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# Note on Properties of Noninformative Priors in the One-Way Random Effect Model

Sang Gil Kang[1], Dal Ho Kim[2] and Jang Sik Cho[3]

## Abstract

For the one-way random model when the ratio of the variance components is of interest, Bayesian analysis is often appropriate. In this paper, we develop the noninformative priors for the ratio of the variance components under the balanced one-way random effect model. We reveal that the second order matching prior matches alternative coverage probabilities up to the second order (Mukerjee and Reid, 1999) and is a HPD (Highest Posterior Density) matching prior. It turns out that among all of the reference priors, the only one reference prior (one-at-a-time reference prior) satisfies a second order matching criterion. Finally we show that one-at-a-time reference prior produces confidence sets with expected length shorter than the other reference priors and Cox and Reid (1987) adjustment.

*Keywords* : Matching prior, reference prior, expected length, variance component

## 1. Introduction

Consider a balanced one-way random effect model

$$Y_{ij}=\mu+\alpha_i+\varepsilon_{ij}, \quad \text{for } i=1,\cdots,I \text{ and } j=1,\cdots,J, \tag{1}$$

where $\mu$ is an unknown constant, and the $\alpha_i$ and $\varepsilon_{ij}$ are independent normal variables with 0 means and variances $\sigma^2_\alpha$ and $\sigma^2$, respectively. Let $\phi=J\sigma^2_\alpha/\sigma^2$ be our parameter of interest.

The present paper focuses on noninformative priors for $\phi$. We consider Bayesian priors such that the resulting credible intervals for $\phi$ have coverage probabilities equivalent to their frequentist counterparts. Although this matching can be justified only asymptotically, our simulation results indicate that this is indeed achieved for small or moderate sample sizes as well.

1) Full-time Lecturer, Department of Liberal Arts, Kyungsung University, Pusan, 608-736, Korea.
E-mail: sangkg@star.kyungsung.ac.kr
2) Associate Professor, Department of Statistics, Kyungpook National University, Taegu, 702-701, Korea.
E-mail: dalkim@knu.ac.kr
3) Assistant Professor, Department of Statistical Information Science, Kyungsung University, Pusan, 608-736, Korea.
E-mail: jscho@star.kyungsung.ac.kr

This matching idea goes back to Welch and Peers (1963). Interest in such priors revived with the work of Stein (1985) and Tibshirani (1989). Among others, we may cite the work of Mukerjee and Dey (1993), DiCiccio and Stern (1994), Datta and Ghosh (1995a,b, 1996), Mukerjee and Ghosh (1997) and Mukerjee and Reid (1999).

On the other hand, Ghosh and Mukerjee (1992), and Berger and Bernardo (1989, 1992) extended Bernardo's (1979) reference prior approach, giving a general algorithm to derive a reference prior by splitting the parameters into several groups according to their order of inferential importance. This approach is very successful in various practical problems. Quite often reference priors satisfy the matching criterion described earlier.

The ratio of variance components in the random effect model has been of interest for a long time, especially in animal science, where this ratio is usually used to estimate the genetic heritability of a certain trait of livestock breeders (Graybill et al., 1956). One difficult part of the analysis of the model (1) from the sampling theory of view is the possible negative estimates for $\sigma^2_\alpha$ as well as for $\phi$. Thus a Bayesian analysis for this model is desirable, not only because of its intrinsic merit, but also because it can resolve this problem. Also the commonly used intraclass correlation $\rho = \sigma_\alpha/(\sigma^2_\alpha + \sigma^2)$ is one-to-one transformation of $\phi$. Therefore, when a good prior distribution for $\phi$ is obtained, its transformed prior distribution for $\rho$ can be used as well. Moreover, the probability matching priors are invariant for the transformation between $\phi$ and $\rho$. (cf. Datta and Ghosh, 1996, Mukerjee and Ghosh, 1997).

The problem of estimating variance components in the one-way random effect model has been investigated by many authors from the Bayesian point of view. We may refer to Hill (1965), Box and Tiao (1973), Palmer and Broemeling (1990), among others. Ye (1994) developed the reference priors for $\phi$, examined frequentist coverage probabilities for various $\phi$ and compared risk functions of the Bayes estimators for the reference priors. Also Chung and Dey (1998) derived the reference priors and first order probability matching priors for $\rho$ and examined the frequentist coverage probabilities for various $\rho$. Kim, Lee and Kang (2001) provided a class of second order probability matching priors for $\phi$ and $\rho$. It is shown that among all of the reference priors, the only one reference prior (one-at-a-time reference prior) satisfies a second order matching criterion.

