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$(\mathcal{T}_i, \mathcal{T}_j)$-FUZZY α-(r, s)-SEMIOPEN SETS AND FUZZY PAIRWISE α-(r, s)-SEMICONTINUOUS MAPPINGS

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ABSTRACT. We introduce and investigate the concepts of $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy α-(r, s)-semiopen (α-(r, s)-semiclosed) sets and fuzzy pairwise α-(r, s)-semicontinuous mappings in smooth bitopological spaces.

1. Introduction

The concept of fuzzy sets was introduced by Zadeh [12] in his classical paper. Using the concept of fuzzy sets Chang [2] introduced fuzzy topological spaces and several other authors continued the investigation of such spaces. Chattopadhyay et al. [4] and Ramadan [8] introduced new definition of smooth topological spaces as a generalization of fuzzy topological spaces. Kandil [6] introduced and studied the notion of fuzzy bitopological spaces as a natural generalization of fuzzy topological spaces. Lee et al. [7] introduced the concept of smooth bitopological spaces as a generalization of smooth topological spaces and Kandil's fuzzy bitopological spaces.

In this paper, we introduce the concepts of $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy α-(r, s)-semiopen sets and fuzzy pairwise α-(r, s)-semicontinuous mappings in smooth bitopological spaces and then we investigate some of their characteristic properties.

2. Preliminaries

Let I be the closed unit interval $[0, 1]$ of the real line $\mathbb{R}$ and let I_0 be the half open interval $(0, 1]$ of the real line $\mathbb{R}$. For a set X, I^X denotes

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the collection of all mapping from X to I. A member μ of I^X is called a fuzzy set of X. By $\tilde{0}$ and $\tilde{1}$ we denote constant mappings on X with value 0and 1, respectively. For any $\mu \in I^X$, μ^c denotes the complement $\tilde{1} - \mu$. All other notations are the standard notations of fuzzy set theory.

A *Chang's fuzzy topology* on X [2] is a family T of fuzzy sets in X which satisfies the following properties:

(1) $\tilde{0}, \tilde{1} \in T$.
(2) If $\mu_1, \mu_2 \in T$ then $\mu_1 \wedge \mu_2 \in T$.
(3) If $\mu_k \in T$ for all k, then $\bigvee \mu_k \in T$.

The pair (X, T) is called a *Chang's fuzzy topological space*. Members of T are called T-fuzzy open sets of X and their complements T-fuzzy closed sets of X.

A system (X, T_1, T_2) consisting of a set X with two Chang's fuzzy topologies T_1 and T_2 on X is called a *Kandil's fuzzy bitopological space*.

A *smooth topology* on X [4, 8] is a mapping $\mathcal{T} : I^X \to I$ which satisfies the following properties:

(1) $\mathcal{T}(\tilde{0}) = \mathcal{T}(\tilde{1}) = 1$.
(2) $\mathcal{T}(\mu_1 \wedge \mu_2) \geq \mathcal{T}(\mu_1) \wedge \mathcal{T}(\mu_2)$.
(3) $\mathcal{T}(\bigvee \mu_k) \geq \bigwedge \mathcal{T}(\mu_k)$.

The pair $(X, \mathcal{T})$ is called a *smooth topological space*. For $r \in I_0$, we call μ a $\mathcal{T}$-fuzzy r-open set of X if $\mathcal{T}(\mu) \geq r$ and μ a $\mathcal{T}$-fuzzy r-closed set of X if $\mathcal{T}(\mu^c) \geq r$.

A system $(X, \mathcal{T}_1, \mathcal{T}_2)$ consisting of a set X with two smooth topologies $\mathcal{T}_1$ and $\mathcal{T}_2$ on X is called a *smooth bitopological space*. Throughout this paper the indices i, j take values in $\{1, 2\}$ and $i \neq j$.

Let $(X, \mathcal{T})$ be a smooth topological space. Then it is easy to see that for each $r \in I_0$, an r-cut

$$\mathcal{T}_r = \{\mu \in I^X \mid \mathcal{T}(\mu) \geq r\}$$

is a Chang's fuzzy topology on X.

