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Analysis of Strategies for Problem Solving Presented in Elementary School Mathematics Textbooks

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I. Introduction

Due to the rapid pace of change in today's world, no education can prepare children for solving all problems in advance. Schools need to arm students with general problem solving skills(Baroody, 1993), and schools of tomorrow will need to put much more emphasis on problem solving than before(Lindquist, 1989; NCTM, 1980, 1989; Schmalz, 1994). If the purpose of teaching problem solving is to improve students' mathematical thinking, we must ask why problem solving lessons are still being presented in ways that encourage children to look for individual words or phrases as the key to selecting the proper operation(Davis & McKillip, 1980). After such instruction, first graders who had previously utilized a variety of strategies to solve addition and multiplication, tended to constrain themselves to strategies taught in such class(Carpenter, 1979). Nibbelink et. al.(1987), after an analysis on problems in the textbooks used from 1950 to 1987, concluded that textbooks now going into service do at least provide work (practice) for children similar to that provided in the late 1950s and early 1960s and in greater quantity than ever. Such approaches seem ineffective for teaching complicated thinking processes mathematics requires. Therefore, we might need a new curriculum reflecting the development of students' considerable and often surprising potential for problem solving.

Certain considerations, if we are to achieve such a goal, arise out of research, and should be kept in mind by curriculum developers, especially when creating textbooks for elementary school students. One consideration is that "at what age should problem solving strategies be taught to children?" Many educators have concluded that it is advantageous to teach problem solving to children as early as possible, possibly as early as kindergarten (Charles, et. al. 1987; Lester, 1994).

The earlier the teaching of problem solving starts, the better for students. Strategies for problem solving should not be introduced to children suddenly, in the middle grades, but rather be taught as soon as children start school. This is because such strategies should not be isolated from students' existing knowledge gained from experiences before starting school(Carpenter & Fennama, 1991; Carpenter, Hiebert & Moser, 1983; Carpenter & Moser, 1982). Also important to recognize is that strategies taught in school not

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be separated from the potent strategies students already know so that students feel confident that they can learn, much as they learn things in everyday life. In addition, children typically invent new strategies while getting acquainted with the obstacles presented by a given problem. By solving several problems similar to the given one, they further develop and refine their rough strategy into a more powerful one until it becomes fully internalized and results in a solution.

Therefore, strategies for problem solving could well be taught in first grade. Yet, only recently did textbooks of elementary school mathematics begin to include chapters for teaching strategies for problem solving in grade 1 in South Korea and in the U.S. In this paper, I will discuss the effectiveness of the following textbook styles for problem solving taught at the elementary level:

(1) One chapter entirely devoted to problem solving;
(2) One problem solving section in each chapter;
(3) Problems designed to enhance problem solving interspersed throughout every chapter of the textbooks.

Then, analysis of elementary school mathematics textbooks used in South Korea based on 6th national mathematics curriculum and in the U.S. will be made for the purpose of comparison of the models above with that used presently in the textbooks. Based on the findings from the comparison, we can get some implications of how to present chapters for teaching strategies for solving problems.

II. Means of Presenting Strategies for Solving Mathematical Problems

What is to be taught to students in order to help them not only to solve mathematical problems in and out of school but also to be able to think mathematically? One response to the question by many researchers is: teach students strategies for solving a variety of problems. Teaching how to solve problems has been shown to improve students' ability to solve them(Broom & Broader, 1950; Brophy & Good, 1986; Shavelson, et. al., 1989). If so, because many teachers consider the textbook as a basic resource for introducing the strategies to their students, the question of how to allocate space for problem solving in a textbook arises. Inasmuch as many teachers have relied heavily on mathematics textbooks in teaching mathematics(Bell, 1978), and considering that most students learn what is contained in a textbook(Begel, 1973), and that most important factor in determining what mathematics is taught is the textbook used(Willoughby, 1984), the decision is an important one which merits debate.

