# Identification of the Distribution Function of the Preisach Model using Inverse Algorithm

# Chang Seop Koh\* and Jae Seop Ryu\*

**Abstract** - A new identification algorithm for the Preisach model is presented. The algorithm treats the identification procedure of the Preisach model as an inverse problem where the independent variables are parameters of the distribution function and the objective function is constructed using only the initial magnetization curve or only the major loop of the hysteresis curve as well as the whole reversal curves. To parameterize the distribution function, the Bezier spline and Gaussian function are used for the coercive and interaction fields axes, respectively. The presented algorithm is applied to the ferrite permanent magnets, and the distribution functions are correctly found from the major loop of the hysteresis curve or the initial magnetization curve.

Keywords: identification process, Preisach model, distribution function, Everett function

#### 1. Introduction

With a view to more accurate magnetic field analysis, the development of an algorithm for hysteresis modeling becomes a major research topic [1-5]. The Preisach model, one of the phenomenological hysteresis models, is being widely applied to the modeling of the anisotropic magnetic materials. The model, for example, is adopted to analyze the distribution of the residual magnetic flux density of the permanent magnet for the BLDC motor [2], and used to calculate the iron loss of the electric machine [6]. For the Preisach model to be used in practical applications, the distribution function of the material with which the hysteretic behavior is concerned should be determined [7,8]. The distribution function is, generally, determined using the measured data, and this procedure is called the identification of the Preisach model.

In the conventional identification method, the Everett function values are first computed from the measured reversal curves and then the distribution function values are calculated [2,7,8]. This method is generally considered the most dependable method, at least theoretically, because it adopts the measured data directly, but it requires a lot of time and difficulty for the measurement of the reversal curves [4,8,9]. Furthermore, when the reversal curves are not very accurately measured, this method supplies negative values of the distribution function, which is physically unacceptable. Usually, to remove these negative attributes, the measured data is modified manually [9]. However, proper modification of the measured data is very time con-

suming and difficult.

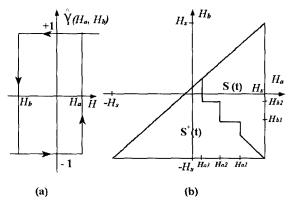
For this reason, some simplified identification methods are suggested [4,10]. S.R. Naidu suggested a simple analytic formula using only the descending branch of the limiting hysteresis loop [10]. Although the formula is simple and easy to implement, it is less accurate than required because it adopts only the limiting hysteresis loop.

In this paper, a new identification algorithm for the Preisach model is developed, considering the identification procedure as an inverse problem. In the algorithm, the distribution function is approximated using the Bezier spline and Gaussian function for the coercive and interaction fields axes, respectively. The control points of the Bezier spline and the variances of the Gaussian function are taken as the parameters for the distribution function and optimized using the inverse algorithm to get the proper distribution function. As an optimizer, the (1+1) evolution strategy is adopted.

#### 2. Preisach Model

In the Preisach model, the magnetic material is assumed to be composed of interacting magnetic domains that have their own interacting and coercive fields. When the magnetic field is externally applied to the domain in which the up and down switching fields are  $(\alpha,\beta)$ , the magnetic behavior is modeled, as shown in Fig. 1(a), using the elementary hysteresis operator  $\gamma(\alpha,\beta)$  [8]. The distribution function relates the Preisach model to a particular material and is physically the number of magnetic domains having the same switching fields. It is defined from the measured reversal curves as follows [8]:

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**Fig. 1** The Preisach plane: (a) the elementary hysteresis operator, (b)  $S^+(t)$  and  $S^+(t)$  when the applied field varies  $H_{al} - H_{bl} - H_{a2} - H_{b2} - H_{a3}$ 

$$\mu(\alpha, \beta) = \frac{1}{2} \frac{\partial^2 M_{\alpha\beta}}{\partial H_a \partial H_b} \tag{1}$$

where  $M_{\alpha\beta}$  is the magnetization when the applied field is decreased to  $H_b=\beta$  after being increased to  $H_a=\alpha$  from the negative saturation field (-H<sub>s</sub>).

