Transport Phenomena in Solid State Fermentation: Oxygen Transport in Static Tray Fermentors

P. K. A. Muniswaran[1]*, S. Sundara Moorthy[2], and N. C. L. N. Charyulu[3]

[1] Department of Chemical Engineering, Manipal Institute of Technology, Manipal, India
[2] Chemical Eng., Pondicherry Eng. College, Pondicherry, India
[3] Chemical Eng., C.B.I.T., Hyderabad, India

Abstract A mathematical model has been developed for describing the oxygen concentration during the exponential growth of microorganisms, in a static solid substrate bed supported on a tray fermentor. The model equations comprise of one partial differential equation for mass transfer and an ordinary differential equation of growth. After nondimensionlisation, analytical solution to the model has been obtained by the method of Laplace transforms. An expression for critical thickness of bed is deduced from the model equation. The significance of the model in the design of tray fermentors is discussed. The validity of the discussion is verified by taking an illustration from the literature.

Keywords: solid state fermentation, mathematical model, tray fermentor, oxygen concentration, critical bed depth

INTRODUCTION

Solid state fermentation (SSF) is becoming increasingly important as a potential tool for the commercial production of many chemicals and enzymes, due to its numerous advantages [1,2]. In aerobic fermentations, oxygen is required for the microbial growth. In SSF, this vital oxygen requirement is supplied by the oxygen present in the gaseous air with which the organism is in physical contact, and as well as by that present in dissolved form in the water associated with the solids. The following is the sequence of events that occur in the bed.

1. The oxygen concentration and temperature are uniform in the bed at the time of inoculation.
2. With the initiation and progress of the bioreaction, oxygen is consumed and CO_2 and heat are evolved.
3. The depletion of oxygen at the growth regions by microbial consumption produces a gradient causing the oxygen from the air above the bed to diffuse into the bed,
4. The CO_2 and heat evolved due to respiration travels up into the gas phase from the interior of the bed.

The oxygen depletion rate depends upon the bioreaction rate which is a function of the conditions prevailing at the reaction sites. The oxygen diffusion rate depends upon the transport properties of the bed. Further, the shrinkage of substrate bed due to mycelial growth that changes the bed porosity and hence the effective diffusivity as well as the CO_2 traveling in the opposite direction hampers the oxygen transport into the bed. Under these circumstances, the original uniform oxygen concentration slowly gives way to the formation of a gradient. At some locations deeper into the bed, the oxygen concentrations become smaller, approaching a value of zero concentration in the course of bioreaction. The existence of concentration gradient in static SSF beds have been reported by many workers [3-5]. Bioreaction can not proceed at the locations, where oxygen concentration falls to zero. For the efficient design of such reactors, it is necessary to predict the concentration profiles in the bed and identify these locations. Mathematical models, dealing with the transport of mass and heat, and their interaction with the bioreaction are few in the literature. The model proposed by Finger et al. [3] for aerobic microbial growth coupled with mass transfer in compost piles and that of Raghav Rao et al. [6] for concentration profiles and critical depth under pseudo steady sate approximation are noteworthy contributions in this regard. In view of the significance of mass transfer in SSF and the paucity of information regarding the interaction of mass transfer with bioreaction, an attempt is made in this paper to develop a model for describing the oxygen concentrations in a fermenting solid substrate bed supported on a tray type fermentor. The purpose is to shed more light on the oxygen transport aspects involved in SSF through the model as well as to develop a criteria for the efficient design of tray reactors.

---
* **Corresponding author**
Tel: +91-0825-571061   Fax: +91-0825-571071
e-mail: munishpka@yahoo.com

MODEL

Fig. 1 shows the schematic diagram of a controlled volume of substrate bed of uniform thickness, supported on a tray.

The following assumptions have been made in developing the model equation.

) The diffusional oxygen flow prevails only in the 'Z' direction $C \equiv C(Z,t)$.

i) Only the exponential growth phase is considered for the purpose of the derivation.

ii) There is neither induced convective flow, nor natural convective flow of oxygen in the bed. There is no mass flow at the bottom end of the bed; Resistance to mass transfer in the stagnant gas film above the bed, is negligible.

v) The transport properties of the bed remain constant.