The issue of how to teach problem solving strategies most effectively remains a central part of textbook design; to this end, one must contemplate strategic placement of problem solving discussions. The first mode is to assign a chapter for problem solving in the textbook. In this way, some strategies necessary and appropriate for each grade will be selected and then some problems for each strategy are posed in a section devoted to each strategy.

The merit of this method is that many strategies can be covered in a single chapter so that students can be expected to learn a number of approaches to solving problems. However, this method has some flaws. First, treating strategies for solving problems as an independent topic gives students the view that problem solving skills are yet another piece of knowledge to acquire, not an opportunity to acquire thinking strategies. Second, students have only a limited time to learn strategies for problem solving while studying mathematics for an entire year. Students need many opportunities to use problem solving strategies, to remember each, and to learn to use them when they need to use them. Third, strategies introduced in a single chapter tend to be very much unrelated to each other and hence, the curriculum of introducing problem solving strategies is disconnected as a whole. The lack of any opportunity for the student to develop a gradually more sophisticated sense of using problem solving strategies limits the refinement of students' thinking abilities.

The second textbook mode is one in which a section for problem solving is put in every chapter. There are two ways to present a section within this mode. One is that a section for only one strategy for problem solving lies in every chapter, while the other is that a section in a chapter is to contain the several selected strategies together. The merit of the first approach is that students can learn one strategy in a section, which leads them to recognize clearly the given strategy. However, this approach is subject to the same flaws as the first mode. The merits of the second approach of the second mode is that students can experience solving the problems using various strategies more often than in the first mode. Frequently encountering the strategies helps students use them appropriately when they face a problem. However, this approach also has some flaws. First, only a few problems can be included in a section. Although students face a variety of strategies in a section, young students tend to forget the methods they have learned, as they have to solve only a few problems using each strategy in a section. Second, having learned one strategy a long time ago, students have to exert extra effort to internalize and to redevelop a sense of connection between the strategy they had once learned and the one with which they have became newly acquainted.

In the third mode, the textbook does not feature a designated section for teaching strategies for problem solving. Problems are assigned on every page or every few pages in each chapter. Assuming the merit of this mode, the question of how ideally to make space for problem solving without heavily sacrificing space for other mathematical content demands attention. NCTM (1989, 2000) recommend that some topics, such as complex paper-and-pencil computations or isolated treatment of paper and pencil computations and the like, be removed from the textbooks for grades K through 4. In this way, we have some space on every page or every few pages for problem solving. One of the benefits of this mode is that students can face some problem solving strategies on almost every school day and thus become more soundly acquainted with the strategies. This mode encourages the student to continue to think about problem solving strategies more frequently throughout their school year, and such a series of continuous considerations will enhance students' ability to think more clearly.

The third approach is based on recommendations by researchers and by an experimental result. First, NCTM(1989) and Reys, Suydam, Lindquist, and Smith(1998) stated that problem solving is not a distinct topic but a process that should permeate that entire program and provide the context in which concepts and skill can be learned. Also, it is said that a problem solving approach should pervade the mathematics curriculum. Although such recommendations do not ignore content (see, NCTM 1989, p. 23), what is more focused in the recommendations is that students frequently learn problem solving strategies in order that they acquire and use them whenever required to solve a variety of problems that can be met in their lives.

Second, Shin(1993) investigated in Korea whether it is more effective to teach one strategy after another or to teach several strategies at the same time. Shin devised 72 problems that were divided into 12 categories of strategies, containing six problems in each category. The students in the experiment were divided into three groups of equal size. Every week, she gave group A 6 problems involving one strategy selected from the 12 strategies, group B three problems from one strategy and three problems from another strategy, and group C six problems from six distinct categories of strategies. This was done continuously for 12 weeks. The post-tests showed that group C contained more effective problem solvers than group B, and that group B contained more than group A. That is, this experiment suggested that continuously teaching several problem solving strategies at the same time is more effective than teaching one strategy after another. Shin's finding coincides with what is called a "disperse learning theory" in educational psychology. The basic notion of the theory is that more effective learning occurs when the instruction is dispersed over time as opposed to instruction of material in large sum at one time (Anderson, 1990. pp. 206-209).