When the applied magnetic field is H(t), the total magnetization of the material is computed using the elementary hysteresis operator and distribution function as follows [7,8]:

$$\begin{split} M(t) &= \iint\limits_{H_a \geq H_b} \mu(H_a, H_b) \hat{\gamma}(H_a, H_b) H(t) dH_a dH_b \\ &= \iint\limits_{S^+} \mu(H_a, H_b) dH_a dH_b - \iint\limits_{S^-} \mu(H_a, H_b) dH_a dH_b \end{aligned} \tag{2}$$

where  $S^+$  and  $S^-$  represent the regions where the elementary hysteresis operator operates at the up and down positions, respectively, as shown in Fig. 1(b), and are defined as

$$S^{+}(t) = \{ (H_a, H_b) \in T : \hat{\gamma}(H_a, H_b)H(t) = +1 \},$$
  

$$S^{-}(t) = \{ (H_a, H_b) \in T : \hat{\gamma}(H_a, H_b)H(t) = -1 \},$$
(3)

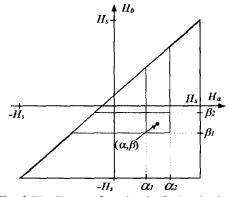


Fig. 2 The Everett function in Preisach plane

where T is the Preisach plane.

From (2), the magnetization of a material can be thought of as a function of only the distribution function if the elementary hysteresis operator and the applied field are given.

## 3. The Identification Algorithms

## 3.1 Conventional Identification Algorithm

Theoretically the distribution function can be computed from the measured reversal curves using (1). However, the numerical implementation is difficult and is computed, in reality, using the Everett function [2]. The Everett function is computed from the reversal curves as follows [2,7,8]:

$$E(\alpha, \beta) = (M_{\alpha} - M_{\alpha\beta})/2 \tag{4}$$

where  $M_{\alpha}$  is the magnetization when the applied field is increased to  $H_a = \alpha$  from negative saturation field  $-H_s$  and  $M_{\alpha\beta}$  is defined as in (1). After the Everett function is found, the distribution function can be computed using the following equation.

$$E(\alpha, \beta) = \iint_{T(H_a \le \alpha, H_b \ge \beta)} \mu(H_a, H_b) dH_a dH_b$$
 (5)

The physical meaning of the Everett function  $E(\alpha,\beta)$  is the area of the triangle in the Preisach plane weighted by the distribution function as shown in Fig. 2. If the distribution function is assumed to be constant at the small region  $(\alpha_1 \leq H_a \leq \alpha_2, \ \beta_1 \leq H_b \leq \beta_2)$ , it can be approximated, as shown in Fig. 2, as follows:

$$\mu(\alpha,\beta) = \frac{E(\alpha_2,\beta_1) - E(\alpha_1,\beta_2) - E(\alpha_2,\beta_2) + E(\alpha_1,\beta_2)}{(\alpha_2 - \alpha_1)(\beta_2 - \beta_1)} \tag{6}$$

where  $\alpha = (\alpha_1 + \alpha_2)/2$ , and  $\beta = (\beta_1 + \beta_2)/2$ .

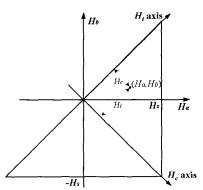


Fig. 3 The coercive and interaction fields axes in Preisach plane

## 3.2 Identification Method using Inverse Algorithm

The Preisach plane can also be defined with the coercive field  $H_c$  and interaction field  $H_i$  axes as shown in Fig. 3, and their values at the point  $(H_a, H_b)$  are calculated as follows [1,8]:

$$H_{i}(H_{a}, H_{b}) = (H_{a} + H_{b})/2$$
  
 $H_{c}(H_{a}, H_{b}) = (H_{a} - H_{b})/2$  (7)

For a particular  $H_c$ , the variation of the distribution function along the interaction field axis, is known to be Gaussian [7,8], and it can be determined uniquely if the values at any two points are given. In this paper, the normalized interaction field  $h_i$  is defined as

$$h_i = H_i / (\sqrt{2}H_s - H_c)$$
 (8)

and the two points,  $h_i = 0$  and  $h_i = 0.25$ , are taken as shown in Fig. 4(a), and the corresponding values,  $D_0(H_c)$  and  $D_{0.25}(H_c)$ , are taken as the parameters for the distribution function. Hence, with the two parameters, the distribution function at a particular  $H_c$  can be written as [8]

$$\mu(H_i, H_c) = D_0(H_c) Exp\{\frac{-H_i^2}{2\sigma^2(H_c)}\}$$

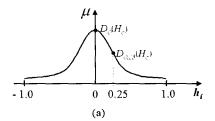
$$2\sigma^2(H_c) = -\frac{H_{im}^2(H_c)}{16\log\{D_{0.25}(H_c)/D_0(H_c)\}}$$
(9)

