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Estimation of the Parameter of a Bernoulli Distribution Using a Balanced Loss Function

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Abstract

In decision theoretic estimation, the loss function usually emphasizes precision of estimation. However, one may have interest in goodness of fit of the overall model as well as precision of estimation. From this viewpoint, Zellner(1994) proposed the balanced loss function which takes account of both "goodness of fit" and "precision of estimation". This paper considers estimation of the parameter of a Bernoulli distribution using Zellner's(1994) balanced loss function. It is shown that the sample mean $\overline{X}$, is admissible. More general results, concerning the admissibility of estimators of the form $a\overline{X}+b$ are also presented. Finally, minimax estimators and some numerical results are given at the end of paper.

Keywords : Admissibility, Balanced loss function Bayes estimator, Inadmissibility, Minimax estimator.

1. Introduction

Let $X_1, \cdots, X_n$ be a random sample from a Bernoulli distribution with p.m.f,

$$f(x|p) = p^x \ (1-p)^{1-x}, \quad x=0,1 \text{ and } 0 < p < 1$$

This paper considers estimation of p under the balanced loss function (BLF),

$$L(\hat{p}, p) = \frac{\omega}{n} \sum_{i=1}^{n} (X_i - \hat{p})^2 + (1-\omega)(p-\hat{p})^2, \tag{1.1}$$

where $\hat{p}$ is any estimator of p, and ω is a nonstochastic weight such that $0 < \omega < 1$. This loss function, introduced by Zellner(1994), is formulated to reflect two criteria, namely goodness of fit and precision of estimation. In the past, loss functions reflect-ing one or the other of these criteria, but not both, have been employed in decision theoretic estimation. For example, least squares estimation reflects goodness of fit considerations whereas use of

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quadratic loss functions involves a sole emphasis on precision of estimation. As is well known, sole emphasis on a precision of estimation criterion, for example mean squared error can often lead to biased estimators. In some circumstances bias is not important but in others it is critical. On the other hand, use of a goodness of fit criterion leads to an estimate which gives good fit and is an unbiased estimator; however it may not be as precise as an estimator which is somewhat biased. Thus there is a need to provide a framework which combines goodness of fit and precision of estimation formally. The BLF framework meets this need. As mentioned above, the first term allows for "goodness of fit" and the second term "precision of estimation". For estimation under the BLF, for some standard distributions, see Zellner(1994), Rodrigues and Zellner(1995), Chung and Kim(1997), Chung, Kim and Song(1998), Chung, Kim and Dey(1999).

The admissibility of linear estimators of the form $a\overline{X}+b$, for estimating a Bernoulli mean and in general for estimating the mean of distributions of the one parameter exponential family under a squared error loss has been studied by Karlin(1958) and Gupta(1966).

In this paper, we consider estimation of p under the loss(1.1). In section 2, we obtain a Bayes estimator of p relative to the loss(1.1) and compute the risk and Bayes risk functions of $a\overline{X}+b$. In section 3, the class of inadmissible linear estimators of the form $a\overline{X}+b$ is classified. In section 4, the class of admissible linear estimators of the form $a\overline{X}+b$ is obtained and admissibility of $\overline{X}$ is proved. Finally, in section 5, minimax estimators of p under the loss(1.1) are tabulated for different values of n.

2. Bayes Estimators

For later use, in this section, we consider Bayesian estimation of p. The conjugate family of prior distribution for p is the family of Beta distribution, $B(\alpha,\beta)$, with density

$$\pi(p)=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\,p^{\alpha-1}\,(1-p)^{\beta-1},\quad 0<p<1 \tag{2.1}$$

where $\alpha>0$ and $\beta>0$. Note that the limiting case, $\alpha,\beta\to 0$ gives the usual "noninformative" prior for p, $\pi(p)\propto p^{-1}\,(1-p)^{-1}$.

It is easy to verify that the posterior distribution of p is

$$B(\alpha+\textstyle\sum_{i=1}^{n}x_i,\, n-\sum_{i=1}^{n}x_i+\beta).$$

The posterior risk of an estimator of $\hat{p}$ under the BLF is

$$E[L(\hat{p},p)|X]=\frac{\omega}{n}\sum_{i=1}^{n}(X_i-\hat{p})^2+(1-\omega)E[(p-\hat{p})^2|X],$$

where $X=(X_1,\cdots,X_n)$.