III. When is the Most Appropriate Time to Introduce Various Strategies to Children?

Yet another issue to be settled is the question: At what level should strategies for problem solving be taught to children? Children have acquired a lot of mathematical knowledge before entering the first grade classroom, depending mainly on their own creation of mathematical knowledge, and less on being helped by others. In addition, they enjoy the activities of learning in their everyday lives. Even though they are young, children less than 7 years old can invent their own knowledge and thinking methods through interaction with their peers and with materials from their play(Thornton, 1995). In an experiment conducted by Kim(1994), second graders, who have just learned multiplication facts, were asked to answer multiplication problems such as 2*10, 2*11, 2*19 and 10*11, 10*12, 10*19, and so on, which they didn't learn in class. The second graders can easily manage to generate answers to such problems. Such generation of new knowledge not learned through a formal instruction seems to be harder than solving problems. It is likely to mean that students have abilities beyond what we expect them to have.

It is important to encourage first graders to use the potential strategies that they already know and to lead them to smoothly link them with ones that are currently being taught. Through this kind of instruction, children come to believe in their ability to learn and invent mathematics. For students, the most powerful benefit gained from learning mathematics is being able to create their own strategies, rather than merely memorize procedures for solving problems. Most important is that students learn to generate strategies that help them make headway(Mokros et. al. 1995).

To achieve such goals, problem-solving strategies should be taught from kindergarten (Charles, et. al. 1987; Lester, 1994). Many educators have concluded that it is good to teach problem solving to children as early as possible. The processes of thinking how to solve problems consist of complicated activities. For example, it appears that a lot of problem spaces exist in solving a problem. In addition, how to choose one or more factor(s) among the wide spaces in order to progress to the next step is a difficult decision that demands the use of much information and the understanding of the various relations among this data. For students to be familiar with such processes, instruction in the schools should closely be connected with their knowledge acquired prior to entering school.

The ability to solve many kinds of problems cannot be acquired within a short period of time and by a small amount of practice. It takes a long time for students to acquire the ability to solve problems. The development of thinking ability occurs gradually through daily experiences related to solving problems in and out of school and with the exchange of thinking processes with each other.

Ⅳ. Analysis of Strategies Presented in Elementary School Mathematics Textbooks Used in Korea and in the U.S.

I have mentioned three pedagogical modes for teaching and learning problem solving strategies, and also have discussed the importance of teaching problem solving strategies to children as soon as they enter elementary school. Of the three modes I think, for reasons discussed above, that the third mode is most appropriate for students to learn these strategies. I am convinced that teaching the strategies to students is desirable as soon as they enter in elementary school.

Based on these two criteria, I will analyze the textbooks used in South Korea and the United States. But, first, I would like to discuss various strategies employed in elementary school mathematics lessons. Unfortunately, there is no consensus on the issue among general educators, psychologists and mathematics educators. For example, Dalton(1985) recommended solving non-routine problems strategies as diverse as listing sources, organizing and classifying sources, using tables, charts, models or diagrams, and comparing. Krulik and Rudnick(1987) suggested the following strategies for elementary school mathematics lessons: recognizing patterns, guess and check, working backward, and the like. And, Lenchner (1983) mentioned as many as 12 strategies to be taught in elementary school as general strategies, including drawing a picture or diagram, finding a pattern, making an organized list, and the like.

There might be reasons for disagreement, raised from different points of view, about what problem solving is, or about children's ability to think, and the like. For the first issue, see the article written by Branca(1980). The second issue is determined by one's view on the appropriateness of teaching logic or deductive reasoning as problem solving strategies to first graders, for example. According to Piaget's theory, teaching this strategy to first graders is not appropriate. Even in research conducted by Kim et. al.(1994), reasoning ability was not treated in problems for first graders, while some problems were treated for this strategy from second to sixth graders. In that research, seven strategies were included: finding out the pattern, experimenting, trial and error, using the equation, working backward, drawing, and simplifying.