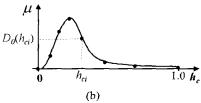
where  $H_{im}$  is the maximum interaction field for  $H_c$  defined as

$$H_{im}(H_c) = (\sqrt{2} H_s - H_c).$$
 (10)

On the other hand, the variation of the distribution function along the coercive field is not Gaussian and can be approximated using the Bezier spline with several control points as shown in Fig. 4(b). In the figure, the coercive field  $H_c$  is normalized by its maximum value  $\sqrt{2}H_s$  and the points  $h_i = 0.0, 0.2, 0.3, 0.4, 0.5, 0.7$ , and 1.0 are taken as the control points. Each control point corresponds to the  $D_0(H_c)$  in (9). Therefore, the distribution function can be accurately defined if the two parameters,  $D_0(H_c)$  and  $D_{0.25}(H_c)$ , are determined for sampling  $D_0(H_c)$  points.

As mentioned before, in the Preisach model, the magnetization is a function of only the distribution function if the elementary hysteresis operator and the applied field are given. The identification procedure, hence, can be considered an inverse problem where the independent variables are the parameters of the distribution function and the object is to get proper outputs (magnetizations) for the given inputs (applied fields).





**Fig. 4** The parameters of the distribution function of the Preisach model: (a) along the interaction field axis, where  $h_i$  is the normalized value of  $H_i$  by  $(\sqrt{2}H_s - H_c)$ , (b) along the coercive field axis, where  $h_c$  is normalized value of  $H_c$  by  $\sqrt{2}H_s$ 

Therefore, the objective function to be minimized can be defined using the calculated and measured magnetizations for the applied magnetic fields as follows:

$$F_{obj}(\mu) = \sum_{i=1}^{N} (M_m(H_{t_i}) - M_c(H_{t_i}))^2$$
 (11)

where N is the number of measured data and  $M_m(H_{ti})$  and  $M_c(H_{ti})$  are the measured and calculated magnetizations for the applied magnetic field intensity  $H_{ti}$ . It should be noted that, in the construction of the objective function, the measured data may not be the reversal curves. The major loop of the hysteresis curve or the initial magnetization curve can possibly be used. The overall algorithm of the identification method using the inverse algorithm can be summarized as follows.

- Step 1 Read the applied magnetic field and measured magnetization data.
- Step 2 Set the initial parameters for the distribution function
- Step 3 Construct the distribution function using parameters.
- Step 4 Compute the objective function using the calculated magnetizations.
- Step 5 If the objective function is small enough, stop.

  Otherwise, modify the parameters and go to Step 3.

As an optimizer, the nondeterministic method, such as the evolution strategy, genetic algorithm, is preferred because the relationship between the distribution function and the objective function is implicit and the gradient vector is difficult to calculate.

# 4. Numerical Examples

The (1+1) evolution strategy is adopted as an optimizer and incorporated with the developed identification algorithm.

## 4.1 Identification Using Major Loop of Hysteresis Curve

The developed identification algorithm using the inverse algorithm is applied to calculate the magnetization of the anisotropic ferrite permanent magnet (PM1), which is constructed so that almost all of the magnetic domains are aligned in one direction [1]. The reversal curves measured using VSM are shown in Fig. 5, where the curves contain some oscillations near the full magnetization. Using the conventional identification method, first the distribution function is computed, shown in Fig. 6. Negative values of the distribution function can be seen.

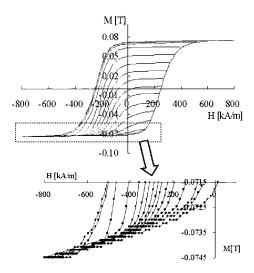
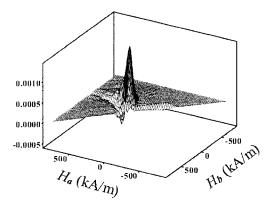
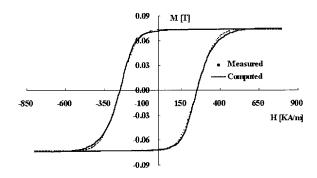


Fig. 5 The reversal curves of the anisotropic ferrite permanent magnet PM1



**Fig. 6** The distribution function for the permanent magnet PM1 computed using the conventional identification method

Secondly, the suggested identification algorithm is applied. To construct the objective function, only the major loop from the measured reversal curves are used. Fig. 7 shows the computed major loop compared with the measured one of the hysteresis curve. The two curves are consistent with each other.



