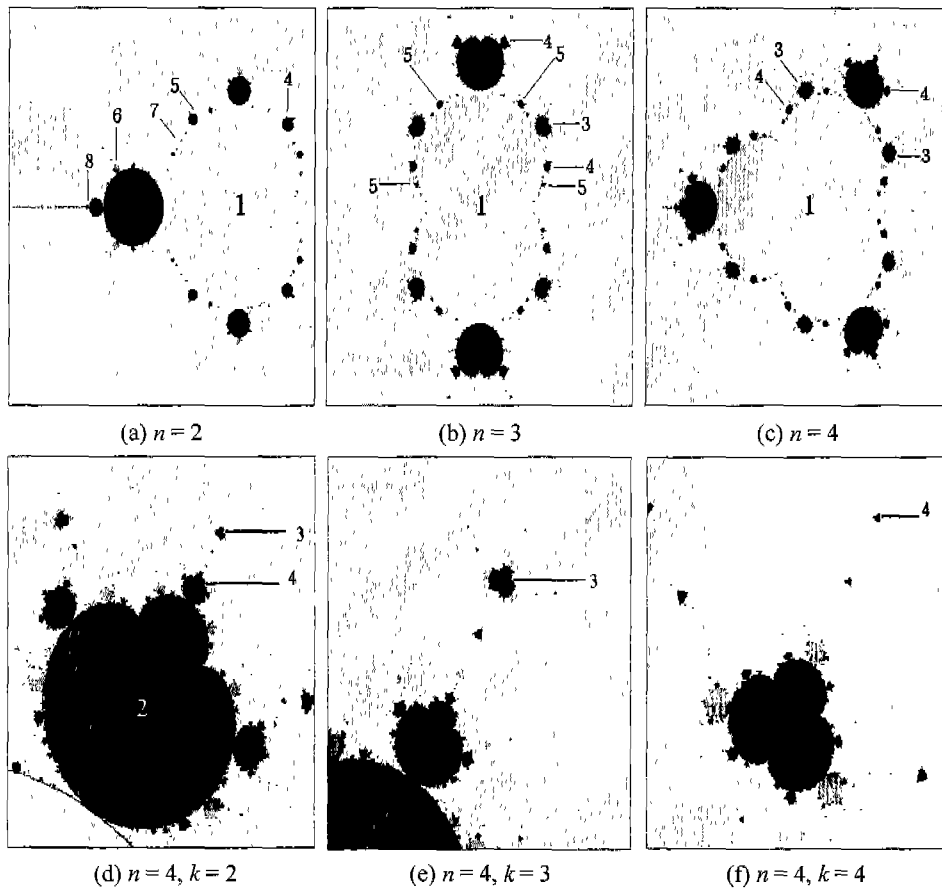


Figure 1. degree-2 bifurcation set.

The symmetry of the degree- n bifurcation set is clearly seen and guaranteed by Theorem 1. As expected from Theorem 2, the degree- n bifurcation set is shown to be simply connected. The following figure shows a magnification of a region (dashed square cut) of the degree- n bifurcation set. Although the magnification seems to suggest that disconnected components appear, they are in fact connected by thin filaments that are invisible at this resolution. See pp. 120-124, Devaney [5].

- 3) Finding the component center of the DNBS
 - (a) Click at an interior point of a component. This draws a cross line and gives the coordinates for the clicked point in one of the Computational Result Boxes.
 - (b) Click "FindCenter" in the top menu to give

the coordinates of the component center as shown in Figure 4.

The coordinates for the component center are computed via Newton's method using the governing equation. The result shows a relatively accurate values.

- 4) Finding the bifurcation point of the main component in the DNBS
 - (a) Click at an interior point of a component attached to the main component. This draws a cross line and gives the coordinates for the clicked point in one of the computational result boxes.
 - (b) Click "FindMainBifPt" in the top menu to produce the coordinates of the bifurcation

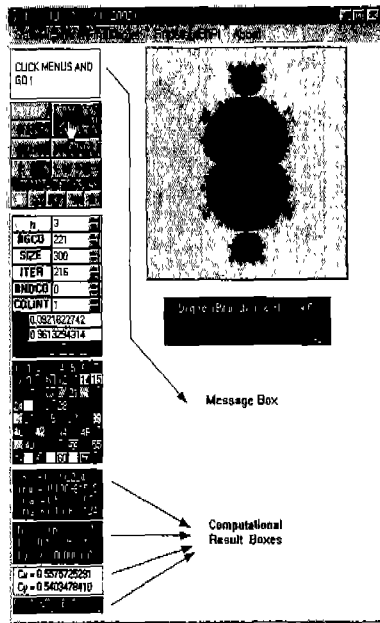


Figure 2. The Construction of the DNBS.

point as shown in Figure 5.