Let (X, T) be a Chang's fuzzy topological space and $r \in I_0$. Then the map $T^r : I^X \to I$ is defined by

$$T^r(\mu) = \begin{cases} 1 & \text{if } \mu = \tilde{0}, \tilde{1}, \\ r & \text{if } \mu \in T - \{\tilde{0}, \tilde{1}\}, \\ 0 & \text{otherwise} \end{cases}$$

becomes a smooth toplogy.

Hence, we obtain that if $(X, \mathcal{T}_1, \mathcal{T}_2)$ is a smooth bitopological space and $r, s \in I_0$, then $(X, (\mathcal{T}_1)_r, (\mathcal{T}_2)_s)$ is a Kandil's fuzzy bitopological space. Also, if (X, T_1, T_2) is a Kandil's fuzzy bitopological space and $r, s \in I_0$, then $(X, (T_1)^r, (T_2)^s)$ is a smooth bitopological space.

DEFINITION 2.1 ([7]). Let $(X, \mathcal{T})$ be a smooth topological space. For each $r \in I_0$ and for each $\mu \in I^X$, the *fuzzy r-closure* is defined by

$$\mathcal{T}\text{-Cl}(\mu, r) = \bigwedge\{\rho \in I^X \mid \mu \leq \rho, \mathcal{T}(\rho^c) \geq r\}$$

and the *fuzzy r-interior*

$$\mathcal{T}\text{-Int}(\mu, r) = \bigvee\{\rho \in I^X \mid \mu \geq \rho, \mathcal{T}(\rho) \geq r\}.$$

THEOREM 2.2 ([7]). *Let μ be a fuzzy set of a smooth topological space $(X, \mathcal{T})$ and let $r \in I_0$. Then we have:*

(1) $\mathcal{T}\text{-Int}(\mu, r)^c = \mathcal{T}\text{-Cl}(\mu^c, r)$.
(2) $\mathcal{T}\text{-Cl}(\mu, r)^c = \mathcal{T}\text{-Int}(\mu^c, r)$.

DEFINITION 2.3 ([7]). Let μ be a fuzzy set of a smooth bitopological space $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $r, s \in I_0$. Then μ is said to be

(1) a *$(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy (r,s)-semiopen* set if there is a $\mathcal{T}_i$-fuzzy r-open set ρ in X such that $\rho \leq \mu \leq \mathcal{T}_j\text{-Cl}(\rho, s)$,
(2) a *$(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy (r,s)-semiclosed* set if there is a $\mathcal{T}_i$-fuzzy r-closed set ρ in X such that $\mathcal{T}_j\text{-Int}(\rho, s) \leq \mu \leq \rho$,
(3) a *$(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy (r,s)-preopen* set if $\mu \leq \mathcal{T}_i\text{-Int}(\mathcal{T}_j\text{-Cl}(\mu, s), r)$,
(4) a *$(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy (r,s)-preclosed* set if $\mathcal{T}_i\text{-Cl}(\mathcal{T}_j\text{-Int}(\mu, s), r) \leq \mu$.

DEFINITION 2.4 ([7]). Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a smooth bitopological space X to another smooth bitopological space Y and $r, s \in I_0$. Then f is said to be

(1) a fuzzy pairwise (r,s)-continuous mapping if the induced mapping $f : (X, \mathcal{T}_1) \to (Y, \mathcal{U}_1)$ is a fuzzy r-continuous mapping and the induced mapping $f : (X, \mathcal{T}_2) \to (Y, \mathcal{U}_2)$ is a fuzzy s-continuous mapping,
(2) a *fuzzy pairwise (r,s)-semicontinuous* mapping if $f^{-1}(\mu)$ is a $(\mathcal{T}_1, \mathcal{T}_2)$-fuzzy (r,s)-semiopen set of X for each $\mathcal{U}_1$-fuzzy r-open set μ of Y and $f^{-1}(\nu)$ is a $(\mathcal{T}_2, \mathcal{T}_1)$-fuzzy (s,r)-semiopen set of X for each $\mathcal{U}_2$-fuzzy s-open set ν of Y,

(3) a *fuzzy pairwise* (r,s)*-precontinuous* mapping if $f^{-1}(\mu)$ is a $(\mathcal{T}_1,\mathcal{T}_2)$-fuzzy (r,s)-preopen set of X for each $\mathcal{U}_1$-fuzzy r-open set μ of Y and $f^{-1}(\nu)$ is a $(\mathcal{T}_2,\mathcal{T}_1)$-fuzzy (s,r)-preopen set of X for each $\mathcal{U}_2$-fuzzy s-open set ν of Y.