Solving the equation

$$\frac{\partial E[L(\hat{p}, p)|X]}{\partial \hat{p}} = 0,$$

we conclude that the Bayes estimator of p under the loss(1.1) is

$$\hat{p}_B = \omega \overline{X} + (1-\omega)\overline{p}$$

where $\overline{p}$ is the posterior mean. Hence, the Bayes estimator with respect to the Beta prior(2.1) is

$$\begin{aligned} \hat{p}_B &= \omega \overline{X} + (1-\omega)\frac{\alpha + \sum_{i=1}^{n} X_i}{n+\alpha+\beta} \\ &= \frac{n+\alpha\omega+\beta\omega}{n+\alpha+\beta}\overline{X} + (1-\omega)\frac{\alpha}{n+\alpha+\beta} \end{aligned} \tag{2.2}$$

Note that $\omega < \frac{n+\alpha\omega+\beta\omega}{n+\alpha+\beta} < 1$.

The risk of $\hat{p}_B$ may be derived directly or deduced from proposition 2.1 below where, for later use, we also give the risk function and Bayes risk of the linear estimator of $a\overline{X}+b$. Since, the derivations are straightforward it is omitted.

Proposition 2.1. The risk function of the estimator $a\overline{X}+b$, relative to the BLF loss function(1.1) is

$$\begin{aligned} R(p, a\overline{X}+b) &= [(a-1)p+b]^2 + \frac{p(1-p)}{n}[(a-\omega)^2 + \omega(n-\omega)], \\ &= [(a-1)^2 - \frac{(a-\omega)^2+\omega(n-\omega)}{n}]p^2 \\ &+ [2b(a-1) + \frac{(a-\omega)^2+\omega(n-\omega)}{n}]p + b^2. \end{aligned}$$

and the Bayes risk of $a\overline{X}+b$, relative to the Beta prior is

$$\begin{aligned} r(\pi, a\overline{X}+b) &= [(a-1)^2 - \frac{(a-\omega)^2+\omega(n-\omega)}{n}]\frac{\alpha(\alpha+1)}{(\alpha+\beta)(\alpha+\beta+1)} \\ &+ [2b(a-1) + \frac{(a-\omega)^2+\omega(n-\omega)}{n}]\frac{\alpha}{\alpha+\beta} + b^2. \end{aligned}$$

3. Inadmissibility

In this section, the class of inadmissible linear estimators of the form $a\overline{X}+b$, is obtained. An improved estimator is exhibited in each of the following six disjoint subclasses.

Theorem 3.1. The estimator $a\overline{X}+b$ is inadmissible under the loss function(1.1) whenever one of the following conditions hold:

(i) $a > 1$,
(ii) $a < \omega$,
(iii) $\omega < a < 1$ and $a + b > 1$,
(iv) $\omega < a < 1$ and $b < 0$,
(v) $a = 1$ and $b \neq 0$,
(vi) $a = \omega$ and $b < 0$.

Proof : (i) If a > 1, then $(a-\omega)^2 > (1-\omega)^2$ and hence by Proposition (2.1),

$$\begin{aligned} R(p, a\overline{X}+b) &\geq \frac{p(1-p)}{n}[(a-\omega)^2+\omega(n-\omega)] \\ &> \frac{p(1-p)}{n}[(1-\omega)^2+\omega(n-\omega)] \\ &= R(p, \overline{X}). \end{aligned}$$

Thus, $a\overline{X}+b$ is dominated by $\overline{X}$.

(ii) If $a < \omega$, then $(a-1)^2 > (\omega-1)^2$ and hence

$$\begin{aligned} R(p, a\overline{X}+b) &= [(a-1)p+b]^2+\frac{p(1-p)}{n}[(a-\omega)^2+\omega(n-\omega)] \\ &= (a-1)^2\, [p+\frac{b}{a-1}]^2 + \frac{p(1-p)}{n}[(a-\omega)^2+\omega(n-\omega)] \\ &> (\omega-1)^2\, [p+\frac{b}{a-1}]^2 + \frac{p(1-p)}{n}[(a-\omega)^2+\omega(n-\omega)] \\ &> (\omega-1)^2\, [p+\frac{b}{a-1}]^2 + \frac{p(1-p)}{n}\,\omega(n-\omega) \\ &= [(\omega-1)p+\frac{b(\omega-1)}{a-1}]^2+\frac{p(1-p)}{n}\,\omega(n-\omega) \\ &= R(p, \omega\overline{X}+\frac{b(\omega-1)}{a-1}). \end{aligned}$$

Thus in this case, $a\overline{X}+b$ is dominated by $\omega\overline{X}+\frac{b(\omega-1)}{a-1}$.