In spite of the disagreement, many efforts have been made to include at each grade level units for problem solving in textbooks. Some textbooks have tried to allot space either for a chapter or for a section in a chapter. Many contain problems developed by mathematics educators and educational psychologists specifically for the purpose of student improvement of problem solving(Krulik & Rudnick, 1987; Baroody, 1993). However, Greenes and Schulman(1993) indicated that at the present time, elementary school textbooks begin the formal development of problem solving in the third grade. Even a first grade textbook authored by an author who argues professionally that problem solving instruction should begin in grade one did not include a chapter on problem solving(Lester, et. al, 1994).

Some textbooks published relatively recently in both South Korea by the Minister of Education (1996) and the U.S. such as Champagne, Ginsburg, et. al.(1998) include chapters and sections for problem solving beginning at the first grade level. Moreover, in the U.S., the sections begin put at the kindergarten level. I will describe and then compare the various ways in which problem solving sections were presented of these the chapters in the textbooks.

Analysis of problem solving strategies taught through a series of textbooks in Korea

In South Korea, developers of mathematics textbooks for elementary school began to be interested in teaching problem solving and changed the curriculum for each grade so that a chapter for problem solving, entitled "Various Problems", was added to textbooks from second through sixth grade in 1982. In 1987 and 1996, some problems in the "Various Problems" chapter were replaced by a variety of activities asking students to question, pose and transform given situations. Yet, both the original and the revised versions still included a lot of content previously learned and which therefore might appear to be merely a review from the student's perspective. That might explain why one of my students once said to me, "Teacher, you do not need to teach this unit to me. I will just ask you some questions for the problems I cannot get a solution to." In this light, I would like to recommend that the review contents of the problem-solving unit be relegated to the supplementary companion workbook, and that the unit itself be solely devoted to the introduction and further development of problem solving strategies.

All of the strategies discussed in the following passage (see Table 1) are taken from either the teacher's edition or the students' edition of the series that target students from first to sixth grade. The textbooks were relatively recently revised in 1996[1]. Each chapter among the series of textbooks contains only a few problem solving strategies. The reasons only a few strategies per chapter are included seem to be twofold. One is because much space is used for enhancing algebraic skills such as "Making Equations with Using Triangle and Squire", especially, in lower grades. And a lot of space is allotted for the purpose of reviewing material that was learned in some previous chapters, and for the purpose of teaching new mathematical concepts, especially, in upper grades.

As discernible in Table 1, the goal of continuously and progressively developing the students' thinking abilities through teaching problem solving strategies is rather poorly achieved by the currently practiced arrangement of problem solving sections in the textbooks. The limited opportunities thus presented to the students to familiarize them with the strategies and to hone their skills in using them is in my opinion the weakest aspect of the textbook design. By being exposed to one and only one chapter per semester devoted to learning problem solving strategies in a given academic term, the students do not receive continuous encouragement to improve their skills. A full year or more will pass before the students get to reacquaint themselves with a particular strategy learned. There is almost no duplication of strategies learned in either the previous semester or grade.

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1) In this paper instead of analyzing strategies presented in mathematics textbooks based on the 7th national curriculum, they are analyzed based on 6th national curriculum. It seems to be unlike that analyzing strategies presented in currently used mathematics textbook. Making a judgment of success and failure of the curriculum should be delayed until next version is published. Probably, the results of analysis of them in present textbooks might be influenced on teaching them in current classroom, which might be one of effects researcher does not hope.