3. $(\mathcal{T}_i,\mathcal{T}_j)$-fuzzy α-(r,s)-semiopen sets

DEFINITION 3.1. Let μ be a fuzzy set of a smooth bitopological space $(X,\mathcal{T}_1,\mathcal{T}_2)$ and $r,s\in I_0$. Then μ is said to be

(1) a $(\mathcal{T}_i,\mathcal{T}_j)$*-fuzzy* α*-*(r,s)*-semiopen* set if there is a $\mathcal{T}_i$-fuzzy r-open set ρ in X such that $\rho\le\mu\le\mathcal{T}_i\text{-Int}(\mathcal{T}_j\text{-Cl}(\rho,s),r)$,

(2) a $(\mathcal{T}_i,\mathcal{T}_j)$*-fuzzy* α*-*(r,s)*-semiclosed* set if there is a $\mathcal{T}_i$-fuzzy r-closed set ρ in X such that $\mathcal{T}_i\text{-Cl}(\mathcal{T}_j\text{-Int}(\rho,s),r)\le\mu\le\rho$.

REMARK 3.2. It is clear that every $\mathcal{T}_i$-fuzzy r-open set is a $(\mathcal{T}_i,\mathcal{T}_j)$-fuzzy α-(r,s)-semiopen set and every $(\mathcal{T}_i,\mathcal{T}_j)$-fuzzy α-(r,s)-semiopen set is not only a $(\mathcal{T}_i,\mathcal{T}_j)$-fuzzy (r,s)-semiopen set but also a $(\mathcal{T}_i,\mathcal{T}_j)$-fuzzy (r,s)-preopen set. However, the following examples show that all of the converses need not be true.

EXAMPLE 3.3. Let $X=\{x,y\}$ and μ_1, μ_2, μ_3 and μ_4 be fuzzy sets of X defined as

$$\mu_1(x)=0.3,\quad \mu_1(y)=0.4;$$

$$\mu_2(x)=0.8,\quad \mu_2(y)=0.2;$$

$$\mu_3(x)=0.6,\quad \mu_3(y)=0.9;$$

and

$$\mu_4(x)=0.9,\quad \mu_4(y)=0.4.$$

Define $\mathcal{T}_1:I^X\to I$ and $\mathcal{T}_2:I^X\to I$ by

$$\mathcal{T}_1(\mu)=\begin{cases}1 & \text{if } \mu=\tilde{0},\tilde{1},\\ \frac{1}{2} & \text{if } \mu=\mu_1,\\ 0 & \text{otherwise;}\end{cases}$$

and

$$\mathcal{T}_2(\mu)=\begin{cases}1 & \text{if } \mu=\tilde{0},\tilde{1},\\ \frac{1}{3} & \text{if } \mu=\mu_2,\\ 0 & \text{otherwise.}\end{cases}$$

Then clearly $(\mathcal{T}_1, \mathcal{T}_2)$ is a smooth bitopology on X. The fuzzy set μ_3 is $(\mathcal{T}_1, \mathcal{T}_2)$-fuzzy α-$(\frac{1}{2}, \frac{1}{3})$-semiopen set which is not a $\mathcal{T}_1$-fuzzy $\frac{1}{2}$-open set. Also μ_4 is a $(\mathcal{T}_2, \mathcal{T}_1)$-fuzzy α-$(\frac{1}{3}, \frac{1}{2})$-semiopen set which is not a $\mathcal{T}_2$-fuzzy $\frac{1}{3}$-open set.