5) The construction of the JULIA

- (a) Click at an interior point ($c = 0.560068588 + 0.6244464726i$ in this example) of a component in the DNBS. This draws a cross line and gives the coordinates for the clicked boxes.
- (b) Click parameter values from the Data Input and Output Menu.
- (c) Click "JULIA" button to construct the JULIA set as shown in Figure 6.

The portion in solid black represents the filled-in

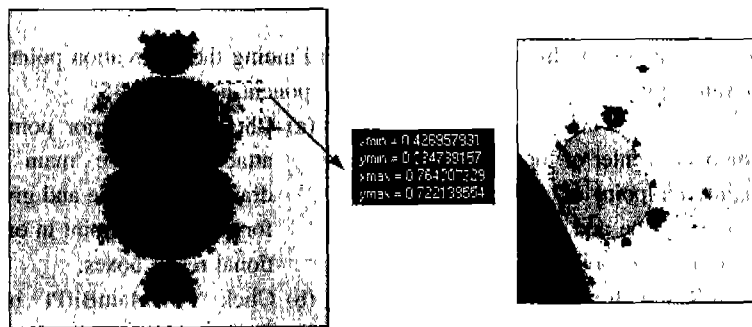


Figure 3. Magnifying a region of the DNBS.

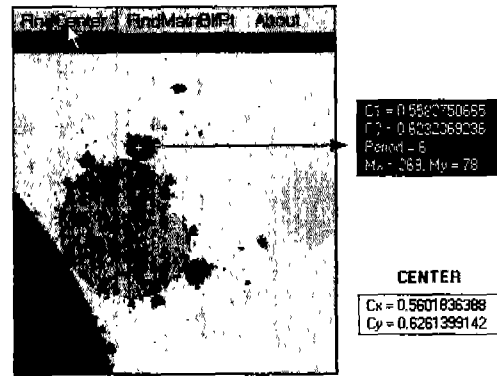


Figure 4. Finding the component center of the DNBS.

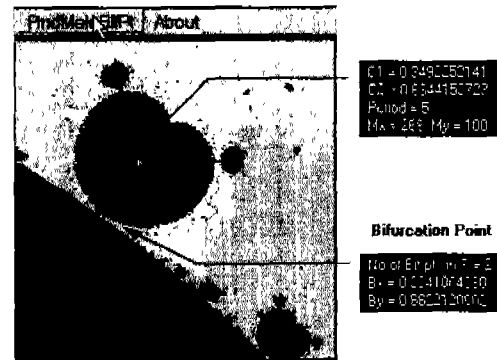


Figure 5. Finding the bifurcation point of the main component.

Julia set. In view of Theorem 4, it is clear that the filled-in Julia set is bounded and connected. Next we will show a magnification of a region (dashed square cut) for the filled-in Julia set.

6) Magnifying a region of the JULIA

- (a) Select a region ($0.3433734940 \leq x \leq$

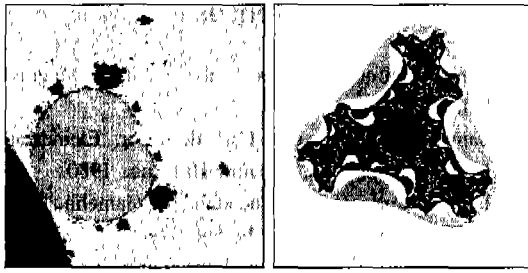


Figure 6. The construction of the JULIA.

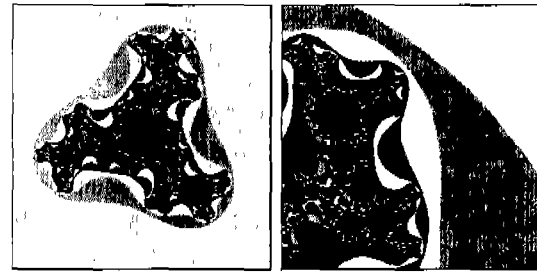


Figure 7. Magnifying a region of the JULIA.