Table 1. Strategies taught through the textbooks from 1st through 6th grades in South Korea

Grade / Strategies for problem solving		2	3	4	5	6
Writing Equations and Reading Them	1					
Expressing Equations in Different Ways	1			1	1	
Making Equations	1	1				
Comparing Objects without Counting	1					
Classifying in a Variety of Ways	1	1				
Making Equations from Pictures	1		1	1		
Making Problems from Given Equations	1	1	2			
Making Equations and Solving Them	1	1	1	1		2
Finding Patterns		2	3	1	1	2
Using a Model Clock		1		1		
Solving in Various Ways		2	1	1	1	1
Building Shapes		1			1	
Calculating Mentally		2	1			
Estimating		2	1			2
Transforming Words (or Sentences) into Equations		1	2	2	1	
Making problems from Given Equations and Solving Them		2	2			
Making Numbers or Equations using Number Cards		4			1	
Choosing Operations		1		1		
Solving Problems by Drawing or Finding Patterns		1				
Making Equations with Using Triangle and Squire			2			
Using Simpler Problems to Solve Given Problems			2		1	1
Finding Units			2			
Solving Puzzles			1		1	
Transforming Problems into Equations			1			
Working Forward			2	2	1	1
Explaining How the Procedure Is Developed			2			
Measuring Weight and Liquid			1			
Choosing Proper Numbers in Calculation			1	1		
Finding Relation between Two Numbers				1		
Expressing Sentences in a Variety of Ways				1	1	
Transforming Equations into Various Words or Sentences				1	1	
Using Drawings					2	1
Making Tables					1	1
Working Backward					1	1
Guess and Check					1	1
Using Drawings or Making Tables						1
Guess and Check or Working Backward						1
Making Trees or Using Simpler problems						1
Solving Multi-Step problems						1
Making Trees						1

- Strategies written either bold or italicized denote ones recommended by educators mentioned above.
- Strategies written bold only are dealt with as strategies in both countries.
- Strategies written both bold and italicized means that they appear only in the textbooks used in S. Korea
- The remainders written by plain style show ones, which have not been mentioned by educators above.
- The numbers in each cell denote the number of times the strategy appeared in each grade.

The arrangement of problem solving strategies in the textbooks is not without some weaknesses. For example, let's consider one strategy, "Solving Multi-Step problems", which appears first in the sixth grade and not in the lower grades. Students have often shown difficulty in solving multi-step problems. We cannot expect them to get answers to problems relevant to suddenly appearing strategies such as "Solving Multi-Step problems", if they are suddenly first shown such strategies in the sixth grade. Therefore, it is required that second-and third step problems are presented to them before they are confronted directly with multi-step problems as in the U.S(see Table 3.). There is one more weakness that is the order of presenting strategies. For example, in one case, the introduction of a new skill, "Making Trees", is offered after the application of it, "Making Trees or Using Simpler problems".

What the textbooks want to emphasize seems to enhance problem posing ability through activities such as Making Equations, Making Problems from Given Pictures, Transforming Words (or Sentences) into Equations, and the like, rather than focusing on teaching problem solving strategies.

Now let's see the details through Table 2 in which strategies only dealt with in South Korea will be discussed.

Table 2. Strategies which are dealt with in textbooks used in Korea

Working Forward	Making equations and Solving Them
Making Equations	***Calculating Mentally***
Making Equations from Pictures	***Solving Problems by Drawing or Finding Patterns***

As indicated in the analysis of the Table 1, most strategies are relevant to computational skills. It can be reached from the strategies, once again, that the purpose of teaching strategies for problems solving is to encourage learners to acquire skills relevant to computations, not thinking ability.

When new versions of the series of textbooks are created in the future, authors should make an effort to put more and diverse strategies into the series. Also, refinement of strategies from grade to grade is required. For example, suppose "Making Table"as a strategy. The making of small table to big table might be an order in which the strategy is presented through the whole grades, and the complexity of the table to be made should be considered. An analysis of the strategies used in both countries is made later in describing ones used in the U.S.

Analysis of problem solving strategies taught through a series of textbooks in the U.S.