(iii) If $\omega < a < 1$ and $a + b > 1$, then

$$(a-1)p+b > (a-1)p+1-a = (a-1)(p-1) > 0,$$

and hence $[(a-1)p+b]^2 > [(a-1)p+1-a]^2$. We have

$$\begin{aligned} R(p, a\overline{X}+b) &= [(a-1)p+b]^2+\frac{p(1-p)}{n}[(a-\omega)^2+\omega(n-\omega)] \\ &> [(a-1)p+1-a]^2+\frac{p(1-p)}{n}[(a-\omega)^2+\omega(n-\omega)] \\ &= R(p, a\overline{X}+1-a). \end{aligned}$$

Thus $a\overline{X}+b$ is dominated by $a\overline{X}+1-a$, when condition (iii) holds.

(iv) For $\omega < a < 1$ and $b < 0$, the estimator $a\overline{X}+b$ is dominated by $a\overline{X}-b$. To see this, note that

$$R(p, a\overline{X}+b)-R(p, a\overline{X}-b) = [(a-1)p+b]^2-[(a-1)p-b]^2 = 4b(a-1)p > 0,$$

since $a < 1$ and $b < 0$.

(v) In this case, $a\overline{X}+b = \overline{X}+b$ is dominated by $\overline{X}$, since

$$\begin{aligned} R(p,\overline{X}+b) &= b^2+\frac{p(1-p)}{n}[(1-\omega)^2+\omega(n-\omega)] \\ &> \frac{p(1-p)}{n}[(1-\omega)^2+\omega(n-\omega)] \\ &= R(p,\overline{X}). \end{aligned}$$

(vi) For condition (vi), it is easily seen that $\omega\overline{X}+b$ is dominated by $\omega\overline{X}$ as regarding the difference functions of $\omega\overline{X}+b$ and $\omega\overline{X}$. It follows that

$$R(p,\omega\overline{X}+b)-R(p,\omega\overline{X}) = [(\omega-1)p+b]^2-[(\omega-1)p]^2 = b[2(\omega-1)p+b] > 0,$$

Hence $\omega\overline{X}+b$ is dominated by $\omega\overline{X}$.

Remark 3.2. Thus we see that in every case we are to look for admissible estimators of the form $a\overline{X}+b$ with $(a+b)$ lying in the following strip of the $a-b$ plane:

$$\{(a,b) : \omega \le a < 1,\ 0 \le b < 1-a\} \cup \{(1,0)\}.$$

4. Admissibility

In this section, admissible linear estimators are obtained. They are either proper Bayes estimators or generalized Bayes estimators relative to an appropriate limiting Beta prior.

Theorem 4.1. The estimator $a\overline{X}+b$ is admissible whenever $\omega < a < 1$ and $0 \le b < 1-a$.

Proof : Take $\omega < a < 1$, and consider first $0 \le b < 1-a$. Define

$$\alpha^* = \frac{nb}{a-\omega},\quad \beta^* = \frac{n(1-a-b)}{a-\omega}.$$

The conditions $\omega < a < 1$ and $0 \le b < 1-a$ ensure that $\alpha^* > 0$ and $\beta^* > 0$.
From (2.2), since

$$\frac{n+\alpha^*\omega+\beta^*\omega}{n+\alpha^*+\beta^*} = a,\quad \frac{(1-\omega)\alpha^*}{n+\alpha^*+\beta^*} = b,$$

It follows that $a\overline{X}+b$ is Bayes estimator of p relative to the prior distribution $B(\alpha^*, \beta^*)$. Also since the loss (1.1) is strictly convex, (2.2) is the unique Bayes estimator and hence admissible. It follows that $a\overline{X}+b$ is admissible when $\omega < a < 1$ and $0 < b < 1-a$.

