A Single DOF Magnetic Levitation System using Time Delay Control and Reduced-Order Observer

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Magnetic levitation systems are required to have a large operating range in many applications. As one method to solve this problem, Time Delay Control (TDC) is applied to a single-axis magnetic levitation system in this paper. A reduced-order observer is utilized to estimate states excluding measurable states in the control law. The system consists of a square air-core solenoid and a circular permanent magnet attached on a plastic ball. Theoretical magnetic forces of the system are obtained on the basis of the location of the magnet around the solenoid. The magnetic levitation force is obtained by the experiment, and then compared with the theoretical one. As the results of the control experiments, the nonlinear controller (TDC : 1-2 mm) has a larger operating range than the linear controller (PD control : 1-1.4 mm), and is superior to linear control in the robustness to the modeling uncertainty and the performance of the disturbance rejection.

Key Words : Magnetic Levitation, Magnetic Force, Estimator, Observer, Time Delay Control, Air-Core Solenoid

1. Introduction

As the semiconductor industry is developed rapidly, high-precision and high-speed actuators, such as chip mounters, wafer stages, and probe stations in the semiconductor-manufacturing processes, are researched in need of the higher performance and the larger travel range. Most of the existing high-precision actuators have a required high stiffness and a satisfactory positioning accuracy, but suffer from a limitation of the operating range, Coulomb friction, and lubrication (Mittal and Menq, 1997).

High-precision levitation actuators are popular for their non-contact nature. Their driving sources are mainly pneumatic (Tomita and Koyanagawa, 1995), electrostatic (Busch-Vishniac, 1992), magnetic (Kim and Trumper et al, 1998 ; Son and Park, 1997) forces. However, air supplied to the pneumatic levitation actuators contains particular matter that is improper for clean room applications, and electrostatic fields of the electrostatic levitation actuators trend to attract dust (Chen and Busch-Vishniac, 1995). But magnetic levitation actuators don't have these kinds of problems. Their control is easier than the mechanical actuators owing to the elimination of Coulomb friction and lubrication, and also there is neither dust nor contamination that is fatal to semiconductor-manufacturing processes. They have the simplicity of the expansion to a multi degree of freedom actuator through the various combinations of several single degree of freedom actuators as well. However, it is the problem that non-contact magnetic levitation actuators are more expensive than contact actuators, and have a limitation of the travel range to the levitated direction due to their narrow gap principle.

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Magnetic levitation systems are originally unstable, and their force equation is nonlinear. Their control algorithms generally use linear control techniques that have fixed gains selected at a nominal point and are based on the force model linearzied at the nominal point by a Taylor's series expansion. Therefore, as the deviation from the nominal point increases, the performance of the systems degrades owing to the fixed gains (Trumper et al, 1997).

To obtain a larger t'avel range, either a sufficient and constant magnetic field or a robust nonlinear control is required. By using linear controllers and two different solenoids to supply the sufficient and constant magnetic field, Chen and Busch-Vishniac (1995) tried to obtain a large travel range. An iron-core solenoid (PD control) is used for the large and coarse travel, and an air-core solenoid (P control) for the narrow and fine travel to the levitated direction. By using the sufficient magnetic field, this system has a large travel range. But it needs to control dually for two solenoids.

Gain scheduling control, feedback linearization control, and sliding mode control are applied to obtain satisfactory performances, robustness to modeling uncertainties, or disturbance rejection instead of the linear control algorithm. Firstly, the gain scheduling control algorithm (C. Y. Kim and K. H. Kim, 1994) utilizes the force models that are linearized at the various operating points with proper controller gains for each point. It has a larger travel range than the linear control algorithm, but requires large look-up tables of controller gains with fine intervals.

An alternative to the gain scheduling algorithm is the feedback linearizion control algorithm (Trumper et al, 1997 ; Slotine and Li, 1991) that introduces a desired linear dynamics by canceling out the nonlinear dynamics in a plant model. In this case, a larger travel range is obtained. If there are modeling uncertainties or parameter variations in a system, the system is suffered from them. Therefore, the stability of the system isn't guaranteed, and there is a robustness problem in the system.