In America, as indicated before, most mathematics textbooks also tend to include either a chapter on problem solving or a section for a strategy in a chapter. The teaching of problem solving strategies used to begin at the third grade level. More recently, however, textbooks in which the authors attempted to include two sections of problem solving in every chapter from kindergarten through grade tenth have been published. "Silver Burdett Ginn Mathematics: Exploring Your World," published in 1998, is such an example(See Table 3).

Table 3. Strategies taught through the textbooks from K to 6th grade in the U. S

Strategies for problem solving \ Grades	K	1	2	3	4	5	6
Finding Facts from Pictures	1	1	1				
Using a Model	3	2					
Classifying	1						
Using a logic (or Logic)	4	1	1	1		2	2
Finding Patterns	1			1	1		
Making a List	1	1		1			
Estimating	1						
Making a Graph	1						
Using Pictograph	1						
Using a Model Clock	1						
Making a Table	1	1					
Using a Drawing	1						
Guess and Check	1		2				
Patterns in addition	1						
Choosing the Operation	1	3	2				
Choosing Addition and Subtraction	1						
Making Predictions	1						
Making a Real Graph		1					
Making and Using a Drawing (or Drawings)		2	1	1	1	1	
Making a Pictograph		1					
Number Patterns		1		2		1	1
Finding Units				2			
Using Money		1					
Collecting Data and Graphing		1	1				
Checking for a Reasonable Answer		1					
Sorting and Classifying		1	1	2			
Measuring Weight and Liquid				1			
Using a Map		1	1				
Exploring Area		1					
Building Shapes		1					
Making and Using a Table (or Tables)		1		1	1	1	1
Too Much Information		1	1				
Two-Step problems		1		1	1		
Using a Chart			1				
Using a Model to Find Missing			1				
Two Uses for Subtraction			1				
Patterns with Odd and Even Numbers			1				
Estimating Money			1				
Estimating Sums			1				
Number and Letter patterns			1				
Using a Schedule			1				
Patterns in Geometry			1				
Question That Make Sense			1				
Lists for Combinations			1				
Comparing Addition and Multiplication			1				
Making Ordered Pairs			1				
Facts from Pictures and Text				1	1	1	
Experiment				1		1	
What's Extra?				1	1		
What's Missing?				1			
Alternative Solutions				1	1	1	2
Guess and Test				1	1	1	1
Too Much or Too Much Little information					1	1	1
What's the Operation?					1		
Experimentation					1		1
Organized listing					1		
Simulation					1	1	1
Multi-Step problems						1	1
Working Backward						1	1
Patterns							1
Solving a Simpler Problem							1

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- Strategies written bold only are dealt with as strategies in both countries.
- Strategies written bold and italicized means that they appear only in the textbooks used in the U.S.
- The remainders written by plain style show ones, which have not been mentioned by educators above.
- The numbers in each cell denote the number

of times the strategy appeared in each grade.

Generally speaking, that which is being newly learned can easily be internalized in a learning situation through frequently repeated exposure, especially in the case of strategies for problem solving. Especially with regard to learning thinking processes like problem solving, learners must progress from simple processes to complicated ones, such as second- and third-step problems to multi-step problems. In this sense, the books are more advanced than those used in Korea. Students can learn many strategies in a year, and students encounter one strategy several times. For example, in the first grade, the sections for the strategies such as Making and Using a Drawing, Using a Model, Choosing the Operation, and Using a Model were presented two to three times in the book. Therefore, the attempt made by the authors to provide continuous exposure is noticeable.

There is one additional feature of this approach. The authors of the textbooks attempted to reflect suggestions for making connections, as recommended by NCTM (1989). According to NCTM, "the standard's purpose is help children see how mathematical ideas are related. ... When mathematical ideas are also connected to everyday experiences, both in and out of school, children become aware of the usefulness of mathematics". A typical example of a mathematical connection can be seen in the strategy called "comparing addition and multiplication". In general, addition and multiplication have been taught individually in schools without any linkage between them, unfortunately. However, the strategy "comparing addition and multiplication" attempts to restore the missing linkage between these two mathematical concepts.