EXAMPLE 3.4. Let $X = \{x, y\}$ and μ_1, μ_2, μ_3, μ_4, μ_5 and μ_6 be fuzzy sets of X defined as

$$\mu_1(x) = 0.1, \quad \mu_1(y) = 0.7;$$

$$\mu_2(x) = 0.8, \quad \mu_2(y) = 0.2;$$

$$\mu_3(x) = 0, \quad \mu_3(y) = 0.6;$$

$$\mu_4(x) = 0.1, \quad \mu_4(y) = 0.8;$$

$$\mu_5(x) = 0.5, \quad \mu_5(y) = 0.6;$$

and

$$\mu_6(x) = 0.9, \quad \mu_6(y) = 0.2.$$

Define $\mathcal{T}_1 : I^X \to I$ and $\mathcal{T}_2 : I^X \to I$ by

$$\mathcal{T}_1(\mu) = \begin{cases} 1 & \text{if } \mu = \tilde{0}, \tilde{1}, \\ \frac{1}{2} & \text{if } \mu = \mu_1, \\ 0 & \text{otherwise;} \end{cases}$$

and

$$\mathcal{T}_2(\mu) = \begin{cases} 1 & \text{if } \mu = \tilde{0}, \tilde{1}, \\ \frac{1}{3} & \text{if } \mu = \mu_2, \\ 0 & \text{otherwise.} \end{cases}$$

Then clearly $(\mathcal{T}_1, \mathcal{T}_2)$ is a smooth bitopology on X. The fuzzy set μ_3 is a $(\mathcal{T}_1, \mathcal{T}_2)$-fuzzy $(\frac{1}{2}, \frac{1}{3})$-preopen set which is not a $(\mathcal{T}_1, \mathcal{T}_2)$-fuzzy α-$(\frac{1}{2}, \frac{1}{3})$-semiopen set and μ_4 is a $(\mathcal{T}_1, \mathcal{T}_2)$-fuzzy $(\frac{1}{2}, \frac{1}{3})$-semiopen set which is not a $(\mathcal{T}_1, \mathcal{T}_2)$-fuzzy α-$(\frac{1}{2}, \frac{1}{3})$-semiopen set. Also μ_5 is a $(\mathcal{T}_2, \mathcal{T}_1)$-fuzzy $(\frac{1}{3}, \frac{1}{2})$-preopen set which is not a $(\mathcal{T}_2, \mathcal{T}_1)$-fuzzy α-$(\frac{1}{3}, \frac{1}{2})$-semiopen set and μ_6 is a $(\mathcal{T}_2, \mathcal{T}_1)$-fuzzy $(\frac{1}{3}, \frac{1}{2})$-semiopen set which is not a $(\mathcal{T}_2, \mathcal{T}_1)$-fuzzy α-$(\frac{1}{3}, \frac{1}{2})$-semiopen set.

THEOREM 3.5. *Let μ be a fuzzy set of a smooth bitopological space $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $r, s \in I_0$. Then the following statements are equivalent:*

(1) *μ is a $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy α-(r, s)-semiopen set.*
(2) *μ^c is a $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy α-(r, s)-semiclosed set.*
(3) $\mu \leq \mathcal{T}_i\text{-Int}(\mathcal{T}_j\text{-Cl}(\mathcal{T}_i\text{-Int}(\mu, r), s), r)$.
(4) $\mu^c \geq \mathcal{T}_i\text{-Cl}(\mathcal{T}_j\text{-Int}(\mathcal{T}_i\text{-Cl}(\mu^c, r), s), r)$.
(5) *μ is a $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy (r, s)-semiopen and $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy (r, s)- preopen set.*
(6) *μ^c is a $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy (r, s)-semiclosed and $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy (r, s)-preclosed set.*

Proof. (1) $\Leftrightarrow$ (2), (3) $\Leftrightarrow$ (4) and (5) $\Leftrightarrow$ (6) follow from Theorem 2.2.

(1) $\Rightarrow$ (3) Let μ be a $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy α-(r, s)-semiopen set of X. Then there is a $\mathcal{T}_i$-fuzzy r-open ρ in X such that $\rho \leq \mu \leq \mathcal{T}_i\text{-Int}(\mathcal{T}_j\text{-Cl}(\rho, s), r)$. Since $\mathcal{T}_i(\rho) \geq r$ and $\rho \leq \mu$, we have $\rho = \mathcal{T}_i\text{-Int}(\rho, r) \leq \mathcal{T}_i\text{-Int}(\mu, r)$. Thus

$$\mu \leq \mathcal{T}_i\text{-Int}(\mathcal{T}_j\text{-Cl}(\rho, s), r) \leq \mathcal{T}_i\text{-Int}(\mathcal{T}_j\text{-Cl}(\mathcal{T}_i\text{-Int}(\mu, r), s), r).$$