The present paper focuses on noninformative priors for $\phi$. In Section 2, we provide the second order matching prior and reference prior. In Section 3, we reveal that the second order matching prior matches the alternative coverage probabilities up to the same order (Mukerjee and Reid, 1999), and is a HPD matching prior. We show that one-at-a-time reference prior satisfying the second order matching criterion produces confidence sets with expected length shorter than the other reference priors (Mukerjee and Reid, 1999; Datta and DiCiccio, 2001). Some concluding remarks are made in Section 4.

## 2. The Noninformative Priors

For a prior $\pi$, let $\theta_1^{1-\alpha}(\pi;\, \boldsymbol{Y})$ denote the $(1-\alpha)$th percentile of the posterior distribution of $\theta_1$, that is,

$$P^{\pi}[\,\theta_1 \le \theta_1^{1-\alpha}(\pi;\, \boldsymbol{Y}) \mid \boldsymbol{Y}] = 1-\alpha, \tag{2}$$

where $\boldsymbol{\theta} = (\theta_1, \cdots, \theta_k)$ and $\theta_1$ is the parameter of interest. We want to find priors $\pi$ for which

$$P[\,\theta_1 \le \theta_1^{1-\alpha}(\pi,\, Y) \mid \theta] = 1-\alpha + o(n^{-u}). \tag{3}$$

for some $u > 0$, as $n$ goes to infinity. Priors $\pi$ satisfying (3) are called matching priors. If $u = 1/2$, then $\pi$ is referred to as a first order matching prior, while if $u = 1$, $\pi$ is referred to as a second order matching prior.

In order to find such matching priors $\pi$, it is convenient to introduce orthogonal parametrization, (Cox and Reid, 1987; Tibshirani, 1989). To this end, let

$$\theta_1 = J\sigma_a^2/\sigma^2,\ \ \theta_2 = \sigma^2\left(\frac{\sigma^2 + J\sigma_a^2}{\sigma^2}\right)^{1/J},\ \ \theta_3 = \mu \tag{4}$$

With this parametrization, the likelihood function of parameters $(\theta_1, \theta_2, \theta_3)$ for model (1) is given by

$$L(\theta_1, \theta_2, \theta_3) \propto \theta_2^{-\frac{IJ}{2}} \exp\{-\frac{1}{2(1+\theta_1)^{-1/J}\theta_2}\,[S_2 + \frac{S_1 + IJ(\overline{Y} - \theta_3)^2}{1+\theta_1}]\}, \tag{5}$$

where $S_1 = J\sum_{i=1}^{I}(\overline{Y}_{i\cdot} - \overline{Y})^2$, $S_2 = \sum_{i=1}^{I}\sum_{j=1}^{J}(Y_{ij} - \overline{Y}_{i\cdot})^2$, $\overline{Y}_{i\cdot} = \sum_{j=1}^{J} Y_{ij}/J$ and $\overline{Y} = \sum_{i=1}^{I}\overline{Y}_{i\cdot}/I$.
Based on (5), the Fisher information matrix is given by

$$I_F = \begin{pmatrix} \frac{I(J-1)(1+\theta_1)^{-2}}{2J} & 0 & 0 \\ 0 & \frac{IJ}{2\theta_2^2} & 0 \\ 0 & 0 & \frac{IJ(1+\theta_1)^{(1-J)/J}}{\theta_2} \end{pmatrix}.$$

Thus $\theta_1$ is orthogonal to $\theta_2$ and $\theta_3$ in the sense of Cox and Reid(1987). Kim, Lee and Kang (2001) derived the class of second order probability matching priors as follows.

$$\pi_m^{(2)}(\theta_1, \theta_2, \theta_3) = (1+\theta_1)^{-1}\theta_2^{-1}h(\theta_3), \tag{6}$$

where $h(\theta_3)$ is any smooth function of $\theta_3$. Also we are interest in the intraclass correlation, $\rho = \sigma_a^2/(\sigma_a^2 + \sigma^2)$. Especially, $\rho = \theta_1/(\theta_1 + J)$ is a one to one function of $\theta_1$. Thus by using invariance of probability matching prior under one to one transformation of the parameter vector (Datta and Ghosh, 1996; Mukerjee and Ghosh, 1997), we obtain the second order probability matching prior for $\pi_m^{(2)}(\rho, \theta_2, \theta_3)$ from the matching prior (6) as follows.