An alternative to the feedback linearization control algorithm is sliding mode control (Slotine and Li, 1991 ; Cho et al, 1993) that introduces Lyapunov stability, feedback linearization technique, and sliding surface. Sliding mode control is a robust nonlinear control algorithm, but the range of the modeling uncertainties should be known.

Time delay control (TDC) has been used robustly to control the systems with unknown dynamics and unpredicted disturbances. It eliminates unknown terms through the control input and the system's output at the previous instant (delay time), and then renders the state vector of a plant to follow that of a reference model. There are successful applications of TDC : a robot (Youcef-Toumi and Shortlidge, 1991 ; Chang and Lee, 1996), a magnetic bearing (Youcef-Toumi and Reddy, 1992), and a brushless DC motor (Song and Byun, 2000).

In this paper, a single dof magnetic levitation system was intended to demonstrate the feasibility of using magnetic levitation and applying to a multi dof high-precision stage composed of several air-core solenoids. Time delay control is implemented in order to realize a wider operating range and cope with parameter variations and modeling uncertainties of a single dof magnetic levitation system. In Section 2, the magnetic force distribution between a square air-core solenoid and a circular permanent magnet is briefly presented to describe the operating principle of the system. And a magnetic levitation force equation is obtained by an experiment. In Section 3, modeling of the magnetic levitation system and linear and nonlinear control are shown. And a reduced-order observer to estimate non-measurable variables in the control law is described. In Section 4, experimental results are shown. Section 5 summarizes the results.

2. Magnetic Force Distribution

We find the magnetic field B at a point P in a square wire of dimensions $2a \times 2a$ (in the xy plane) by using the Biot-Savart law.

$$B=\frac{2\mu_0 i a^2}{\pi(a^2+l^2)\sqrt{2a^2+l^2}} \tag{1}$$

where i is a flowing current. P is located at any point on the z axis, and a distance from a center of the wire to P is l. μ_0 is a permeability of the free space ($4\pi\times10^{-7}$ N/A^2).

If a permanent magnet is located in the magnetic field generated by supplying current to a wire, the magnet has forces by interaction between the magnetic dipole moment **m** and the magnetic field **B**. The force **F** on the permanent magnet (Wangsness, 1979) is represented as

$$\mathbf{F}=(\mathbf{m}\cdot\nabla)\mathbf{B} \qquad (2)$$

The force applied on the permanent magnet is calculated numerically.

Figure 1 illustrates the levitation force on the permanent magnet in the magnetic field of a rectangular air-core solenoid with current. A wire of the solenoid is an AWG 24 wire (diameter: 0.5 mm). A solenoid and its air-core are 23 mm×23 mm×19 mm and 11 mm×11 mm×19 mm in dimension, respectively. A permanent magnet (NdFeB, residual induction: 0.321 T) has dimensions of 10 mm×5 mm (diameter×height) and a dipole moment of 0.2243 Am^2. It is assumed that its dipole moment is a collection of dipole moments, and the magnet is uniformly magnetized.

Figure 2 (a) and (b) represent the force curves when the magnet is located at a point in the z-and x-axis, respectively. The levitation force lifts off the magnet. The stabilizing force occurs when the magnet is deviated from the z-axis, and it renders the magnet to be located in the z-axis.

Force distributions on the basis of the direction and the location of the magnetic dipole moment are shown in Figs. 3 and 4. Magnetic actuators consisting of air-core solenoids and permanent magnets can have a repulsive or attractive-type force, and they have simplicity in the combination of the magnetic forces generated by solenoids and magnets. These features are very useful to generate various forces for multi dof actuators.