(3) $\Rightarrow$ (1) Let $\mathcal{T}_i\text{-Int}(\mathcal{T}_j\text{-Cl}(\mathcal{T}_i\text{-Int}(\mu, r), s), r) \geq \mu$ and take $\rho = \mathcal{T}_i\text{-Int}(\mu, r)$. Then ρ is a $\mathcal{T}_i$-fuzzy r-open set. Also

$$\begin{aligned}\rho = \mathcal{T}_i\text{-Int}(\mu, r) \leq \mu &\leq \mathcal{T}_i\text{-Int}(\mathcal{T}_j\text{-Cl}(\mathcal{T}_i\text{-Int}(\mu, r), s), r) \\ &= \mathcal{T}_i\text{-Int}(\mathcal{T}_j\text{-Cl}(\rho, s), r).\end{aligned}$$

Hence μ is a $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy α-(r, s)-semiopen set.

(1) $\Rightarrow$ (5) It is obvious.

(5) $\Rightarrow$ (3) Let μ be a $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy (r, s)-semiopen and $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy (r, s)-preopen set of X. Then $\mu \leq \mathcal{T}_j\text{-Cl}(\mathcal{T}_i\text{-Int}(\mu, r), s)$ and $\mu \leq \mathcal{T}_i\text{-Int}(\mathcal{T}_j\text{-Cl}(\mu, s), r)$. Therefore,

$$\begin{aligned}\mu \leq \mathcal{T}_i\text{-Int}(\mathcal{T}_j\text{-Cl}(\mu, s), r) &\leq \mathcal{T}_i\text{-Int}(\mathcal{T}_j\text{-Cl}(\mathcal{T}_j\text{-Cl}(\mathcal{T}_i\text{-Int}(\mu, r), s), s), r) \\ &= \mathcal{T}_i\text{-Int}(\mathcal{T}_j\text{-Cl}(\mathcal{T}_i\text{-Int}(\mu, r), s), r).\end{aligned}$$

This completes the proof. □

THEOREM 3.6. (1) *Any union of $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy α-(r, s)-semiopen sets is a $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy α-(r, s)-semiopen set.*

(2) *Any intersection of $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy α-(r, s)-semiclosed sets is a $(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy α-(r, s)-semiclosed set.*

Proof. (1) Let $\{\mu_i\}$ be a collection of $(\mathcal{T}_i,\mathcal{T}_j)$-fuzzy α-(r,s)-semiopen sets. Then for each i, there is a $\mathcal{T}_i$-fuzzy r-open set ρ_i such that $\rho_i \le \mu_i \le \mathcal{T}_i\text{-Int}(\mathcal{T}_j\text{-Cl}(\rho,s),r)$. Since $\mathcal{T}_i(\bigvee \rho_i) \ge \bigwedge \mathcal{T}_i(\rho_i) \ge r$, $\bigvee \rho_i$ is a $\mathcal{T}_i$-fuzzy r-open set. Also

$$\begin{aligned}\bigvee \rho_i \le \bigvee \mu_i &\le \bigvee \mathcal{T}_i\text{-Int}(\mathcal{T}_j\text{-Cl}(\rho_i,s),r)\\ &\le \mathcal{T}_i\text{-Int}(\mathcal{T}_j\text{-Cl}(\bigvee \rho_i,s),r).\end{aligned}$$

Thus $\bigvee \mu_i$ is a $(\mathcal{T}_i,\mathcal{T}_j)$-fuzzy α-(r,s)-semiopen set.

(2) It follows from (1) using Theorem 3.5. □

4. Fuzzy pairwise α-(r,s)-semicontinuous mappings

DEFINITION 4.1. Let $f:(X,\mathcal{T}_1,\mathcal{T}_2)\to(Y,\mathcal{U}_1,\mathcal{U}_2)$ be a mapping from a smooth bitopological space X to another smooth bitopological space Y and $r,s\in I_0$. Then f is called a *fuzzy pairwise α-(r,s)-semicontinuous* mapping if $f^{-1}(\mu)$ is a $(\mathcal{T}_1,\mathcal{T}_2)$-fuzzy α-(r,s)-semiopen set of X for each $\mathcal{U}_1$-fuzzy r-open set μ of Y and $f^{-1}(\nu)$ is a $(\mathcal{T}_2,\mathcal{T}_1)$-fuzzy α-(s,r)-semiopen set of X for each $\mathcal{U}_2$-fuzzy s-open set ν of Y.