$$\pi_m^{(2)}(\rho, \theta_2, \theta_3) = [1+(J-1)\rho]^{-1}(1-\rho)^{-1}\theta_2^{-1}h(\theta_3). \tag{7}$$

Reference priors introduced by Bernardo (1979), and extended further by Berger and Bernardo (1992) have become very popular over the years for the development of noninformative priors. Due to the orthogonality of the parameters, following Datta and Ghosh (1995), choosing rectangular compacts for each $\theta_1$, $\theta_2$ and $\theta_3$ when $\theta_1$ is the parameter of interest, the reference priors are given by as follows. (Kim, Lee and Kang, 2001). Notation such as $\{\theta_1, (\theta_2, \theta_3)\}$ will be used to specify the groups and the importance of the parameters; $\{\theta_1, (\theta_2, \theta_3)\}$ means that there are two groups, with $\theta_1$ being more important than the group $(\theta_2, \theta_3)$.

| Group ordering | Reference prior |
|---|---|
| $\{(\theta_1, \theta_2, \theta_3)\}$, | $\pi_1 \propto (1+\theta_1)^{-(3J-1)/2J}\theta_2^{-3/2}$ |
| $\{\theta_1, (\theta_2, \theta_3)\}$, | $\pi_2 \propto (1+\theta_1)^{-1}\theta_2^{-3/2}$ |
| $\{\theta_1, \theta_2, \theta_3\}, \{\theta_1, \theta_3, \theta_2\}, \{(\theta_1, \theta_2), \theta_3\}$, | $\pi_3 \propto (1+\theta_1)^{-1}\theta_2^{-1}$ |
| $\{(\theta_1, \theta_3), \theta_2\}$, | $\pi_4 \propto (1+\theta_1)^{-(3J-1)/2J}\theta_2^{-1}$ |

## 3. Main Results

### 3.1 The Probability Matching Priors: Matching the Alternative Coverage Probabilities

Mukerjee and Reid (1999) studied that a prior satisfying (3) matches $P[\theta_1 + \beta(I^{11})^{1/2} \le \theta_1^{1-\alpha}(\pi, Y) \mid \boldsymbol{\theta}]$ with the corresponding posterior probability, up to the same order and for each $\beta$ and $\alpha$, where the scalar $\beta$ is free from $n$, $\boldsymbol{\theta}$ and $\boldsymbol{Y}$. If a matching prior matches the alternative coverage probabilities then there is a stronger justification for calling it noninformative in so far as agreement with a frequentist is concerned. In general a second order matching prior may or may not match the alternative coverage probabilities up to the same order of approximation.

Under orthogonal parametrization, Mukerjee and Reid (1999) gives the simple differential equations that a second order probability matching prior matches alternative coverage probabilities up to the second order. The differential equations are given by

$$\sum_{i=2}^{t}\sum_{j=2}^{t} D_i\{L_{11j}I^{ij}I_{11}^{-1/2}d(\theta_2, \cdots, \theta_t)\} = 0, \tag{8}$$

$$\sum_{i=2}^{t}\sum_{j=2}^{t} D_i\{L_{j,11}I^{ij}I_{11}^{-1/2}d(\theta_2, \cdots, \theta_t)\} = 0, \tag{9}$$

$$D_1\{I_{11}^{-3/2}L_{111}\} = 0, \quad D_1\{I_{11}^{-3/2}L_{1,11}\} = 0, \tag{10}$$

where $\boldsymbol{\theta} = (\theta_1, \cdots, \theta_t)^T$, $\theta_1$ is the parameter of interest, $I_{ij}$ is the element that is in the $i$th row and $j$th column of $I_F$, $I^{ij}$ is the element that is in the $i$th row and $j$th column of the inverse matrix of $I_F$, $D_i = \partial/\partial\theta_i$, $i=1,\cdots,t$, $L_{11j} = E[D_1 D_1 D_j \log L]$ and $L_{j,11} = E[D_j \log L \times D_1 D_1 \log L]$, $j=1,\cdots,t$.

**Theorem 1.** The second order probability matching prior,

$$\pi_m^{(2)}(\theta_1, \theta_2, \theta_3) = (1+\theta_1)^{-1}\theta_2^{-1}h(\theta_3),$$

where $h(\theta_3)$ is any smooth function of $\theta_3$, matches the alternative coverage probabilities up to the second order.