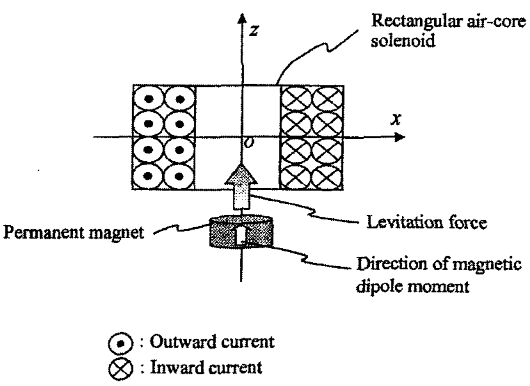
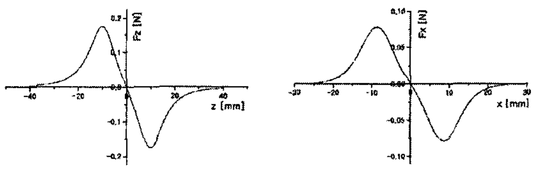


Fig. 1 Permanent magnet and solenoid



(a) Levitation force at x=0, y=0 mm (b) Stabilizing force at y=0, z=−14.5 mm

Fig. 2 Forces on the permanent magnet (0.5 A)

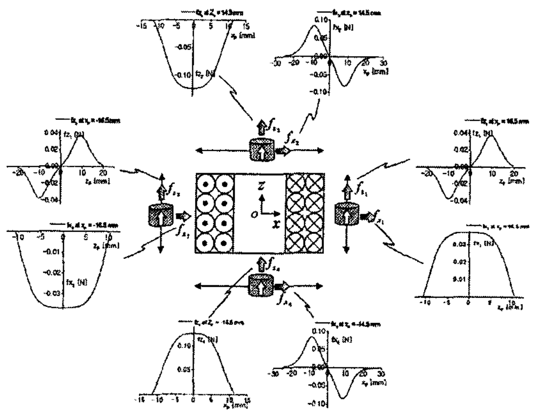


Fig. 3 Force distribution at a point in the xz plane (z-direction magnetic dipole moment, 0.5 A)

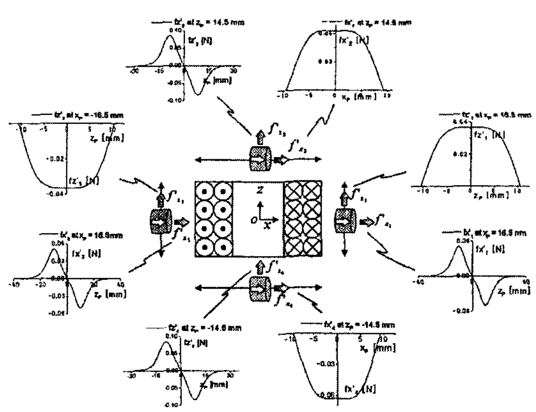


Fig. 4 Force distribution at a point in the xz plane (x-direction magnetic dipole moment, 0.5 A)

The theoretical magnetic forces are obtained as follows. A magnetic field established by a wire with current is calculated by using the Biot-Savart law, and then the sum of the magnetic field of each wire leads to the total magnetic field of the solenoid. The magnetic force is obtained by the inner product of the gradient of the total magnetic field and the magnetic dipole moment.

The levitation force is a function of the supplied current and the distance between the magnet and the solenoid. The theoretical force equation of (2) is complex and nonlinear. So the experimental levitation force equation is obtained through a simple experiment:

$$F_{\exp}=\frac{Ci}{0.04161+4.7638(h-z)+924.5454(h-z)^2} \quad (3)$$

where C is a constant (0.01786) determined by the system, h is a initial distance (5 mm) between the top of the magnet and the bottom of the solenoid, and z is a displacement. The equation is based on the relationship between the initial distance and the least current for lifting off various objects attached on the magnet. As the initial distance is increased in the experiment, the larger lift-off current is required for the fixed weight of an object that is equal to the magnetic levitation force. This experiment is repeated for the various weights of the objects. As a result, the relationship between the current and the distance is quadratic for the fixed weight, and force-current relationship is linear at a fixed distance. Accuracy of the force equation is dependent on the resolution of the analog signal from the DA converter and the power supply.

Figure 5 shows theoretical and experimental force curves. The theoretical force curve is calculated from (2). There are differences between theoretical and experimental force curves due to the assumption that the wire is compactly wounded on the solenoid and the magnet is a collection of dipole moments. And the experimental force curve is obtained in manual.