For the case $b=0$, consider an estimator $a\overline{X}$ where a belongs to $(\omega, 1)$. We can show that $a\overline{X}$ is admissible when a is in $(\omega, 1)$. Let us consider the estimator as follows

$$\hat{p} = \left(\frac{n+\beta\omega}{n+\beta}\right)\overline{X}.$$

It is seen that $\hat{p}$ is the limit of Bayes estimators relative to the Beta prior $B(\alpha,\beta)$ as $\alpha \to 0$. We establish the admissibility of $\hat{p}$ by applying a limiting Bayes argument, a technique due to Blyth(1951).

Consider the improper prior

$$\pi_k(p) = p^{\frac{1}{k}-1}(1-p)^{\beta-1},$$

where $\beta > 0$. If A is a nondegenerate convex subset of (0,1), it can be shown that there exists a k_0 such that $\int_A \pi_k(p)dp \geq \varepsilon$ for some $\varepsilon > 0$ and all $k \geq k_0$.

The Bayes estimator with respect to π_k can be derived as in (2.2) which is given as

$$\widehat{p_k} = \frac{nk+\omega+\beta k\omega}{nk+1+\beta k}\overline{X} + \frac{1-\omega}{nk+1+\beta k}.$$

By proposition (2.1), the risk function of $\widehat{p_k}$ is

$$\begin{aligned} R(p,\widehat{p_k}) &= [(\frac{nk+\omega+\beta k\omega}{nk+1+\beta k}-1)^2 - \frac{1}{n}(\frac{nk+\omega+\beta k\omega}{nk+1+\beta k}-\omega)^2 - \frac{\omega(n-\omega)}{n}]p^2 \\ &+ \{\frac{2(1-\omega)}{nk+1+\beta k}[\frac{nk+\omega+\beta k\omega}{nk+1+\beta k}-1] + \frac{1}{n}(\frac{nk+\omega+\beta k\omega}{nk+1+\beta k}-\omega)^2 + \frac{\omega(n-\omega)}{n}\}\,p \\ &+ \frac{(1-\omega)^2}{(nk+1+\beta k)^2}. \end{aligned}$$

The Bayes risk of $\widehat{p_k}$ with respect to π_k is

$$\begin{aligned} r(\pi_k,\widehat{p_k}) &= [(\frac{nk+\omega+\beta k\omega}{nk+1+\beta k}-1)^2 - \frac{1}{n}(\frac{nk+\omega+\beta k\omega}{nk+1+\beta k}-\omega)^2 - \frac{\omega(n-\omega)}{n}]\frac{\Gamma(\frac{1}{k}+2)\Gamma(\beta)}{\Gamma(\frac{1}{k}+\beta+2)} \\ &+ \{\frac{2(1-\omega)}{nk+1+\beta k}[\frac{nk+\omega+\beta k\omega}{nk+1+\beta k}-1] + \frac{1}{n}(\frac{nk+\omega+\beta k\omega}{nk+1+\beta k}-\omega)^2 \\ &+ \frac{\omega(n-\omega)}{n}\}\frac{\Gamma(\frac{1}{k}+1)\Gamma(\beta)}{\Gamma(\frac{1}{k}+\beta+1)} + \frac{(1-\omega)^2}{(nk+1+\beta k)^2}\frac{\Gamma(\frac{1}{k})\Gamma(\beta)}{\Gamma(\frac{1}{k}+\beta)} \end{aligned}$$

The risk and Bayes risk of $\hat{p}$ are respectively

$$\begin{aligned} R(p,\hat{p}) &= [(\frac{n+\beta\omega}{n+\beta}-1)^2 - \frac{1}{n}(\frac{n+\beta\omega}{n+\beta}-\omega)^2 - \frac{\omega(n-\omega)}{n}]^2 \\ &+ [\frac{1}{n}(\frac{n+\beta\omega}{n+\beta}-\omega)^2 + \frac{\omega(n-\omega)}{n}]p, \end{aligned}$$

and

$$r(\pi_k, \hat{p}) = [(\frac{n+\beta\omega}{n+\beta}-1)^2-\frac{1}{n}(\frac{n+\beta\omega}{n+\beta}-\omega)^2-\frac{\omega(n-\omega)}{n}]\frac{\Gamma(\frac{1}{k}+2)\Gamma(\beta)}{\Gamma(\frac{1}{k}+\beta+2)}$$
$$+ [\frac{1}{n}(\frac{n+\beta\omega}{n+\beta}-\omega)^2+\frac{\omega(n-\omega)}{n}]\frac{\Gamma(\frac{1}{k}+1)\Gamma(\beta)}{\Gamma(\frac{1}{k}+\beta+1)}$$