**Proof.** Due to the orthogonality of $\theta_1$ with $\theta_2$ and $\theta_3$, the differential equations (8)-(10) are simplified to

$$D_2\{L_{112}I^{22}I_{11}^{-1/2}d(\theta_2,\theta_3)\} + D_3\{L_{113}I^{33}I_{11}^{-1/2}d(\theta_2,\theta_3)\} = 0,$$
$$D_2\{L_{2,11}I^{22}I_{11}^{-1/2}d(\theta_2,\theta_3)\} + D_3\{L_{3,11}I^{33}I_{11}^{-1/2}d(\theta_2,\theta_3)\} = 0,$$
$$D_1\{I_{11}^{-3/2}L_{111}\} = 0, D_1\{I_{11}^{-3/2}L_{1,11}\} = 0.$$

Since

$$d(\theta_2,\theta_3) = \theta_2^{-1}h(\theta_3),$$
$$L_{111} = I\left(\frac{J-1}{J}\right)\left(\frac{2J-1}{J}\right)(1+\theta_1)^{-3},\quad L_{112} = \frac{I(J-1)}{2J}(1+\theta_1)^{-2}\theta_2^{-1},\quad L_{113} = 0$$
$$L_{1,11} = c_1(1+\theta_1)^{-3}, c_1 = \text{ a constant},\ L_{2,11} = c_2(1+\theta_1)^{-2}\theta_2^{-1}, c_2 = \text{ a constant},\ L_{3,11} = 0,$$
$$I_{11} = \frac{I(J-1)(1+\theta_1)^{-2}}{2J},\quad I_{22} = \frac{IJ}{2\theta_2^2},\quad I_{33} = \frac{IJ(1+\theta_1)^{(1-J)/J}}{\theta_2},$$

thus

$$D_2\left\{\frac{I(J-1)}{2J}(1+\theta_1)^{-2}\theta_2^{-1}\cdot\frac{2\theta_2^2}{IJ}\cdot\sqrt{\frac{2J}{I(J-1)(1+\theta_1)^{-2}}}\cdot\theta_2^{-1}h(\theta_3)\right\} = 0,$$
$$D_2\left\{c_2(1+\theta_1)^{-2}\theta_2^{-1}\cdot\frac{2\theta_2^2}{IJ}\cdot\sqrt{\frac{2J}{I(J-1)(1+\theta_1)^{-2}}}\cdot\theta_2^{-1}h(\theta_3)\right\} = 0,$$
$$D_1\left\{\left[\frac{I(J-1)(1+\theta_1)^{-2}}{2J}\right]^{-3/2}\cdot I\left(\frac{J-1}{J}\right)\left(\frac{2J-1}{J}\right)(1+\theta_1)^{-3}\right\} = 0,$$
$$D_1\left\{\left[\frac{I(J-1)(1+\theta_1)^{-2}}{2J}\right]^{-3/2}\cdot c_1(1+\theta_1)^{-3}\right\} = 0.$$

Therefore the second order matching prior matches the alternative coverage probabilities up to the second order. This completes the proof. □

### 3.2 HPD Matching Priors

There are alternative ways through which matching can be accomplished. One such approach (DiCiccio and Stern, 1994; Ghosh and Mukerjee, 1995) is matching through the HPD region. Specifically, if $\tilde{\pi}$ denotes the posterior distribution of $\theta_1$ under a prior $\pi$, and $k_\alpha \equiv k_\alpha(\pi, Y)$ is such that

$$P^\pi[\,\tilde{\pi}(\theta_1 \mid Y) \geq k_\alpha \mid Y] = 1-\alpha+o(n^{-u}),$$

then the HPD region for $\theta_1$ with posterior coverage probability $1-\alpha+o(n^{-u})$ is given by

$$H_\alpha(\pi, Y) = \{\theta_1: \tilde{\pi}(\theta_1 \mid Y) \geq k_\alpha\}.$$

DiCicco and Stern (1994) and Ghosh and Murkerjee (1995) characterized priors $\pi$ for which

$$P[\theta_1 \in H_\alpha(\pi, Y) \mid \boldsymbol{\theta}] = 1-\alpha+o(n^{-u}), \tag{11}$$

for all $\boldsymbol{\theta}$ and all $\alpha \in (0,1)$. They found necessary and sufficient conditions under which $\pi$ satisfies (11). Due to the orthogonality of $\theta_1$ with $(\theta_2, \cdots, \theta_k)$, from equation (33) of DiCicco and Stern (1994) or equation (4.1) of Ghosh and Mukerjee (1995), a prior $\pi$ is a HPD matching prior if and only if it satisfies

$$\frac{\partial^2}{\partial\theta_1^2}\{I^{11}\pi\} - \frac{\partial}{\partial\theta_1}\{L_{111}(I^{11})^2\pi\} - \sum_{v=2}^{k}\frac{\partial}{\partial\theta_v}\{(\sum_{s=2}^{k}L_{11s}I^{vs})I^{11}\pi\} = 0. \tag{12}$$