/) The kinetic parameters μ, Y_x/O_2 and X_o are constant.

Writing the oxygen mass balance in the differential depth ΔZ and simplifying.

Rate of O_2 in $|_{Z^*-\Delta Z}$ = rate of O_2 out $|_{Z^*}$ + Rate of consumption of O_2 + Rate of accumulation of O_2

$$\cdot\ AD_e \frac{\delta}{\delta Z^*}|_{Z^*-\Delta Z} = -AD_e \frac{\delta c}{\delta Z}|_{Z^*} = R/Y(\rho_m.A.\Delta Z^*) + \delta/\delta t(A.\Delta Z. \in .C)$$

$$\overline{D}_e \frac{\delta^2 c}{\delta z^{*2}} = \frac{R}{Y} * \frac{\rho_m}{\in} + \frac{\delta c}{\delta t} \tag{1}$$

The biomass production rate 'R' is taken to follow the Malthus equation,

$$R = dX / dt = \mu X \tag{2}$$

Where μ is the specific growth rate and X is the biomass concentration.

Rewriting Eq. (1), and rearranging,

$$\overline{D}_e \frac{\delta^2 c}{\Delta z^{*2}} - \frac{\delta c}{\delta t} - \frac{\rho_m \mu}{Y \epsilon} * X = 0 \tag{3}$$

The initial and boundary conditions are,

At time $t = 0$, $C(Z^*,0) = C_b$

$X(0) = X_o$

At $Z^* = 0$, $C(0,t) = C_b$

AT $Z^* = L$, $\frac{\delta c}{\delta Z^*}|_{Z^*=L} = 0$

where C_b is the concentration of O_2 in the bulk of air above the bed. As it is desirable to rewrite Eqs. (2) and (3) in dimensionless form, the following dimensionless variable are defined;

$$\overline{C} = \frac{C}{C_b} \tag{4}$$

$$\overline{X} = \frac{X}{X_o} \tag{5}$$

$$\psi^* = \frac{Z^*}{L} \tag{6}$$

$$\theta = \frac{t.\overline{D}e}{L^2} \tag{7}$$

$$\psi = \frac{Z}{L} = 1 - \psi^* \tag{8}$$

where L is the total bed height. It may be noted that the dimensionless time θ is the well known Fourier number for mass transfer. Substitution of these dimensionless variables in Eqs. (2) and (3) results, respectively in,

$$\frac{d\overline{X}}{d\theta} = q\overline{X} \tag{9}$$

$$\frac{\delta^2 c}{\delta \psi^{*2}} - \frac{\delta c}{\delta \theta} - \beta \overline{X} = 0 \tag{10}$$

$$q = \frac{L^2 \mu}{De} \tag{11}$$

$$\beta = \frac{\rho_m \mu L^2 X_o}{D_e Y.\epsilon C_b} \tag{12}$$

$$= \frac{\rho_m.X_o}{Y.\epsilon.C_b.} * \frac{L^2 \mu}{De}$$

$$= \beta^1 q \tag{13}$$

The dimensionless number q is the ratio of the resistance to oxygen transfer in the bed to the resistance for the growth of the organism. It reflects the relative magnitude of the two rates, and may be termed as "growth number". β^1, is the ratio of the initial quantity of biomass available, to that of biomass producible from the available oxygen, and thus may be termed as "Oxygen number". The boundary conditions in dimensionless form become,

At $\theta = 0$
$$\left.\begin{array}{l} \overline{C}(\psi^*,0) = 1 \\ \\ X(0) = 1 \end{array}\right\} \tag{14}$$

