기능저하 시스템에서의 최적 교체 정책

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On Optimal Replacement Policies for a Deteriorating System

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Abstract

In this paper, replacement problems for a deteriorating system are considered. In the system under consideration, the successive lifetimes after repair become shorter and shorter, while the consecutive repair times become longer and longer. More specifically, the lifetimes of the system form a nonhomogeneous Poisson process, whereas the consecutive repair times constitute a stochastically increasing geometric process. Optimal replacement policies for the long-run average cost rate and the steady state availability are considered. Also taking the cost and the availability into consideration at the same time, the properties of optimal policies under the Cost Priority Policy and the Availability Priority Policy are obtained.

1. Introduction

Many maintenance models assume that a repaired system is returned to the 'as good as new' state. In this type of maintenance models, the successive lifetimes generate a renewal process provided that the repair times are ignored. However in many practical situations, because of deterioration or friction in the system, the systems are deteriorating, in which the lifetime after each repair tends to be shorter and shorter whereas, in view of the ageing and cumulative wear, the repair times increase.

For modelling deteriorating systems several methods have been proposed. For example, in Ohnishi et al. (1986) and Lam and Yeh (1994a, 1994b), the deterioration was modelled by a "multi-state continuous-time Markov process", in which the system of state $i(0 \le i \le N)$ can transit to and only to state $i+1$ due to deterioration or state $N+1$(failed state). In the system, the lifetimes are independent and their distributions are limited only to the exponential.

Recently, a "geometric process" has been used for modelling deteriorating systems. Yeh (1988) suggested a repair replacement model, in which the nth lifetime X_n and the nth repair time Y_n are distributed as $a^{-(n-1)}X_1$ and $b^{-(n-1)}Y_1$, respectively, where $a>1$ and $b<1$, and $X_1, Y_1, X_2, Y_2, \cdots$ is an independent sequence. Thus the lifetimes(repair times) are stochastically decreasing(increasing) at a geometric rate. Yeh (1988) considered two kinds of replacement policies, one based on the working age T of the system and the other based on the failure number N of the system. The explicit expressions of the long-run average cost per unit time under these two kinds of policies were derived. This basic replacement model was generalized by Stadje and Zuckerman (1990), in which the independent successive lifetimes $\{X_n, n=1,2,\cdots\}$ of the system form a stochastically decreasing process and each lifetime has an increasing(non-decreasing) failure rate(IFR), and also the consecutive repair times $\{Y_n, n=1,2,\cdots\}$ constitute a stochastically increasing process and each repair time has the property that the new is better than used in expectation(NBUE). More recently, Zhang (1994) generalized Yeh (1988)'s work by a bivariate policy (T, N) under which the system is replaced at working age T or at the time of Nth failure, whichever occurs first.

However, in general, for most repairable systems the lifetimes are more likely to be dependent rather than being independent. Hence, in spite of its shortcomings, in many practical situations, a "nonhomogeneous Poisson process(NHPP)" is most commonly adopted for modelling a deteriorating system because of its mathematical tractability and its property of goodness of fit to real data(See also, Thompson (1981), Lawless (1982) and Asher and Feingold (1984)).

In this paper, it is assumed that the lifetimes of the system form a nonhomogeneous Poisson process with an increasing intensity function, whereas the consecutive repair times constitute a stochastically increasing geometric process. Hence the system failure rate is an increasing failure rate(IFR) and by a repair action the failed system is restored to its condition just prior to failure. The problem is to find an optimal working age T^* at which system is replaced by a new system. In many researches, to determine an optimal policy only economic criterion is considered. But, in some instances, the availability criterion is more serious than the economic consideration. For example, when our interest is in studying power supplies for a hospital or a steel manufacturing complex, or an electric security systems, or a protection systems, high availability is required. In this paper, optimal replacement policies for the long-run average cost rate and the steady state availability are considered. Also taking the cost and the availability into consideration at the same time, the properties of optimal policies under the Cost-Priority-Policy, in which optimal policy is determined so that the long-run average cost rate is low enough and the steady state availability is maximized, and those under the Availability-Priority-Policy are obtained.

2. Model Description

We make the following assumptions about the replacement model.