Datta, Ghosh and Mukerjee (2000) provided a theorem which establishes the equivalence of second order matching priors and HPD matching priors within the class of first order matching priors. The equivalence condition is that $I_{11}^{-3/2}L_{111}$ dose not depend on $\theta_1$. Since

$$I_{11}^{-3/2}L_{111} = [\,\frac{I(J-1)(1+\theta_1)^{-2}}{2J}\,]^{-3/2} \cdot I(\frac{J-1}{J})(\frac{2J-1}{J})(1+\theta_1)^{-3},$$

it does not depend on $\theta_1$. Thus the second order probability matching prior,

$$\pi_m^{(2)}(\theta_1, \theta_2, \theta_3) = (1+\theta_1)^{-1}\theta_2^{-1}h(\theta_3),$$

where $h(\theta_3)$ is any smooth function of $\theta_3$, is a HPD matching prior up to the same order.

**Remark 1.** The reference prior (one-at-a-time reference prior), $\pi_3$, satisfies a second order matching criterion. Therefore the $\pi_3$ is a HPD matching prior and matches the alternative coverage probabilities up to the second order.

### 3.3 Expected Lengths

We compare HPD credible sets based on various noninformative priors and the various adjusted likelihoods via asymptotic expected volumes of confidence sets. Mukerjee and Reid (1999) and Datta and DiCiccio (2001) considered likelihood ratio (LR) confidence sets by inverting approximate $1-\alpha+o(n^{-1})$ acceptance regions of LR tests obtained via maximization of the adjust likelihoods. Let $C_u(X)$ be $1-\alpha+o(n^{-1})$ LR confidence set of $\theta_1$ based on the profile likelihood function of $\theta_1$ and $C(X)$ be a $1-\alpha+o(n^{-1})$ LR confidence set of $\theta_1$

based on adjustments to the profile likelihood. To compare these adjustments, Mukerjee and Reid (1999) and Datta and DiCiccio (2001) have developed an expression for the change in the asymptotic expected volume of $C(X)$ relative to the asymptotic expected volume of $C_u(X)$. We denote this quantity by $T$. It was shown by Mukerjee and Reid (1999) and Datta and DiCiccio (2001) that $T$ remains the same whether one uses adjustment to profile likelihood due to Cox and Reid or Barndorff-Nielsen or McCullagh and Tibshirani or Stern. Therefore, in our comparison, we consider only the Cox and Reid (1987) adjustment.

We write down an expression of $T$ which is obtained from equation (20) of Datta and DiCiccio (2001). Let $l(\theta)$ denote the log-likelihood function of $\theta$. Writing $D_u=\partial/\partial\theta_u$, we define $\lambda_{uvw}=E_\theta[D_uD_vD_wl(\theta)]$. Due to orthogonality of $\theta_1$ with $\theta_i$, $i=2,3$, the expression of $T$ as given by equation (20) of Datta and DiCiccio (2001) simplifies to

$$T=\frac{\delta^2}{2I_{11}}+\sum_{u=1}^{3}I^{1u}D_u\{\delta\}-\frac{1}{2}\sum_{u=1}^{3}\sum_{s=1}^{3}\sum_{t=1}^{3}I^{1u}I^{st}\delta(2D_t\{I_{us}\}+\lambda_{ust})+\frac{\lambda_{111}\delta}{2I_{11}^2}+o(n^{-1}),$$

where $\delta$ depends on the adjustment term and its expression will be given later. Using $I^{12}=I^{13}=0$, $I^{23}=0$, $\lambda_{122}=0$, $T$ reduces to

$$T=\frac{\delta^2}{2I_{11}}+D_1\{\delta I_{11}^{-1}\}-\frac{\delta}{2I_{11}I_{33}}\lambda_{133}+o(n^{-1}).$$