The difference of the Bayes risks with respect to $\widehat{p_k}$ and $\hat{p}$ is

$$r(\pi_k, \hat{p})-r(\pi_k, \widehat{p_k}) = [\frac{(1-\omega)^2\beta^2}{(n+\beta)^2}-\frac{n(1-\omega)^2}{(n+\beta)^2}-\frac{(1+\beta k)^2(1-\omega)^2}{(nk+\beta k+1)^2}$$
$$+\frac{nk^2(1-\omega)^2}{(nk+\beta k+1)^2}]\frac{\Gamma(\frac{1}{k}+2)\Gamma(\beta)}{\Gamma(\frac{1}{k}+\beta+2)}+[\frac{n(1-\omega)^2}{(n+\beta)^2}+\frac{2(1-\omega)^2(1+\beta k)}{(nk+\beta k+1)^2}$$
$$-\frac{nk^2(1-\omega)^2}{(nk+\beta k+1)^2}]\frac{\Gamma(\frac{1}{k}+1)\Gamma(\beta)}{\Gamma(\frac{1}{k}+\beta+1)}-\frac{k(1-\omega)^2}{(nk+\beta k+1)^2}\frac{\Gamma(\frac{1}{k}+1)\Gamma(\beta)}{\Gamma(\frac{1}{k}+\beta)}$$

But,

$$\lim_{k\to\infty}(r(\pi_k, \hat{p})-r(\pi_k, \widehat{p_k})) = 0,$$

therefore, $\frac{n+\beta\omega}{n+\beta}\overline{X}$ is admissible for every $\beta > 0$. This proves that $a\overline{X}$ is admissible for $\omega < a < 1$.

Theorem 4.2. Under the BLF (1.1), $\overline{X}$ is admissible.

Proof : Consider the improper prior $\pi_k(p)$,

$$\pi_k(p)= p^{\frac{1}{k}-1}(1-p)^{\frac{1}{k}-1},$$

where $k > 0$. The Bayes estimator, $\widetilde{p_k}$, with respect to π_k can be derived as in (2.2) which is given as

$$\widetilde{p_k} = \frac{nk+2\omega}{nk+2}\overline{X}+\frac{1-\omega}{nk+2}.$$

The risk and Bayes risk of $\widetilde{p_k}$, with respect to π_k are

$$R(p, \widetilde{p_k}) = [\frac{nk^2(1-\omega)^2}{(nk+2)^2}+\frac{\omega(n-\omega)}{n}-\frac{4(1-\omega)^2}{(nk+2)^2}]p(1-p)+\frac{(1-\omega)^2}{(nk+2)^2},$$

and

$$r(\pi_k, \widetilde{p_k}) = \left[\frac{nk^2(1-\omega)^2}{(nk+2)^2} + \frac{\omega(n-\omega)}{n} - \frac{4(1-\omega)^2}{(nk+2)^2}\right] \frac{\Gamma(\frac{1}{k}+1)\Gamma(\frac{1}{k}+1)}{\Gamma(\frac{2}{k}+2)}$$
$$+ \frac{(1-\omega)^2}{(nk+2)^2} \frac{\Gamma(\frac{1}{k})\Gamma(\frac{1}{k})}{\Gamma(\frac{2}{k})}$$

Also, the risk and Bayes risk of $\overline{X}$ are respectively

$$R(p, \overline{X}) = \frac{(1-\omega)^2 + \omega(n-\omega)}{n} p(1-p)$$

and

$$r(\pi_k, \overline{X}) = \frac{(1-\omega)^2 + \omega(n-\omega)}{n} \frac{\Gamma(\frac{1}{k}+1)\Gamma(\frac{1}{k}+1)}{\Gamma(\frac{2}{k}+2)}$$

Then

$$r(\pi_k, \overline{X}) - r(p_k, \widetilde{p_k}) = \left[\frac{(1-\omega)^2}{n} - \frac{nk^2(1-\omega)^2}{(nk+2)^2} + \frac{4(1-\omega)^2}{(nk+2)^2}\right] \frac{\Gamma(\frac{1}{k}+1)\Gamma(\frac{1}{k}+1)}{\Gamma(\frac{2}{k}+2)}$$
$$- \frac{2k(1-\omega)^2}{(nk+2)^2} \frac{\Gamma(\frac{1}{k}+1)\Gamma(\frac{1}{k}+1)}{\Gamma(\frac{2}{k}+1)}$$

therefore,

$$\lim_{k \to \infty} (r(\pi_k, \overline{X}) - r(\pi_k, \widetilde{p_k})) = 0.$$

Hence, $\overline{X}$ is admissible by Blyth's(1951) Lemma.