(i) The system failure rate is $\lambda(t)$ which is assumed to be IFR and assume that this system is replaced every time its age reaches at T.

(ii) For each intervening failure only minimal repair is done and assume that the minimal repair times and the times for replacement are not negligible.

Define N_i' as the number of minimal repairs in the ith renewal cycle. Let $X_{i,j}$ be the jth lifetime of the system in ith renewal cycle, $i=1,2,\cdots$, $j=1,2,\cdots,N_i$, where N_i is the total number of repairs including a replacement in ith renewal cycle($N_i=N_i'+1$). Also define the corresponding repair times $Y_{i,j}$.

(iii) Assume that the sequence $\{Y_{i,j},\ j=1,2,\cdots,N_i'\}$ forms a geometric process with parameter $a<1$(See the Definition 1 in Yeh (1988)), $i=1,2,\cdots$.

(iv) For $N_i\neq 1$, the distribution of $Y_{i,1}$ is $G_1(y)$ and that of the replacement time Y_{i,N_i} is $G_2(y)$, where the mean of $G_1(y)$ is ν_1 and that of $G_2(y)$ is ν_2. For $N_i=1$, the distribution of $Y_{i,1}$ is $G_2(y)$ and the mean of $Y_{i,1}$ is ν_2.

Let t denote the system operating time and S_i be the ith renewal time, $i=1,2,\cdots$, where $S_0\equiv 0$. Define $V_i=S_i-S_{i-1}$, $i=1,2,\cdots$, then V_i's are times between renewals and $V_i=\sum_{j=1}^{N_i}(X_{i,j}+Y_{i,j})=T+\sum_{j=1}^{N_i}Y_{i,j}$.

(v) Assume that V_i's are mutually independent, $i=1,2,\cdots$.

3. Optimal Policy for the Long-run Average Cost Rate

In this section, optimal replacement policy for the long run expected cost rate is considered. The problem considered in this section was investigated previously in Cha et al. (2000). Let the cost rate of a minimal repair be C_1, and the cost rate of a replacement be C_2. Hereafter throughout this paper we assume that $0 < C_1 < C_2$ then this means that the cost rate of a replacement is higher than that of a minimal repair. Furthermore, let the reward rate whenever the system is working be r. Then from renewal theory the long-run average cost per unit time is given by

$$C(T) \equiv [\textit{the expected cost incurred in a renewal cycle}] \,/\, [\textit{the expected length of a renewal cycle}]. \tag{1}$$

Hence the long-run average cost per unit time could be obtained by calculating the expected cost incurred in a renewal cycle and the expected length of a renewal cycle.

Lemma 1.

The long-run average cost per unit time $C(T)$ under policy T is given by

$$C(T) = [(\exp[(\frac{1}{a}-1)\Lambda(T)]-1)\cdot \frac{a}{1-a}\nu_1\cdot C_1+\nu_2\cdot C_2-rT] \Big/ [T+(\exp[(\frac{1}{a}-1)\Lambda(T)]-1)\cdot \frac{a}{1-a}\nu_1+\nu_2]. \tag{2}$$

Proof.

See Appendix for its poof. ■

The properties regarding the optimal replacement policy T_C^* minimizing $C(T)$ in (2) are given in the following theorem.

Theorem 1.

The optimal replacement policy T_C^* exists and is the unique solution of the equation

$$\exp[(\frac{1}{a}-1)\Lambda(T)]\cdot\{\nu_1\lambda(T)\cdot[(C_1+r)T+(C_1-C_2)\nu_2] - (C_1+r)\frac{a}{1-a}\nu_1\} + (C_1+r)\frac{a}{1-a}\nu_1-(r+C_2)\nu_2 = 0. \tag{3}$$

Furthermore, the lower bound for T_C^* is given by $(C_2-C_1)\nu_2/(C_1+r)$, that is, $T_C^* > (C_2-C_1)\nu_2/(C_1+r)$.

Proof.

See Appendix for its poof. ■

For the case of $C_1 \geq C_2$, see the concluding remarks in Section 8.