In general, two magnetic elements are required in a magnetic levitation system. In this paper, an active magnetic element, an air-core solenoid, is fixed in space, and a passive element, a permanent magnet, is attached on the levitated object. The interaction between these two elements generates a magnetic levitation force. A magnetic field occurs around the solenoid to which voltage is supplied, and then the magnetic force is generated by the relationship between the magnetic field of the solenoid and the magnetic dipole moment of the permanent magnet. Voltage for this force is adjusted by the control program based on data from the sensor to place the levitated object on a desired position.

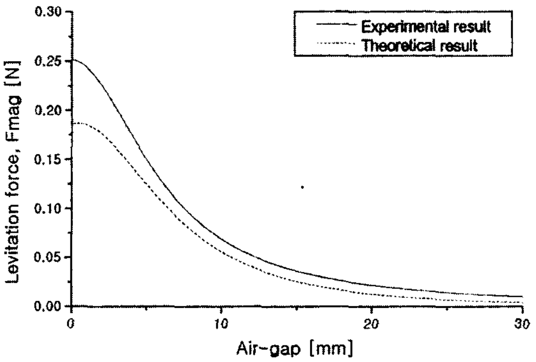


Fig. 5 Theoretical and experimental force curves (0.5 A)

3. System Modeling and Control

3.1 Modeling of the system

Magnetic levitation systems are based on the narrow gap principle. The travel ranges of the systems are narrow and the systems are originally unstable owing to the negative spring force. So they need enough forces to cancel the weight of the levitated object and the negative spring force. The force is generated by the interaction between magnets and solenoids in the magnetic levitation systems, and it has a non-contact characteristic that is useful for clean-room applications.

There are a weight of the levitated object and the levitation force in the 1 dof magnetic levitation system as shown in Fig. 6. The experimental force equation of (3) rather than the equation of (2) is used to derive the equation of the motion.

The system equation can be expressed as

$$m\ddot{z}=F_{\exp}(z,\ i)-mg+f_d \quad (4)$$

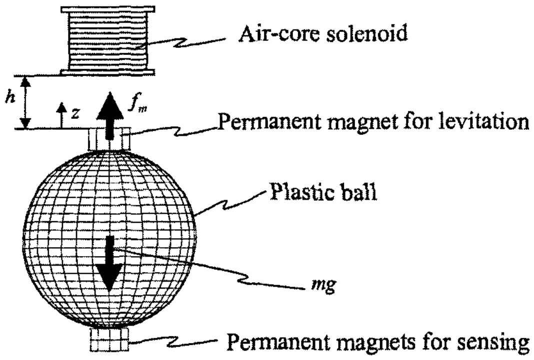


Fig. 6 Forces in the magnetic levitation system

where m is the mass of the levitated object (7.34 g), $F_{\exp}(z, i)$ is the magnetic levitation force equation of (3), and f_d is a disturbance force.

3.2 Proportional-derivative control (PD control)

The equation of the motion is linearized at the operating point since the levitation force equation of (4) is a nonlinear function of the air-gap. The linearized equation is obtained by the Taylor's series expansion and neglecting higher-order terms. Let the perturbations to each variable define $\delta i = i - i_0$ and $\delta z = z - z_0$. The force equation of (3) is written as

$$F_{\exp} = f_0(z_0, i_0) + K_i \big|_{\substack{i=i_0 \\ z=z_0}} \delta i + K_z \big|_{\substack{i=i_0 \\ z=z_0}} \delta z \quad (5)$$

where z_0 and i_0 are the nominal point (1 mm) and the nominal current (0.304 A), respectively. K_i and K_z are the force-current coefficient (0.2367 N/A) and the force-displacement coefficient (11.5954 N/m), respectively. $f_0(z_0, i_0)$ is equivalent to the weight of the levitated object.

Substituting Eq. (5) into Eq. (4) and dividing both sides by m, the motion of the equation can be expressed as

$$\delta\ddot{z} = 1579.75\delta z + 32.25\delta i \quad (6)$$

For convenience, the above equation is rewritten as

$$\ddot{z} = 1579.75z + 32.25i \quad (7)$$

A negative spring force exists in the case of the solenoid with zero current in Eq. (7), and the system is unstable in open loop. The initial value of the levitation force and the nominal force cancel out the negative spring force and the weight of the levitated object. Current is continuously supplied until the levitated object arrives at the desired point.