Remark 4.3. The case not covered yet is when $a = \omega$ and $b = 0$. It is seen that $\omega\overline{X}$ is the limit of Bayes estimators (2.2), when $\alpha \to 0$, $\beta \to \infty$, and it is conjectured that it is admissible, but we do not have a proof. The problem is that the limiting Bayes argument does not work in this case.

5. Minimaxity

To determine a minimax estimator, we consider the Bayes estimator (2.2) and find values of $\alpha > 0$ and $\beta > 0$ such that the risk function of this Bayes estimator is constant. By Proposition(2.1), the risk function of $\hat{p}_B$ is given by

$$R(p, \hat{p}_B) = \left[\frac{(1-\omega)^2(\alpha+\beta)^2}{(n+\alpha+\beta)^2} - \frac{n(1-\omega)^2}{(n+\alpha+\beta)^2} - \frac{\omega(n-\omega)}{n}\right]p^2 + \left[\frac{-2\alpha(\alpha+\beta)(1-\omega)^2}{(n+\alpha+\beta)^2} + \frac{n(1-\omega)^2}{(n+\alpha+\beta)^2} + \frac{\omega(n-\omega)}{n}\right]p + \frac{(1-\omega)^2\alpha^2}{(n+\alpha+\beta)^2} \quad (5.1)$$

Setting the coefficients of p^2 and p in (5.1) equal to zero shows that (5.1) is constant if and only if

$$\begin{cases} n(\alpha+\beta)^2(1-\omega)^2 = n^2(1-\omega)^2 + \omega(n-\omega)(n+\alpha+\beta)^2 \\ 2n\alpha(\alpha+\beta)(1-\omega)^2 = n^2(1-\omega)^2 + \omega(n-\omega)(n+\alpha+\beta)^2 \end{cases} \quad (5.2)$$

Now assume that $0 < \omega < \frac{3n-\sqrt{n(5n-4)}}{2(n+1)}$. Note that $\frac{3n-\sqrt{n(5n-4)}}{2(n+1)} \le \frac{1}{2}$. Solving equations in (5.2) for α and β gives

$$\alpha = \beta = \frac{n\omega(n-\omega) + n(1-\omega)\sqrt{n(1-\omega)^2 + \omega(n-1)(n-\omega)}}{2n(1-\omega)^2 - 2\omega(n-\omega)} = c_n(\omega)(say).$$

From our assumption, we have $c_n(\omega) > 0$, and hence, the estimator

$$\delta = \frac{n+2c_n(\omega)\omega}{n+2c_n(\omega)}\overline{X} + \frac{(1-\omega)c_n(\omega)}{n+2c_n(\omega)},$$

is the unique minimax estimator of p under the loss (1.1) with $0 < \omega < \frac{3n-\sqrt{n(5n-4)}}{2(n+1)}$.

The risk function of δ is

$$R(p,\delta) = \frac{(1-\omega)^2 \; c_n^2(\omega)}{(n + 2\, c_n(\omega))^2}$$

In table 1, we calculate the values of $c_n(\omega)$ for the different values of n when $\omega = \frac{1}{3}, \frac{1}{4}$ and $\frac{1}{5}$.

Table 1: Values of $c_n(\omega)$

	$\omega = \frac{1}{3}$	$\omega = \frac{1}{4}$	$\omega = \frac{1}{5}$
n	$c_n(\omega)$	$c_n(\omega)$	$c_n(\omega)$
2	4.07	2.00	1.50
3	6.97	3.00	2.16
4	10.00	4.00	2.80
5	13.10	5.00	3.44
10	28.95	10.00	6.62
15	45.00	15.00	9.80
20	61.10	20.00	12.97
25	77.22	25.00	16.14
30	93.36	30.00	19.31
50	157.95	50.00	31.99
100	319.51	100.00	63.68

If we want to choose between δ and the MLE $\overline{X}$, which has risk function

$$R(p,\overline{X}) = \frac{p(1-p)}{n}[(1-\omega)^2+\omega(n-\omega)],$$

and is not constant, Figure 1 is helpful. In Figure 1, we compare risk of $\overline{X}$ and δ for sample sizes $n = 4, 30, 100$ when $\omega = \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ respectively. It is seen that, for small value of n, δ is better than $\overline{X}$ for most of the range of p (unless there is a strong belief that p is near 0 or 1) and for large (and even moderate) n, $\overline{X}$ is better than δ.

