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On Optimal Burn-in and Maintenance Policy

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Abstract

Burn-in is a widely used method to eliminate the initial failures. Preventive maintenance policy such as age replacement is often used in field operation. In this paper burn-in and maintenance policy are taken into consideration at the same time. The properties of the corresponding optimal burn-in times and optimal maintenance policy are discussed.

Keywords : burn-in, maintenance policy, repair

1. Introduction

Burn-in is a method used to eliminate the initial failures of components before they are put into field operation. The burn-in procedure is stopped when a preassigned reliability goal is achieved, e.g. when the mean residual life is long enough. Since burn-in is usually costly, one of the major problem is to decide how long the procedure should continue. The best time to stop the burn-in process for a given criterion to be optimized is called the optimal burn-in time. An introduction to this important area of reliability can be found in Jensen and Petersen (1982). In the literature, certain cost structures have been proposed and the corresponding problem of finding the optimal burn-in time has been considered. See, for example, Clarotti and Spizzichino (1991), Mi (1994), Cha (2000) and Block and Savits (1997) for a review of the burn-in procedure.

Let $F(t)$ be a distribution function of a lifetime X. If X has density $f(t)$ on $[0,\infty)$, then its failure rate function $h(t)$ is defined by $h(t)=f(t)/\overline{F}(t)$ where $\overline{F}(t)=1-F(t)$ is the survival function of X.

It is widely believed that many products, particularly electronic products such as silicon integrated circuits, exhibit bathtub-shaped failure rate functions. This belief is supported by much experience and extensive data collection by practitioners and researchers in many industries. The following is one definition of a bathtub-shaped failure rate function which we

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shall use.

Definition. A real-valued failure rate function $h(t)$ is said to be bathtub-shaped failure rate (BTR) with change points t_1 and t_2, if there exist change points $0 \le t_1 \le t_2 < \infty$ such that $h(t)$ is strictly decreasing in $[0, t_1)$, constant in $[t_1, t_2)$ and then strictly increasing in $[t_2, \infty)$.

The time interval $[0, t_1]$ is called the infant mortality period; the interval $[t_1, t_2]$, where $h(t)$ is flat and attains its minimum value, is called the normal operating life or the useful life; the interval $[t_2, \infty)$ is called the wear-out period.

The most common popular maintenance policy might be the age replacement policy. Under this policy a failed component is always replaced at the time of failure or T hours after its installation, where T is a fixed number, whichever occurs first. Once a cost structure is established to model the total cost related to the maintenance policy adopted, an optimal T is determined (denoted by T^* and called the optimal maintenance policy) such that the cost will be minimized. Under the assumption that the underlying distribution F has increasing failure rate function, Barlow and Proschan (1965) have shown that an optimal age replacement policy exists, but may be infinite.

The optimal maintenance policy, however, clearly depends on the distribution function of the component used in operation. It is thus natural to take both burn-in and preventive maintenance into consideration. That is, models are required to describe the total cost incurred by both burn-in and maintenance. To this end we determine the optimal burn-in time and optimal maintenance policy so that our cost function is minimized.

Mi (1994) consider the following procedure. Consider a fixed burn-in time b and begin to burn-in a new component. If the component fails before burn-in time b, then repair it completely with shop repair cost, then burn-in the repaired component again and so on. He assume that the repair is complete, i.e. the repaired component is as good as new. If the component survives the burn-in time b, then it is put into field operation. The cost for burn-in is assumed to be proportional to the total burn-in time. For a burned-in component he consider the age replacement policy. He discuss the properties of the optimal burn-in and optiaml maintenance policies. Cha (2000) consider that the failed component is only minimally repaired rather than being completely repaired during a burn-in period. He adopt block replacement policy with minimal repair at failure as it was in Mi (1994).

In this paper we consider the following burn-in procedure. Consider a fixed burn-in time b and begin to burn-in a new component. If the component fails before burn-in time b, then only minimal repair is done with shop repair cost, and continue the burn-in procedure for the repaired component. Immediately after the fixed burn-in time b, the component is put into field operation. Note that the total burn-in time of this burn-in procedure is a constant b. For a burned-in component, age replacement policy with complete repair is adopted.