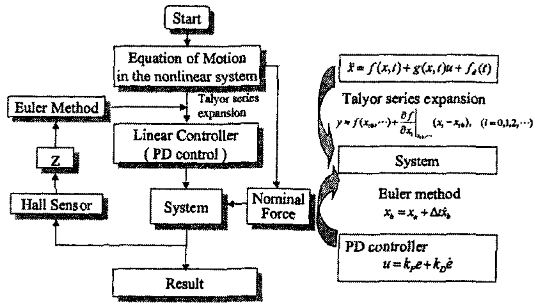


Fig. 7 Process of the linear control

Proportional-derivative control (PD control) as a classical control is applied to the linearized equation of motion. PD control is represented as

$$u = k_P(z_d - z) + k_D\dot{z} \quad (8)$$

where k_P and k_D are the proportional gain and the derivative gain, respectively. z_d is the desired displacement. The process of the linear control is described in Fig. 7.

3.3 Time Delay Control (TDC)

Time delay control (Youcef-Toumi and Ito, 1990; Youcef-Toumi and Reddy, 1992) deals with parameter variations and modeling uncertainty of the system. The nonlinear dynamic equation of (4) is represented as

$$\ddot{x} = f(x, t) + h(x, t) + B(x, t)u + d(t) \quad (9)$$

where x represents a displacement of the object, $f(x, t)$ a known dynamics, $h(x, t)$ an unknown dynamics, $d(t)$ an unknown disturbance, u a control input, and $B(x, t)$ a known function.

A reference model generates a desired trajectory that the system follows. The reference model is chosen as

$$\ddot{x}_m = -a_{m1}x_m - a_{m2}\dot{x}_m + b_m r \quad (10)$$

where a_{m1} and a_{m2} are constants based on the performance of the system, x_m is the reference model position, and r is a command input.

The error dynamics can be written by

$$\ddot{e} = A_1\dot{e} + A_2 e \quad (11)$$

where A_1 and A_2 are constants. By combing Eq. (9) ~ (11), the error dynamics is expressed as

$$\ddot{e}=\ddot{x}_m-\ddot{x}=-a_{m1}e-a_{m2}\dot{e} -\{f+h+d+Bu+a_{m1}x+a_{m2}\dot{x}-b_m r\} \quad (12)$$

If the following equation is always met, the control input u can be obtained :

$$-f-h-d-Bu-a_{m1}x-a_{m2}\dot{x}+b_m r =k_1 e+k_2\dot{e} \quad (13)$$

where k_1 and k_2 are constants. Thus, the control input u is obtained :

$$u=B^{-1}\{-f-h-d-a_{m1}x-a_{m2}\dot{x} +b_m r-k_1 e-k_2\dot{e}\} \quad (14)$$

While $h(x, t)$ and $d(t)$ are unknown, their estimates can be obtained from the information at the previous time $t-L$ since $h(x, t)$ and $d(t)$ are close to $h(x, t-L)$ and $d(t-L)$:

$$\hat{h}(x, t)+\hat{d}(t)\cong h(x, t-L)+d(t-L) \quad (15)$$

where L is a small time delay. Hence, combing Eq. (9) and Eq. (15), the estimates are represented as

$$\hat{h}(x, t)+\hat{d}(t)\cong\ddot{x}(t-L)-f(x, t-L) -B(x, t-L)u(t-L) \quad (16)$$

The TDC control law is obtained by substitution of Eq. (16) into Eq. (14):

$$u(t)=B^{-1}(t)\{-f(t)-\ddot{x}(t-L)+f(t-L) +B(t-L)u(t-L)-a_{m1}x(t)-a_{m2}\dot{x}(t) +b_m r(t)-k_1 e(t)-k_2\dot{e}(t)\} \quad (17)$$

In the implementation of the Time Delay Controller, the delay time is equivalent to a sampling time, and the estimates of the acceleration and the velocity are obtained by using a reduced-order observer.