At $\psi^* = 0$ $\overline{C}(0,\theta) = 1$ \tag{15}

At $\psi^* = 1$, $\frac{d\overline{c}}{d\psi^*}|_{\psi^*=1} = 0$ \tag{16}

In setting the above boundary condition (Eq. (16)) it is presumed that oxygen is available at all bed depths during growth. The model equations are solved simultaneously using Laplace transforms, to result in the final analytical solution as,

$$\overline{C}(\psi,\theta) = 1 - \beta^1 e^{q\theta}\left[1 - \frac{\cosh\sqrt{q}.\psi}{\cosh\sqrt{q}}\right]$$

$$+ \frac{4\beta^1 q}{\pi} \sum_{n=1}^{\alpha} \frac{\cos\{[(2n-1)\pi/2]\psi\} - e^{\frac{-(2n-1)^2 \pi^2\theta}{4}}}{(2n-1)[(2n-1)^2 \pi^2 + 4q]} \tag{17}$$

As the contribution of the sigma term in the equation is relatively negligible, approximation of the equation yields,

$$\overline{C}(\psi,\theta) = 1 - \beta^1 e^{q\theta}\left[1 - \frac{\cosh\sqrt{q}\cdot\psi}{\cosh\sqrt{q}}\right] \qquad (18)$$

which facilitates the prediction of oxygen concentrations in a solid substrate bed, when the bed depth is equal or less than the critical bed depth.

Inspection of Eq. (18) shows that the term $\beta^1 e^{q\theta}$ and q are critical in the determination of the oxygen concentration profiles. As $\beta^1 e^{q\theta}$ increases, the profiles become steeper, eventually resulting in zero oxygen concentration at the bottom of the bed at a certain fermentation time. Beyond a certain value of $\beta^1 e^{q\theta}$ the zero oxygen concentration occurs, not at the bottom of the bed, but at some higher locations in the bed, leaving part of the bed devoid of oxygen. Increase in this value will result in larger zones of zero concentrations in the bed. Similar will be the effect if the total bed height 'L' is increased since q is a function of bed height L. The bed height at which the oxygen concentration at the bottom of the bed just drops to zero can be defined as the critical bed depth. Correspondingly, introducing C = 0 at ψ = 0 in Eq. (18) at which $L = L_c$ and simplifying the resulting equation, the critical bed depth is obtained by the equation,

$$Lc = \sqrt{\frac{\overline{D_e}}{\mu}}\ln\left[b + \sqrt{(b^2 - 1)}\right] \qquad (19)$$

where

$$b = 1/(1\text{-}p) \qquad (20)$$

in which $\quad p = \dfrac{Y\varepsilon C_b - e^{-\mu\Delta t}}{\rho_m X_c} \qquad (21)$

Knowing the kinetic variables Xo, $\underline{Y}$ and μ, transport properties of the bed ρ_m, ε and $\overline{D_e}$ and atmospheric oxygen concentration C_b the critical bed depth can be determined from Eq. (19).

In the design of tray type reactors, the determination of the critical depth is the crucial aspect and Eqs. (18) and (19) are adequate for this purpose.

RESULTS AND DISUCSSION

The parameter 'q' in the Eq. (18) represents the relative significance of biomass growth rate and oxygen diffusion rate. The term β^1 represents the effect of bed porosity, atmospheric oxygen concentration, bulk density and the kinetic parameters. Together, these two decide the magnitude of oxygen concentrations in the bed and the nature of the concentration profile, indicating whether the concentration drops to zero in the bed, and if it does, the time and depth at which it does so. However, it is easier to examine the effect of the vari-
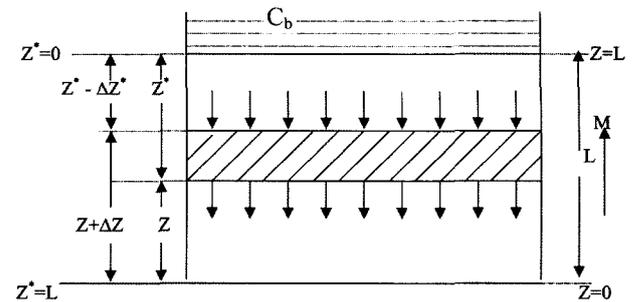

Fig. 1. Schematic sketch of a controlled volume of substrate bed.

ables involved in these two terms, using the expression for critical depth.