**Fig. 7** Comparison of the computed and measured major loops of the hysteresis curve of the permanent magnet PM1

The distribution function and Everett function, computed using the suggested identification method, are shown in Fig. 8 and Fig. 9, respectively. Comparing with Fig. 6, which is obtained using the conventional identification method, the suggested identification algorithm gives, a smoother and more reasonable result and supplies no negative values of the distribution function.

Notice that just the major loop of the hysteresis curve is enough in the suggested identification method while the whole reversal curves are required in the conventional identification method.

## 4.2 Identification Using Initial Magnetization Curve

The suggested identification algorithm is also applied to the material of which only the initial magnetization curve (B-H curve) is available. If the distribution function can be found from the initial magnetization curve, the identification method will be very useful in real applications because, in many real engineering problems, the initial magnetization curve can be easily obtained while the reversal curves are very difficult to obtain.

Fig. 10 shows the B-H curve of the anisotropic rubber ferrite permanent magnet (PM2), which is used in the drum motor of video cassette recorders. Using the B-H data from the curve, the objective function is constructed.

The computed distribution function and the Everett function are shown in Fig. 11 and Fig. 12, respectively, and are not as sharp as these in Fig. 8 and Fig. 9 because, in the authors' opinion, the magnetic domains are not fully aligned in one direction. The calculated B-H curve is shown in Fig. 10 together with the measured curve, where the two curves

are shown to be consistent with each other.

For the arbitrary applied fields, the magnetization curve is also calculated with the distribution function computed by using the suggested identification method. The waveform of the applied magnetic fields is shown in Fig. 13(a), and the corresponding computed magnetizations are shown in Fig. 13(b) together with the measured one. It can be seen the computed magnetizations are almost same with the measured magnetizations.

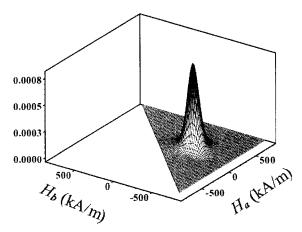


Fig. 8 The calculated distribution function for the permanent magnet PM1

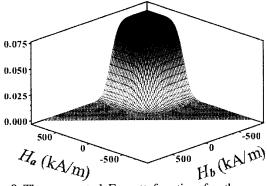


Fig. 9 The computed Everett function for the permanent magnet PM1

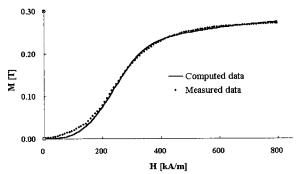
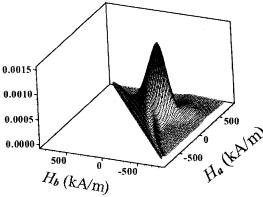


Fig. 10 Comparison of the computed and measured initial magnetization curves of the permanent magnet PM2



**Fig. 11** The computed distribution function for the permanent magnet PM2

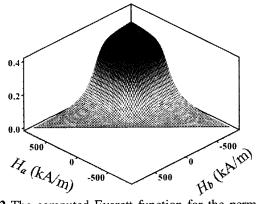


Fig. 12 The computed Everett function for the permanent magnet PM2

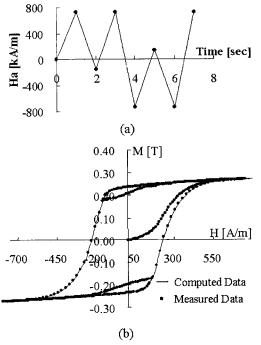


Fig. 13 The hysteresis curves of the permanent magnet PM1 for arbitrary inputs: (a) the waveform of the applied magnetic field, (b) the computed and measured hysteresis curves

#### 5. Conclusions

A novel identification algorithm for the Preisach model is suggested. The suggested algorithm treats the identification procedure as an inverse problem. The distribution function is parameterized using the Bezier spline and Gaussian function for the coercive and interaction fields axes. respectively, and the parameters are used as the independent variables in the inverse problem. As an optimizer, the (1+1) evolution strategy is adopted. Other nondeterministic optimizers, however, can also be adopted. Through numerical examples with permanent magnets, the validity of the suggested algorithm is proven. In the suggested algorithm, the major loop of the hysteresis curve or the initial magnetization curve is enough to find the distribution function of the Preisach model. This makes the suggested algorithm more applicable to real engineering problems. Hence, the Preisach model is thought to be easily applied to the calculation of the magnetization or the iron loss of the magnetic materials with the suggested identification method.

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