In this paper we assume the distribution F of the original component (i.e. before burn-in) has a BTR function $h(t)$. Under this assumption the conclusion obtained is that the optimal burn-in time b^* must occur before the first change point t_1 of $h(t)$ and the sum b^*+T^* must occur the second change point t_2 of $h(t)$.

2. Burn-in and Maintenance Policy

Consider a fixed burn-in time b and begin to burn-in a new component. If the component fails before burn-in time b, then only minimal repair is done with shop repair cost $c_s>0$, and continue the burn-in procedure for the repaired component. Note that the total burn-in time of this procedure is a constant b. The cost for burn-in is assumed to be proportional to the total burn-in time with proportionality constant c_0. Then the total expected cost incurred by burn-in is the sum of the cost for burn-in, $c_0 b$, and the expected cost of minimal repairs, $c_s\int_0^b h(t)dt$;

$$C_1(b) = c_0 b + c_s\int_0^b h(t)dt. \tag{1}$$

where $\int_0^b h(t)dt$ is the expected number of minimal repairs during the burn-in period.

Immediately after the fixed burn-in time b, the component is put into field operation. For a burned-in component we adopt age replacement policy with complete repair described in Barlow and Proschan (1965). Let c_f denote the cost incurred for each failure in field operation and c_r satisfying $0<c_r<c_f$ the cost incurred for each non-failed item which is replaced at age $T>0$ in field operation. If we use only items which have survived the burn-in time b, then total expected replacement cost is the sum of the expected cost incurred by replacement at age T and the expected cost incurred by failure replacement before T;

$$C_2(T) = c_f F_b(T) + c_r\overline{F}_b(T), \tag{2}$$

where $\overline{F}_b(T)$ is the conditional survival function, i.e. $\overline{F}_b(x)=\overline{F}(b+x)/\overline{F}(b)$.

The total expected cycle length is the sum of the expected length of a replacement for non-failed item and the expected length of failure cycle;

$$T\overline{F}_b(T) + \int_0^T t f_b(t)dt = \int_0^T \overline{F}_b(t)dt. \tag{3}$$

Hence, from (1), (2) and (3) the long-run average cost per unit time $C(b, T)$ is given by

$$C(b,T)=\frac{c_0+c_s\int_0^b h(t)dt+c_f F_b(T)+c_r\overline{F}_b(T)}{\int_0^T\overline{F}_b(t)dt}$$
$$=\frac{(c_0+c_s\int_0^b h(t)dt)\overline{F}(b)+c_f(\overline{F}(b)-\overline{F}(b+T))+c_r\overline{F}(b+T)}{\int_0^T\overline{F}(b+t)dt},$$

The result regarding the optimal burn-in time b^* and the optimal age T^* which satisfy

$$C(b^*,T^*)=\min_{b\geq 0,\,T>0} C(b,T)$$

is given in the following theorem.

THEOREM 1 Suppose the failure rate function $h(t)$ is differentiable and BTR with change points t_1 and t_2. Then, the optimal burn-in time b^* and the corresponding optimal age $T^*=T^*_{b^*}$ satisfy

$$0\leq b^*\leq t_1 \quad\text{and}\quad b^*+T^*=b^*+T^*_{b^*}>t_2,$$

where $T^*_{b^*}$ is either the unique solution of Equation (4) or equal to ∞ depending on whether (4) has a solution or not.

$$h(b+T)\int_0^T\frac{\overline{F}(b+t)}{\overline{F}(b)}dt+\frac{\overline{F}(b+T)}{\overline{F}(b)}=\frac{c_f+c_0+c_s\int_0^b h(t)dt}{c_f-c_r}. \tag{4}$$

3. Proof of Theorem 1

For any fixed $b\geq 0$

$$\frac{\partial}{\partial T}C(b,T)=\frac{\overline{F}(b)\overline{F}(b+T)(c_f-c_r)}{(\int_0^T\overline{F}(b+t)dt)^2}\left\{\eta_b(T)-\frac{c_f+c_0+c_s\int_0^b h(t)dt}{c_f-c_r}\right\}, \tag{5}$$

where

$$\eta_b(T)=h(b+T)\int_0^T\frac{\overline{F}(b+t)}{\overline{F}(b)}dt+\frac{\overline{F}(b+T)}{\overline{F}(b)}.$$