REMARK 4.2. It is clear that every fuzzy pairwise (r,s)-continuous mapping is also a fuzzy pairwise α-(r,s)-semicontinuous mapping and every fuzzy pairwise α-(r,s)-semicontinuous mapping is not only a fuzzy pairwise (r,s)-semicontinuous mapping but also a fuzzy pairwise (r,s)-precontinuous mapping. However, the following examples show that all of the converses need not be true.

EXAMPLE 4.3. Let $(X,\mathcal{T}_1,\mathcal{T}_2)$ be a smooth bitopological space as described in Example 3.3. Define $\mathcal{U}_1: I^X\to I$ and $\mathcal{U}_2: I^X\to I$ by

$$\mathcal{U}_1(\mu)=\begin{cases}1 & \text{if } \mu=\tilde{0},\tilde{1},\\ \frac{1}{2} & \text{if } \mu=\mu_3,\\ 0 & \text{otherwise;}\end{cases}$$

and

$$\mathcal{U}_2(\mu)=\begin{cases}1 & \text{if } \mu=\tilde{0},\tilde{1},\\ 0 & \text{otherwise.}\end{cases}$$

Then clearly $(\mathcal{U}_1,\mathcal{U}_2)$ is a smooth bitopology on X. Consider the identity mapping $1_X:(X,\mathcal{T}_1,\mathcal{T}_2)\to(X,\mathcal{U}_1,\mathcal{U}_2)$. Then it is a fuzzy pairwise α-$(\frac{1}{2},\frac{1}{3})$-semicontinuous mapping which is not a fuzzy pairwise $(\frac{1}{2},\frac{1}{3})$-continuous mapping.

EXAMPLE 4.4. Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be a smooth bitopological space as described in Example 3.4. Define $\mathcal{U}_1 : I^X \to I$ and $\mathcal{U}_2 : I^X \to I$ by

$$\mathcal{U}_1(\mu) = \begin{cases} 1 & \text{if } \mu = \tilde{0}, \tilde{1}, \\ \frac{1}{2} & \text{if } \mu = \mu_3, \\ 0 & \text{otherwise;} \end{cases}$$

and

$$\mathcal{U}_2(\mu) = \begin{cases} 1 & \text{if } \mu = \tilde{0}, \tilde{1}, \\ 0 & \text{otherwise.} \end{cases}$$

Then clearly $(\mathcal{U}_1, \mathcal{U}_2)$ is a smooth bitopology on X. Consider the identity mapping $1_X : (X, \mathcal{T}_1, \mathcal{T}_2) \to (X, \mathcal{U}_1, \mathcal{U}_2)$. Then it is a fuzzy pairwise $(\frac{1}{2}, \frac{1}{3})$-precontinuous mapping which is not a fuzzy pairwise α-$(\frac{1}{2}, \frac{1}{3})$-continuous mapping.

Define $\mathcal{V}_1 : I^X \to I$ and $\mathcal{V}_2 : I^X \to I$ by

$$\mathcal{V}_1(\mu) = \begin{cases} 1 & \text{if } \mu = \tilde{0}, \tilde{1}, \\ \frac{1}{2} & \text{if } \mu = \mu_4, \\ 0 & \text{otherwise;} \end{cases}$$

and

$$\mathcal{V}_2(\mu) = \begin{cases} 1 & \text{if } \mu = \tilde{0}, \tilde{1}, \\ 0 & \text{otherwise.} \end{cases}$$

Then clearly $(\mathcal{V}_1, \mathcal{V}_2)$ is a smooth bitopology on X. Consider the identity mapping $1_X : (X, \mathcal{T}_1, \mathcal{T}_2) \to (X, \mathcal{V}_1, \mathcal{V}_2)$. Then it is a fuzzy pairwise $(\frac{1}{2}, \frac{1}{3})$-semicontinuous mapping which is not a fuzzy pairwise α-$(\frac{1}{2}, \frac{1}{3})$-continuous mapping.