$0.6084337349, 0.9698795181 \leq y \leq 1.23493997590$ in this example) while dragging the mouse. This draws a dotted rectangle. The coordinates for the selected region appear in the computational result box.

- (b) Click parameter values from the Data Input and Output Menu.
- (c) Click "JULIA" button to obtain the magnification shown in Figure 7.

4. Implementation of MANJUL and Conclusive Remarks

From the implementation of MANJUL, we list in this section some typical degree- n bifurcation sets and Julia sets and confirm their theoretical properties. The execution of MANJUL constructs the

following set of pictures that exhibit the degree-3 bifurcation set and its related filled-in Julia sets K_c .

Picture (a) shows the degree-3 bifurcation set with period-1 and period-5 components marked by arrow, (b) exhibits magnification of a region $0.0732261661 \leq c_1 \leq 0.0931618544, 0.7591973548 \leq c_2 \leq 0.7791330430$ which contains period-23, period-18 and period-36 components. Pictures (c)- (h) display filled-in Julia sets K_c at $c = 0.384900, c = 0.3849015, c = 0.3555602007 + 0.6803929766i, c = 0.078493455 + 0.765131389i, c = 0.0978740235 + 0.7911672942i, c = 0.1001496129 + 0.7906435896i$, respectively.

As expected from the theory stated in Section 1, picture (c) shows a connected set, while (d) a totally disconnected set. Pictures (e)-(h) show connected, compact subsets of C and show c -values taken from

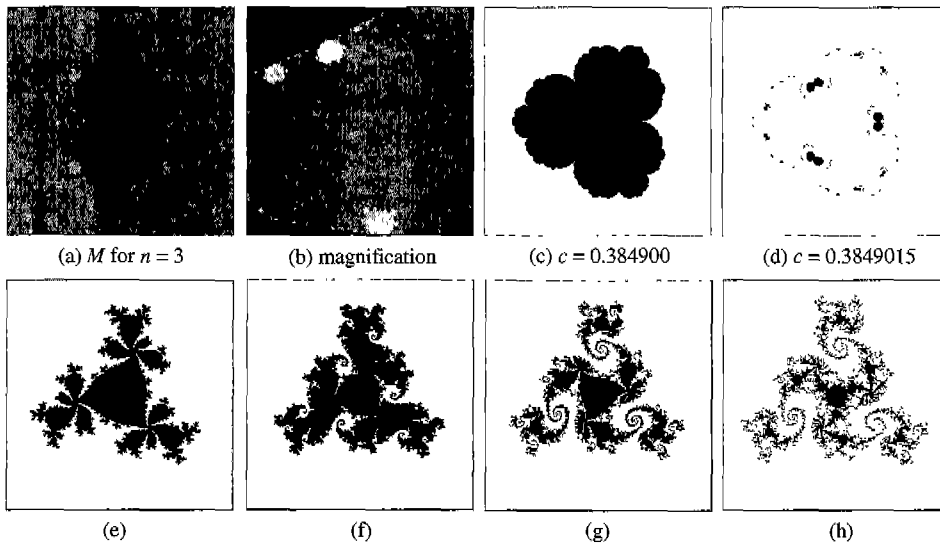


Figure 8. Typical Degree- n Bifurcation Sets and Julia Sets.

the attracting components of period 5, 23, 18 and 36, respectively. Note that Julia sets J_c are the boundaries of these sets.

The theoretical background including the symmetry and escape criteria described in Theorems 1 and 2 has significantly simplified the algorithm constructing the degree- n bifurcation set as well as the Julia sets. The theory for the component centers developed has reduced a great amount of numerical work in writing efficient programming codes in C++ language especially for higher values of n , although not many digits of accuracy is guaranteed as in Maple programming. The MANJUL software developed in this study has successfully constructed the degree- n bifurcation set as well as the Julia sets and has shown its great versatility. Its capability of magnification and computing bifurcation points and component centers will play significant roles in understanding the sophisticated structure of many other fractal sets which arise in the fields of applied and natural sciences. The graphical beauty observed from the magnification by the software might intrigue many graphic designers to view their objects in different aspects.

The graphical user interface and the convenience of MANJUL will be useful in observing and analyzing the complicated structures of many fractal sets arising in science fields. Although the source codes of MANJUL are not listed here, the program Wmanjul.exe that is executable under the windows operating system will be open to the public through the Internet system.

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