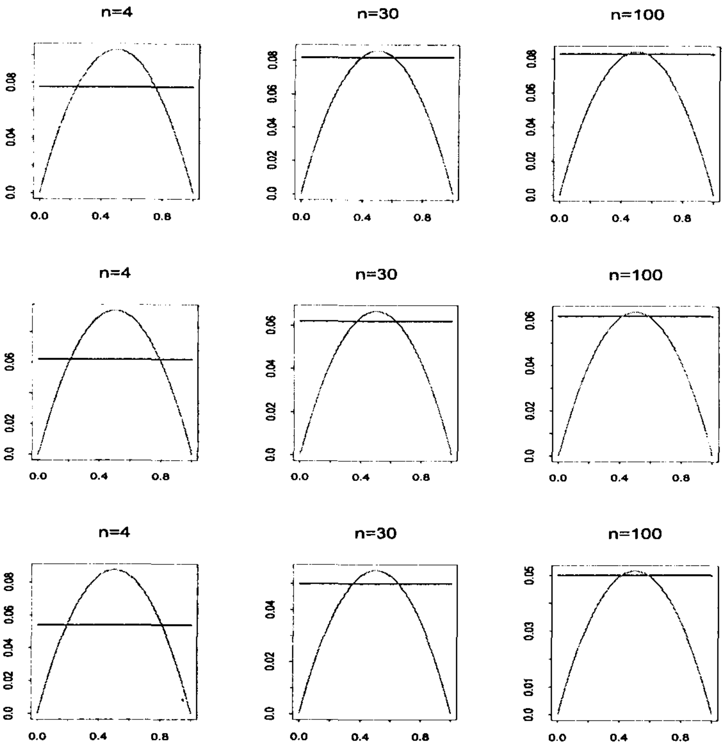

Figure 1: comparison of risk of $\overline{X}$ (curve) and δ (straight line) for sample sizes $n = 4, 30, 100$ when $\omega = \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ respectively.

References

[1] Blyth, C.R. (1951). On minimax statistical decision procedures and their admissibility. Ann. Math. Statist. Vol. **22**, 22-42.

[2] Casella, G. and Berger, R.L. (1990). Statistical Inference. Wadsworth, Belmont, CA.

[3] Chung, Y. and Kim, C. (1997). Simultaneous estimation of the multivariate normal mean under balanced loss function. Communications in Statistics- Theory and Methods. Vol. **26**, 1599-1611.

[4] Chung, Y., Kim, C. and Song, S. (1998). Linear estimators of a Poisson mean under balanced loss functions. Statistics and Decisions. Vol. **16**, 245-257.

[5] Chung, Y., Kim, C. and Dey, D.K. (1999). A new class of minimax estimators of multivariate normal mean vector under balanced loss function. Statistics and Decision. Vol. **17**, 255-266.

[6] Gupta, M.K. (1966). On the admissibility of linear estimates for estimating the mean of distribution of the one parameter exponential family. Calc. Statist. Assoc. Bull. Vol. **15**, 14-19.

[7] Karlin, S. (1958). Admissibility for estimation with quadratic loss. Ann. Math. Statist. Vol. **29**, 406-436.

[8] Lehmann, E.L. and Casella, G. (1998). Theory of point Estimation. Springer Verlag, New York.

[9] Rodrigues, J. and Zellner, A. (1995). Weighted balanced loss function for the exponential mean time to failure. Cummunications in Statistics-Theory and Methods Vol. **23**, 3609-3616.

[10] Zellner, A. (1994). Bayesian and Non-Bayesian estimation using balanced loss functions. Statistical Decision Theory and Related Topics V, (J.O. Berger and S.S. Gupta Eds). New York: Springer-Verlag: 377-390.

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