Finally, although it appears that learners have fewer opportunities to learn strategies in the upper grades than in the lower grades, strategies taught from 3rd grade seem to be a combination of strategies taught in earlier grades. For example, before teaching "Too Much or Too Little Information", "Too Much Information", and "Too Little Information" were taught, and before "Multi-Step problems", "Two step problems were taught. Now let's see the details through Table 4, in which the discussion concerns the strategies used in the U.S(see Table 4).

Table 4. Strategies that are dealt with in textbooks only used in the U.S.

Using a Model	Using a Map
Using a logic (or Logic)	Making and Using a Table (or Tables)
Making a List	Too Much Information
Making a Graph	Two-Step problems
Using Pictograph	Using a Chart
Making Predictions	Using a Model to Find Missing
Making a Real Graph	Two Uses for Subtraction
Making and Using a Drawing (or Drawings)	Patterns with Odd and Even Numbers
Making a Pictograph	Number and Letter patterns
Number Patterns	Patterns in Geometry
Finding Units	Alternative Solutions
Using Money	Guess and Test
Collecting Data and Graphing	Too Much or Too Much Little information
Checking for a Reasonable Answer	*Experimentation*
Measuring Weight and Liquid	Organized listing
Sorting and Classifying	Simulation

Many strategies in the Table 4 are not relevant to operational skills. A strategy such as "Patterns" is treated in diverse forms. For example, there are strategies called "Number Pattern, Patterns with Odd and Even Numbers, Number and Letter Patterns, Patterns in Geometry". We can see the effort to shift points of view in order to provide students with opportunities to understand the pattern search strategy. There are also a lot of strategies that are focused on the uses of thinking abilities, from simple to complicated. In this regard there are many more strategies taught in the U.S. than in S. Korea. Through many activities the U.S. authors tended to provide students with more opportunities to progressively develop their thinking skills.

The table 5 below shows varied strategies taught in both countries. However, one might feel the lack of strategies like inductive and deductive reasoning. Of course, dependent upon the theory author(s) of textbooks apply, the strategies recommended above can be put into textbooks, or not. If we accept the fact that one of reasons children do not reason is due to the lack of information necessary to do it from the theory of cognitive science, we can provide the kind of strategies which can be solved using learners' knowledge level.

Tables 1 and 3 raise some questions. Certain strategies such as "Finding patterns, Solving a Simple Problems and the like" appear in different grades in the textbooks used in South Korea and in the U. S. When is teaching a particular strategy appropriate to children? Certain strategies such as "Calculating Mentally, and the like" appear only at the textbooks used in South Korea and some, such as "Using a Logic (or Logic), Simulation, only at the textbooks in the U. S. The questions is "is it not necessary to teach children in Korea a strategy such as "Using a Logic (or Logic) and children in the U.S. a strategy like "Calculating Mentally"? Mathematics textbook developers should reflect on such questions.

With the three textbook modes discussed above in mind, this approach falls between the second and the third mode as described above which seems to be preferred among textbook authors in the U.S. The approach currently in place in Korea lies in transition from the first to the second mode. In short, the frame for introducing problem solving strategies has been shifting from the first to the second and then to the third mode. However, we have yet to see the third mode in action.

An increasing number of researchers now believe that even young children do think by formal reasoning. Research results of some researchers(e.g. Kim, 1994) support this belief. It

Table 5. Strategies taught in both countries

Classifying in a Variety of Ways: Finding Patterns: Using a Model Clock: Building Shapes: Estimating: Choosing Operations: Using Simpler Problems to Solve Given Problems: Solving Puzzles: Using Drawings: Making Tables: Working Backward: Guess and Check: Using Drawings or Making Tables: Guess and Check or Working Backward: Making Trees or Using Simpler problems: Solving Multi-Step Problems: Making Trees

is promising to see in the textbook "Silver Burdett Ginn Mathematics" an attempt to teach a strategy "using logic"from as early as kindergarten. I foresee a need for educators to shift their points of view in order to reflect children's ability to think logically, should the third mode be reflected in textbook design.