Hence $\partial C(b,T)/\partial T=0$ if and only if

$$\eta_b(T)=\frac{c_f+c_0+c_s\int_0^b h(t)dt}{c_f-c_r}.$$

Taking partial derivative with respect to T one can obtain

$$\eta_b'(T)=h'(b+T)\int_0^T \frac{\overline{F}(b+t)}{\overline{F}(b)}\,dt.$$

Hence from the BTR assumption of $h(t)$ it follows that

$$\eta_b(T) \ \text{ is } \begin{cases} \text{strictly decreasing} & \text{if } \ T\le t_1-b, \\ \text{constant} & \text{if } \ t_1-b\le T\le t_2-b, \\ \text{strictly∈creasing} & \text{if } \ T\ge t_2-b. \end{cases}$$

Note that $(c_f+c_0+c_s\int_0^b h(t)dt)/(c_f-c_r)>1$, $\eta_b(0)=1$ and $\eta_b(T)$ is non-increasing for all T such that $T\le t_2-b$. Thus by (5) we immediately obtain that $\partial C/\partial T<0$ if $T\le t_2-b$. This imply that we have $b+T_b^*>t_2$ for all $b\ge 0$, where T_b^* is either ∞ or the unique solution of equation (4) and satisfies

$$C(b,T_b^*)=\min_{T>0} C(b,T), \ \text{ for all } \ b\ge 0.$$

Define the mean residual life function $\mu(t)=\int_t^\infty \overline{F}(u)du/\overline{F}(t)$. Then it is easy to see that $\eta_b(\infty)\equiv\lim_{T\to\infty}\eta_b(T)=h(\infty)\mu(b)$. Let $B_1\equiv\{b\ge 0: T_b^*<\infty\}$. Then clearly

$$\begin{aligned} B_1 &= \left\{ b\ge 0:\ \eta_b(\infty)>\frac{c_f+c_0+c_s\int_0^b h(t)dt}{c_f-c_r} \right\} \\ &= \left\{ b\ge 0:\ h(\infty)\mu(b)>\frac{c_f+c_0+c_s\int_0^b h(t)dt}{c_f-c_r} \right\}. \end{aligned}$$

Now we need the following lemma.

Lemma 1 (Mi, 1995) Let the mean residual life function $\mu(t)$ achieve its maximum value at $\tilde{b}$. Then

(1) $0\le\tilde{b}\le t_1$

(2) $$\mu(t) \ \text{ is } \begin{cases} \text{strictly increasing} & \text{if } 0\le t\le\tilde{b}, \\ \text{strictly decreasing} & \text{if } t\ge\tilde{b}. \end{cases}$$

From Lemma 1 it is easy to see that $(c_f+c_0+c_s\int_0^b h(t)dt)/\mu(b)$ is strictly increasing in $b\ge\tilde{b}$. Now we need to consider two cases <Case 1> $B_1=\phi$ and <Case 2> $B_1\ne\phi$.

In case 1 we have $T_b^*=\infty$ and for all $b>t_1$,

$$C(b,\ T_b^*) = C(b, \infty) = \frac{c_f + c_0 + c_s \int_0^b h(t)dt}{\mu(b)} > \frac{c_f + c_0 + c_s \int_0^{t_1} h(t)dt}{\mu(t_1)} = C(t_1, \infty)$$

and consequently in this case $b^* \le t_1$ and $T_b^* = \infty$.

For case 2, we can see the optimal burn-in b^* satisfies $b^* \le t_1$ applying the same method in the proof of Theorem 1 in Mi (1994).

Consequently, the optimal burn-in b^* and the corresponding optimal age $T^* = T^*_{b^*}$ satisfy,

$$0 \le b^* \le t_1 \quad \text{and} \quad b^* + T^* = b^* + T^*_{b^*} > t_2.$$

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