The concept of critical depth implies that when the bed depth is greater than the critical value, the excessive bed undergoes oxygen depravation. Therefore an equation to predict the critical bed thickness will be of design significance. Eq. (19), expresses the critical design thickness L_c in terms of the structural, kinetic and transport variables.

The validity of the above discussion is verified by taking an illustration. For this purpose the growth data of Sugama and Okazaki [9] is taken. The initial biomass concentration Xo the yield coefficient Y and the lag time are observed to be 0.28 mg cells/mL. 1.07 mg cells/g O_2 and 10 h respectively. The data is found to fit the Malthus model of growth used in the derivation and the specific growth rate is determined from a plot of $\ln X$ vs. t as 0.2031 h^{-1}. The effective diffusivity value De is also taken as 0.03 cm^2/sec from the literature [10], assuming the value to be valid at any bed porosity. The oxygen concentration in the atmospheric air is calculated as 2.7×10^{-4} g/mL. As the model is applicable only in the exponential phase, the fermentation time, used in the calculations is obtained by subtracting the lag time from the total time. For example fermentation time of 24 h corresponds to (24-10) = 14 h of exponential growth, which is the value used.

The oxygen concentration profiles for beds of porosity 0.4 and total heights 2 cm and 5 cm calculated using Eq. (18) are shown in Figure 2-3. The profiles become steeper as time increases and obtain lower concentration values at the bottom of the bed. For example, the concentration C at the bottom of the bed ($\psi^* = 1$) is 0.69 in 32 h and 0.30 in 36 h (Fig. 2).

Comparison of the concentration profiles for different total bed heights L, (Fig. 2 and 3) reveals that with increase in L, the profiles become steeper and concentration C at a specific location ψ^* and time θ decreases. This means that the zero oxygen concentration is arrived at the bottom of the bed more quickly as the total bed height is increased. For example, the zero concentration occurs at the bottom of the 2 and 5 cm bed in >36 and >28 h respectively.

Critical bed height values calculated using Eq. (19), agreed with those observed from Figures, when the total
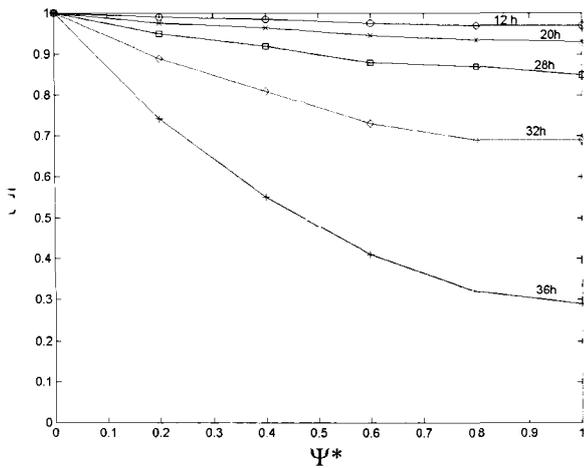
Fig. 2. Variation of oxygen concentration with bed depth at different fermentation times. Total bed depth =2 cm; Bed porosity = 0.4.
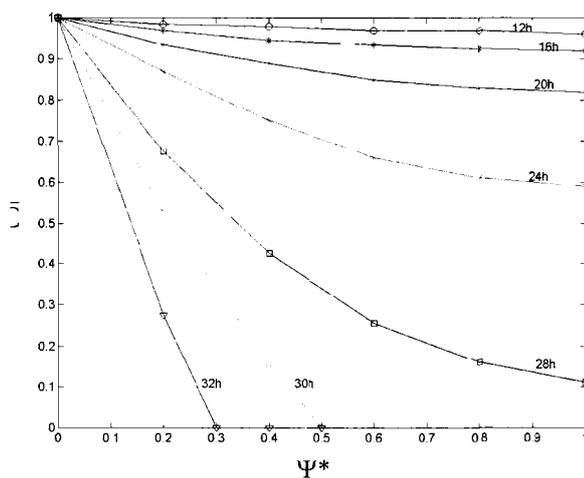

Fig. 3. Variation of oxygen concentration with bed depth at different fermentation times; total bed depth = 5 cm; bed porosity = 0.4.

bed height is less than or equal to the critical height. Another inference regarding L_c is that it decreases with decreasing bed porosity. For example, the value of critical bed depths were 3.38, 2.926 and 2.386 cms respectively for beds of 0.4, 0.3 and 0.2 voidage (Eq. (19)) for a fermentation time of 24 h.