3.4 Reduced-order observer

A reduced-order observer compared to a full-order observer estimates variables excluding measurable variables in the system, and reduces the order of the observer by the number of the measurable variables (Franklin et al, 1994). Most of the existing accelerometers are contact sensors, so the observer is useful to maintain the non-contact nature of the system. Since the system is controllable and observable, the observer can be applicable.

The nonlinear system dynamics directly follows the reference model. So the linear reference model can be used for the design of the reduced-order observer instead of the nonlinear system dynamics. This is similar to the idea of Chang and Lee (1996). Using the matrix form of the reference model, the system dynamics for the observer is rewritten as

$$\begin{pmatrix}\dot{x}_a\\ \dot{x}_b\end{pmatrix}=\begin{bmatrix}0 & 1\\ -a_{m1} & -a_{m2}\end{bmatrix}\begin{pmatrix}x_a\\ x_b\end{pmatrix}+\begin{bmatrix}0\\ b_m\end{bmatrix}r, \quad y=[1\ 0]\begin{bmatrix}x_a\\ x_b\end{bmatrix} \quad (18)$$

where x_a is measured, xb needs to be estimated, and y is the output.

The reduced-order observer is given by

$$\dot{x}_c=-(a_{m2}+K)\hat{x}_b-a_{m1}y+b_m r \quad (19)$$

where K is a proportional gain, $\hat{x}_b$ is an estimate of the velocity, and $x_c=\hat{x}_b-Ky$.

Solving Eq. (19), the estimates are obtained as

$$\hat{x}_b=x_c+Ky, \quad \hat{\dot{x}}_b=-a_{m2}\hat{x}_b-a_{m1}y+b_m r \quad (20)$$

where $\hat{\dot{x}}_b$ is an estimate of the acceleration.

By using Eq. (20), the Time Delay Control law of (17) is written as

$$u(t)=B^{-1}(t)\{-f(t)-\hat{\dot{x}}_b(t-L)+f(t-L) +B(t-L)u(t-L)-a_{m1}x(t)-a_{m2}\hat{x}_b(t) +b_m r(t)-k_1 e-k_2\dot{e}\} \quad (21)$$

Figure 8 describes the process of the Time Delay Control.

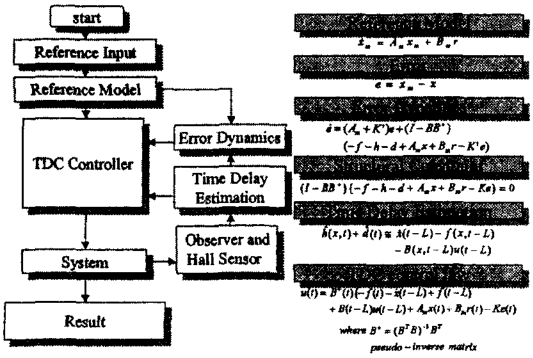


Fig. 8 Process of the Time Delay control

4. Experiment Results

The control architecture of the system is shown in Fig. 9. The position is measured by a hall-effect sensor (SS49 series, Honeywell), and the data acquisition board is a DR1010 that has 12-bit A/D and 12-bit D/A. The control algorithms are implemented on a Pentium 333 MHz with the DR1010 after 0.5 sec. The sampling rate is selected to be 1 KHz. The solenoid is driven by a linear power amplifier whose output voltage is proportional to input voltage. The linear power amplifier is a HP 6023A (DC Power Supply). The specifications of the solenoid are described in Section 2.

The responses of the system with the PD controller are shown in Fig. 10(a). The system has a fast settling time of 0.16 sec, but it becomes unstable over a position of about 1.4 mm owing to the linearization of the nonlinear force equation and the fixed gains. Figure 10(b) shows the results of the modified PD controller. The concept of modified PD control is similar to that of model-following control. A reference model is designed within the performance of the system. The modified PD controller renders the system to follow the reference model, and it has the same reference model as the TDC controller. The system is stable even over the desired reference input of 1.4 mm when the reference model has a natural frequency ω_n of 4.5 rad/s and damping ratio ζ of 0.89, but there is a larger steady-state error as the desired input is larger. As shown in Fig. 11, the operating range of the system with TDC is from 1 mm to 2 mm that is the order of the linear sensing range of the hall sensor.