THEOREM 4.5. *Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping and $r, s \in I_0$. Then the following statements are equivalent:*

(1) *f is a fuzzy pairwise α-(r, s)-semicontinuous mapping.*

(2) *$f^{-1}(\mu)$ is a $(\mathcal{T}_1, \mathcal{T}_2)$-fuzzy α-(r, s)-semiclosed set of X for each $\mathcal{U}_1$-fuzzy r-closed set μ of Y and $f^{-1}(\nu)$ is a $(\mathcal{T}_2, \mathcal{T}_1)$-fuzzy α-(s, r)-semiclosed set of X for each $\mathcal{U}_2$-fuzzy s-closed set ν of Y.*

(3) *For each fuzzy set μ of Y,*

$$\mathcal{T}_1\text{-Cl}(\mathcal{T}_2\text{-Int}(\mathcal{T}_1\text{-Cl}(f^{-1}(\mu), r), s), r) \le f^{-1}(\mathcal{U}_1\text{-Cl}(\mu, r))$$

and

$$\mathcal{T}_2\text{-Cl}(\mathcal{T}_1\text{-Int}(\mathcal{T}_2\text{-Cl}(f^{-1}(\mu), s), r), s) \le f^{-1}(\mathcal{U}_2\text{-Cl}(\mu, s)).$$

(4) *For each fuzzy set* ρ *of* X,

$$f(\mathcal{T}_1\text{-Cl}(\mathcal{T}_2\text{-Int}(\mathcal{T}_1\text{-Cl}(\rho,r),s),r)) \leq \mathcal{U}_1\text{-Cl}(f(\rho),r)$$

and

$$f(\mathcal{T}_2\text{-Cl}(\mathcal{T}_1\text{-Int}(\mathcal{T}_2\text{-Cl}(\rho,s),r),s)) \leq \mathcal{U}_2\text{-Cl}(f(\rho),s).$$

Proof. (1) $\Leftrightarrow$ (2) It follwos from Theorem 3.5.

(2) $\Rightarrow$ (3) Let μ be any fuzzy set of Y. Then $\mathcal{U}_1\text{-Cl}(\mu,r)$ is a $\mathcal{U}_1$-fuzzy r-closed set and $\mathcal{U}_2\text{-Cl}(\mu,s)$ is a $\mathcal{U}_2$-fuzzy s-closed set of Y. By (2), $f^{-1}(\mathcal{U}_1\text{-Cl}(\mu,r))$ is a $(\mathcal{T}_1,\mathcal{T}_2)$-fuzzy α-(r,s)-semiclosed set and $f^{-1}(\mathcal{U}_2\text{-Cl}$ $(\mu,s))$ is a $(\mathcal{T}_2,\mathcal{T}_1)$-fuzzy α-(s,r)-semiclosed set of X. Thus

$$\begin{aligned} f^{-1}(\mathcal{U}_1\text{-Cl}(\mu,r)) &\geq \mathcal{T}_1\text{-Cl}(\mathcal{T}_2\text{-Int}(\mathcal{T}_1\text{-Cl}(f^{-1}(\mathcal{U}_1\text{-Cl}(\mu,r)),r),s),r) \\ &\geq \mathcal{T}_1\text{-Cl}(\mathcal{T}_2\text{-Int}(\mathcal{T}_1\text{-Cl}(f^{-1}(\mu),r),s),r) \end{aligned}$$

and

$$\begin{aligned} f^{-1}(\mathcal{U}_2\text{-Cl}(\mu,s)) &\geq \mathcal{T}_2\text{-Cl}(\mathcal{T}_1\text{-Int}(\mathcal{T}_2\text{-Cl}(f^{-1}(\mathcal{U}_2\text{-Cl}(\mu,s)),s),r),s) \\ &\geq \mathcal{T}_2\text{-Cl}(\mathcal{T}_1\text{-Int}(\mathcal{T}_2\text{-Cl}(f^{-1}(\mu),s),r),s). \end{aligned}$$