We denote the $\delta$ corresponding to Cox and Reid adjustment by $\delta^{CR}$ and the $\delta$ corresponding to a prior $\pi(\theta_1,\theta_2,\theta_3)$ by $\delta^{\pi}$. From equation (23) of Datta and DiCiccio (2001) it follows that

$$\delta^{CR}=\frac{1}{2}\sum_{i=2}^{3}\sum_{j=2}^{3}I^{ij}\lambda_{1ij}.$$

Using $\lambda_{122}=0$, $\lambda_{133}=I(J-1)(1+\theta_1)^{(1-2J)/J}\theta_2^{-1}$, $I^{23}=0$, $I^{33}=\theta_2(1+\theta_1)^{(J-1)/J}/IJ$ it follows that

$$\delta^{CR}=\frac{J-1}{2J}(1+\theta_1)^{-1}.$$

By equation (26) of Datta and DiCiccio (2001), we get

$$\delta^{\pi}=\delta^{CR}+D_1\{\log\pi\}=\frac{J-1}{2J}(1+\theta_1)^{-1}+D_1\{\log\pi\}.$$

Thus for a prior $\pi_a(\theta_1,\theta_2,\theta_3)=(1+\theta_1)^{-a}g(\theta_2,\theta_3)$ for some constant $a$ and for a general function $g(\theta_2,\theta_3)$, we get

$$\delta^{\pi_a}=(\frac{J-1}{2J}-a)(1+\theta_1)^{-1}. \quad (13)$$

Using $I_{11}=I(J-1)(1+w_1)^{-2}/2J$ we get from (13) that the expression of $T$ corresponding to the prior $\pi_a$, denoted by $T_a$, is given by

$$T_a = \frac{J}{I(J-1)} \left[ \left( \frac{J-1}{2J} - a \right)^2 + \left( \frac{J-1}{2J} - a \right) \left( 1 + \frac{1}{J} \right) \right] + o(n)^{-1}.$$

The class of priors given by $\pi_a(\theta_1, \theta_2, \theta_3)$ is general and by suitably choosing $a$ and $g(\theta_2, \theta_3)$ we get Jeffrey's prior, second order matching prior, reference prior for $\theta_1$. Note that the second order matching prior matches alternative coverage probabilities up to the second order (Mukerjee and Reid, 1999) and is a HPD matching prior. And one-at-a-time reference prior satisfies the second order matching criterion. Thus the one-at-a-time reference prior corresponds to $a=1$, while the other reference priors (including Jeffrey's prior) correspond to $a=(3J-1)/2J$. Under theses priors, we show that

$$T_1 < T_{\frac{3J-1}{2J}} < T_0,$$

where $T_0$ corresponds to Cox-Reid adjustments because

$$T_{\frac{3J-1}{2J}} - T_1 = \frac{(J-1)^2}{4J} > 0, \ J > 1.$$

Therefore the one-at-a-time reference prior produces confidence sets with expected length shorter than the other reference priors and Cox and Reid adjustment.

## 4. Concluding Remarks

In a balanced one-way random effect model when the ratio of the variance components and intraclass correlation is of interest, we revealed that the second order matching prior matches alternative coverage probabilities up to the second order (Mukerjee and Reid, 1999) and is a HPD matching prior. We easily derived the matching prior for the ratio of the variance components by orthogonal parametrization. And also by the invariance of probability matching prior under one to one transformation of the parameter vector, we obtained the second order probability matching prior for intraclass correlation. In the study of Ye (1994) for $\phi$, he derived the four reference priors, $\pi_1, \pi_2, \pi_3, \pi_4$, for different groups of ordering of $(\mu, \sigma^2, \phi)$. He revealed that from both the asymptotic frequentist coverage property and the decision theory points of view, the $\pi_3$, the one-at-a-time grouping reference prior, is the best among all of these reference priors. For $\rho$, the one-at-a-time reference prior is the most appealing reference distribution in the sense of the asymptotic frequentist coverage probability in Kim, Lee and Kang (2001). In our results, the only one-at-a-time reference prior satisfied the second order matching criterion.

In comparison expected length of confidence sets, we showed that HPD credible set based on the one-at-a-time reference prior has the smallest expected length of the confidence sets.

Consequently the one-at-a-time reference prior satisfying second order matching criterion seems to be the best appropriate results than the other reference priors for $\phi$ and $\rho$ in the sense of asymptotic frequentist coverage property and expected length of confidence interval.

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