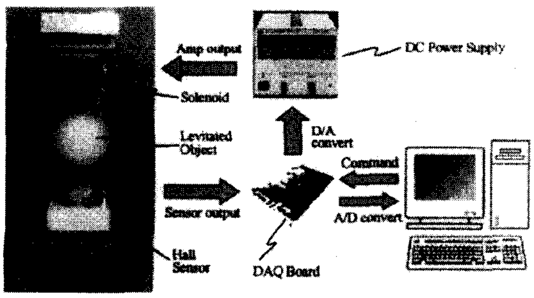
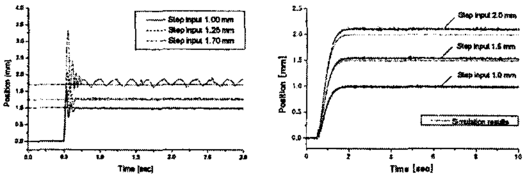


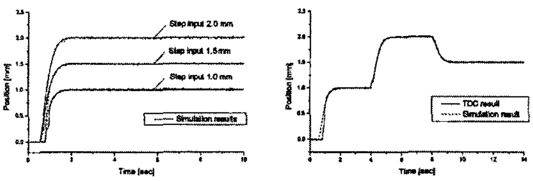
Fig. 9 Control architecture of a single-DOF Magnetic Levitation System

Figure 12 shows the responses of the system with a disturbance (3 mm) at 5 sec. The disturbance as an impulse is made by C program in the control process. The system with the time delay controller becomes stable with the disturbance rejected, but the modified PD controller cannot reject the disturbance. The response of the system loading an additional mass (2.2 g) is shown in Fig. 13. In this case, the system has modeling errors. As a result, the system with the modified PD controller has the steady-state error because



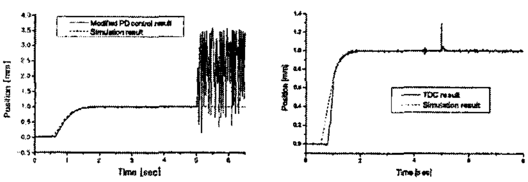
(a) PD control (b) Modified PD control

Fig. 10 Linear control (step input)



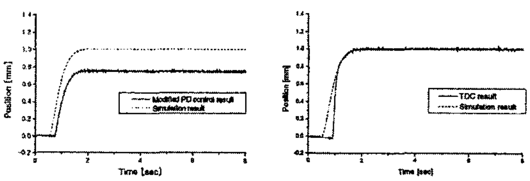
(a) Step input (b) Variable step input

Fig. 11 Nonlinear control (Time Delay Control)



(a) Modified PD control (b) TDC

Fig. 12 Response of the system with disturbance



(a) Modified PD control (b) TDC

Fig. 13 Response of the system loading an additional mass the controller has fixed gains and it cannot compensate for the modeling errors. But the system with the time delay controller follows the reference model well and doesn't suffer from the modeling errors. The Time Delay Controller has the superior performance to the modified PD controller in the case of existing the modeling errors in the system.

5. Conclusions

The single dof magnetic levitation system in this paper has useful potential for multi dof magnetic levitation system designs. The force distributions by the interaction between a rectangular air-core solenoid and a circular permanent magnet are represented, and the levitation force is obtained experimentally and theoretically. The Time Delay Control algorithm is implemented in the single dof magnetic levitation system to realize a large travel. Non-measurable variables in the TDC law are estimated by the reduced-order observer. The performances of the system with the linear control algorithm (PD control) are compared with those of the system with the nonlinear robust control algorithm (TDC) in experiment. The linear control (PD control) suffers from the disturbance, the modeling uncertainty, and parameter variations due to the fixed gains and the linearized force equation, but the nonlinear robust control (TDC) has the good performances of the system in spite of the disturbance, the modeling uncertainty, and parameter variations.

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