(3) $\Rightarrow$ (4) Let ρ be any fuzzy set of X. Then $f(\rho)$ is a fuzzy set of Y. By (3),

$$\begin{aligned} f^{-1}(\mathcal{U}_1\text{-Cl}(f(\rho),r)) &\geq \mathcal{T}_1\text{-Cl}(\mathcal{T}_2\text{-Int}(\mathcal{T}_1\text{-Cl}(f^{-1}f(\rho),r),s),r) \\ &\geq \mathcal{T}_1\text{-Cl}(\mathcal{T}_2\text{-Int}(\mathcal{T}_1\text{-Cl}(\rho,r),s),r) \end{aligned}$$

and

$$\begin{aligned} f^{-1}(\mathcal{U}_2\text{-Cl}(f(\rho),s)) &\geq \mathcal{T}_2\text{-Cl}(\mathcal{T}_1\text{-Int}(\mathcal{T}_2\text{-Cl}(f^{-1}f(\rho),s),r),s) \\ &\geq \mathcal{T}_2\text{-Cl}(\mathcal{T}_1\text{-Int}(\mathcal{T}_2\text{-Cl}(\rho,s),r),s). \end{aligned}$$

Hence

$$\begin{aligned} \mathcal{U}_1\text{-Cl}(f(\rho),r) &\geq ff^{-1}(\mathcal{U}_1\text{-Cl}(f(\rho),r)) \\ &\geq f(\mathcal{T}_1\text{-Cl}(\mathcal{T}_2\text{-Int}(\mathcal{T}_1\text{-Cl}(\rho,r),s),r)) \end{aligned}$$

and

$$\begin{aligned}\mathcal{U}_2\text{-Cl}(f(\rho), s) &\geq ff^{-1}(\mathcal{U}_2\text{-Cl}(f(\rho), s)) \\ &\geq f(\mathcal{T}_2\text{-Cl}(\mathcal{T}_1\text{-Int}(\mathcal{T}_2\text{-Cl}(\rho, s), r), s)).\end{aligned}$$

(4) $\Rightarrow$ (2) Let μ be any $\mathcal{U}_1$-fuzzy r-closed set and ν any $\mathcal{U}_2$-fuzzy s-closed set of Y. Then $f^{-1}(\mu)$ and $f^{-1}(\nu)$ are fuzzy sets of X. By (4),

$$\begin{aligned}f(\mathcal{T}_1\text{-Cl}(\mathcal{T}_2\text{-Int}(\mathcal{T}_1\text{-Cl}(f^{-1}(\mu), r), s), r)) &\leq \mathcal{U}_1\text{-Cl}(ff^{-1}(\mu), r) \\ &\leq \mathcal{U}_1\text{-Cl}(\mu, r) = \mu\end{aligned}$$

and

$$\begin{aligned}f(\mathcal{T}_2\text{-Cl}(\mathcal{T}_1\text{-Int}(\mathcal{T}_2\text{-Cl}(f^{-1}(\nu), s), r), s)) &\leq \mathcal{U}_2\text{-Cl}(ff^{-1}(\nu), s) \\ &\leq \mathcal{U}_2\text{-Cl}(\nu, s) = \nu.\end{aligned}$$

So

$$\begin{aligned}&\mathcal{T}_1\text{-Cl}(\mathcal{T}_2\text{-Int}(\mathcal{T}_1\text{-Cl}(f^{-1}(\mu), r), s), r) \\ &\quad \leq f^{-1}f(\mathcal{T}_1\text{-Cl}(\mathcal{T}_2\text{-Int}(\mathcal{T}_1\text{-Cl}(f^{-1}(\mu), r), s), r)) \\ &\quad \leq f^{-1}(\mu)\end{aligned}$$

and

$$\begin{aligned}&\mathcal{T}_2\text{-Cl}(\mathcal{T}_1\text{-Int}(\mathcal{T}_2\text{-Cl}(f^{-1}(\nu), s), r), s) \\ &\quad \leq f^{-1}f(\mathcal{T}_2\text{-Cl}(\mathcal{T}_1\text{-Int}(\mathcal{T}_2\text{-Cl}(f^{-1}(\nu), s), r), s)) \\ &\quad \leq f^{-1}(\nu).\end{aligned}$$

Thus $f^{-1}(\mu)$ is a $(\mathcal{T}_1, \mathcal{T}_2)$-fuzzy α-(r, s)-semiclosed set and $f^{-1}(\nu)$ is a $(\mathcal{T}_2, \mathcal{T}_1)$-fuzzy α-(s, r)-semiclosed set of X. $\square$

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