V. Conclusion

So far, I have mentioned some issues related to teaching problem solving strategies in elementary school mathematics. Problem solving is a complicated process so that practice and experience are an essential ingredient in developing problem solving ability(Fendel, 1987). We need to think about what we have to do in order that students achieve this objective in practicing and experiencing problem solving. First, we have to start to teach problem solving strategies as soon as children enter elementary school. How to teach them through textbooks has to be pondered as well.

As I have suggested, one model, the third mentioned, might be the best way to teach problem solving because this way makes it possible for students to practice and experience solving problems in which a variety of strategies apply and that students can continually use. Textbook developers need to create new kinds of problem solving chapters or sections. However, to date, no mathematics program has been developed that adequately addresses the issues of making problem solving the central focus of the curriculum(Lester, 1994).

Unfortunately, our current textbooks do not yet include material that is conducive to helping students to learn problem solving skills. We need to begin in the first grade, teaching children a variety of problem solving strategies, allowing them to solve problems which call for applying not just a single, repetitive strategy, but various strategies. My strong hope is that a textbook series incorporating the third mode appears in the near future and becomes available in the elementary school classroom.

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초등학교 수학교과서에 나타난 문제해결 전략의 양식에 대한 분석

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연구자들은 학생들에게 문제해결 전략을 지도하는 것이 학생들의 문제해결력을 신장시켜 준다는 보고하고 있다. 이와 같은 연구결과를 배경으로 수학 교과서를 통하여 문제해결 전략을 지도하려는 시도들이 미국을 비롯하여 한국에서도 있어 왔다. 본 논문은 문제해결 전략을 교과서에 제시할 수 있는 가능한 세 가지 모델들을 논의하고, 미국과 한국의 수학교과서에서 문제해결 전략을 제시하는 방법을 분석하였다. 한 가지 모델은 문제해결 전략에 한 단원을 할애하는 것이다. 두 번째 모델은 각 수학내용을 지도하는 단원에 문제해결 전략의 지도를 위한 하위단원을 할당하는 것이다. 마지막, 세 번째 모델은 문제해결 전략 지도를 위한 특정 단원이나 하위 단원을 설정하는 것이 아니라 가능한 많은 쪽에 전략을 제시하는 것이다. 위에 언급한 세 가지 가능한 모델을 바탕으로 미국과 한국의 초등학교 수학교과서에서 문제해결 전략을 제시하는 양상을 비교하였다.

이 비교를 위하여 각 학년별로 제시되는 모든 전략들을 교과서와 교사용 지도서를 토대로 추출하였다. 각 교과서에서 전략을 제시한 양식을 비교한 결과 다음과 같은 결론을 얻게 되었다. 한국의 수학교과서는 전형적으로 첫 번째 모델의 양식으로 문제해결전략을 제시하고 있었다. 각 단원마다 별개의 문제해결 전략이 제시되었다. 또한, 학년별 지도 전략을 살펴보면 학년별로 연계성이 있게 전략이 제시 되었다기 보다는 학년별로 다른 다양한 전략의 지도에 중점을 둔 듯하다. 미국의 수학교과서는 두 번째 모델과 세 번째 모델의 중간적인 양식으로 문제해결 전략을 제시하고 있다. 즉, 각 단원마다 문제해결 전략 지도를 위한 하위 단원을 지정하였으며 필요한 경우에는 본 단원의 주 학습요소와 관련된 문제해결 전략은 단원 중에도 제시되고 있었다. 따라서, 차기 수학교과서 개정시기에는 세 번째 모델을 그 모형으로 삼아 문제해결 전략들을 제시하는 방안을 강구해야 할 것으로 기대된다.

key words : problem solving, representation, strategy, elementary mathematics textbook
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