The foregoing discussion illustrated that the oxygen concentration falls to zero at some interior location in the bed, depending on the time of fermentation, total bed height and bed porosity under unchanging kinetic, structural and transport conditions. The depth of the bed in which oxygen is available, during exponential phase can be either calculated by Eq. (19) or by observation from the profiles, which are determined by Eq. (13).

CONCLUSION

The model relates the mass transport effect with bioreaction quantitatively, under the actual unsteady state conditions prevailing in the bed during fermentation. The model scientifically explains, and provides answers to many of the facts observed in practice in static tray fermentors.

NOMENCLATURE

C	Oxygen concentration in the bed, g/mL
C_b	Oxygen concentration in atmospheric air, g/mL
$\bar{C}$	Dimensionless oxygen concentration, C/C_b
D_e	Diffusivity of oxygen through bed, cm^2/h
$\bar{D}_e$	Effective diffusivity, D_e/ε, cm^2/h
L	Bed height, cm
q	Dimensionless parameter
R	Bioreaction rate, g cells/g substrate
t	Time of fermentation, hr
X	Microbial concentration, g/g
X_0	Initial microbial concentration, g/g
$\bar{X}$	Dimensionless microbial concentration, X/X_0
Y	Yield coefficient g cells/g O_2
Z	Bed height along the axis measured from Bottom, cm
Z^*	Bed height along the axis measured from top, cm

Greek Letters

ψ	Dimensionless bed height, Z^*/L
ψ^*	Dimensionless bed height, Z/L, $1-Z$
ε	Bed void fraction
θ	Dimensionless time t, $\bar{D}_e/L^2$
μ	Specific growth rate, h^{-1}
ρ_m	Bed bulk density, g/mL
β, β^1	Dimensionless parameters

REFERENCES

[1] Lonsane B. K, N. P. Ghildyal, S. Budiatman, and S. V. Ramakrishnan (1985) Engineering aspects of solid state fermentation. *Enzyme Microb. Technol.* 7: 258-265.

[2] Moo-Young M, A. R. Moreira, and R. P. Tengardy (1983) Principles of solid state fermentation. pp. 117-144. In: Smith J. E., D. A. Berry, and B. Kristainsen (eds.). *The Filamentous Fungi.* Vol. 4, Oxford IBH Publishers, New Delhi, India.

[3] Finger S. M, R. T. Hatch, and T. Reingan (1974) *Biotechnol. Bioeng.* 18: 1193-1218.

[4] Rathbun B. L. and M. L. Schuler (1983) Heat and mass transfer effects in static solid state fermentation: Design of fermentation chambers. *Biotechnol. Bioeng.* 25: 929-938.

[5] Ghildyal N. P, M. Ramakrishna, B. K. Lonsane, and N. G. Karanth (1992) Gaseous concentration gradients in tray type solid state fermentors: Effect on yield and produc-

tivities. *Bioprocess Engineering* 8: 67-72.

[6] Raghava Rao K. S. M. S, M. K. Gowtaman, N. P. Ghildyal, and N. G. Karanth (1993) A mathematical model for solid state fermentation in tray bioreactors. *Bioprocess Eng.* 8: 255-262.

[7] Cannel E. and M. Moo-Young (1980) Solid state fermentaiton systems. *Process Biochem.* 25: 24-28.

[8] Bajracharya R. and R. E. Mudgett (1980) Effect of controlled gas environment in solid state substrate fermentations of rice. *Biotechnol Bioeng.* 22: 2219-2235.

[9] Sugama S. and N. Okazaki (1979) Growth estimation of *Aspergillus oryzae* cultured on solid media. *J. Ferment. Technol.* 57: 408-412.

[10] Masumune S. and J. M. Smith (1982) Pore diffusion in silver catalysts. *AIChE J.* 8: 217-221.