4. Optimal Policy for the Steady State Availability

Let the state of the system be given by the binary variable

$$X(t)=\begin{cases}1 & \text{if } \textit{the system is working at time } t\\ 0 & \textit{otherwise}.\end{cases}$$

An important characteristic of a repairable system is *availability*. The availability at time t is defined by

$$A(t)=P(X(t)=1),$$

which is the probability that the system is working at time t. Because the study of $A(t)$ is too hard except for a few simple cases, other measures have been proposed, and more attention is being paid to the limiting behavior of this quantity, i.e., engineers are more interested in the extent to which the system will be available after it has been run for a long time. The steady state availability (or limiting availability) of the system is, *when the limit exists*, defined by

$$A=\lim_{t\to\infty}A(t),$$

which is a significant measure of performance of a repairable system. Some other kinds of availability which are useful in practical applications can be found in Birolini(1985, 1994) and Hoyland and Rausand (1994).

The following Lemma gives the steady state availability of the system under consideration.

Lemma 2.
The steady state availability of the system under policy T, which is denoted by $A_\infty(T)$, exists and is given by

$$A_\infty(T) = T/[\,T+(\exp[(\frac{1}{a}-1)\cdot \Lambda(T)]-1)\frac{a}{1-a}\nu_1+\nu_2]. \quad (4)$$

Proof.
A brief derivation of (4) is given in Appendix. See also Cha (2002) for a more detailed derivation. ■

The property of the optimal replacement policy T_A^* maximizing $A_\infty(T)$ in (4) is given in the following theorem.

Theorem 2.
The optimal replacement policy T_A^* exists and is the unique solution of the equation

$$\exp[(\frac{1}{a}-1)\Lambda(T)]\{T\lambda(T)\nu_1-\frac{a}{1-a}\nu_1\}+\frac{a}{1-a}\nu_1-\nu_2=0. \quad (5)$$

Proof.
See Appendix for its poof. ■

5. Cost Priority Policy

Now, the problem is to determine the optimal policy T_{CP}^* so that the long-run average cost rate is low enough and the steady state availability is maximized. In advance observe that $\lim_{T\to 0} C(T) = C_2$, $\lim_{T\to\infty} C(T) = C_1$ and there exists the unique T_C^* which minimizes $C(T)$. Also observe that $C(T)$ strictly decreases for $T \le T_C^*$ and strictly increases for $T \ge T_C^*$. Then let us define a set of $T > 0$,

$$S_1(\alpha) \equiv \{T: C(T) \le \alpha\}, \quad (6)$$

where $C(T)$ is given by (2) and α, $C(T_C^*) < \alpha < C_2$, is the predetermined upper bound of the long-run average cost rate. Then the optimal policy T_{CP}^* is defined by the value which satisfies

$$A_\infty(T_{CP}^*) = \max_{T \in S_1(\alpha)} A_\infty(T).$$

The property of the optimal policy T_{CP}^* is given as follows.

Theorem 3. Let T_A^* be the solution of the equation (5) and suppose that the upper bound α satisfies $C(T_C^*) < \alpha < C_2$.

(I) Suppose that $\alpha < C_1$. Then

(i) if $T_A^* < T_1$, then $T_{CP}^* = T_1$,

(ii) if $T_1 \le T_A^* \le T_2$, then $T_{CP}^* = T_A^*$, and

(iii) if $T_2 < T_A^*$, then $T_{CP}^* = T_2$,

where $T_1 < T_2$ are the values satisfying $C(T_1) = C(T_2) = \alpha$.

(II) Suppose that $\alpha \ge C_1$.

(i) if $T_A^* < T_1$, then $T_{CP}^* = T_1$, and

(ii) if $T_A^* \ge T_1$, then $T_{CP}^* = T_A^*$,

where T_1 is the value satisfying $C(T_1) = \alpha$.

proof.

Note that $\lim_{T\to 0} C(T) = C_2$ and $\lim_{T\to\infty} C(T) = C_1$. Then two separate cases are considered.

Case I : $C(T_C^*) < \alpha < C_1$. In this case observe that there exist two values satisfying $C(T_1) = C(T_2) = \alpha$, $T_1 < T_2$ and $S_1(\alpha) = [T_1, T_2]$. Then by the property of $A_\infty(T)$, the results are obvious.

Case II : $C_1 \le \alpha < C_2$. In this case note that there exist only one value satisfying $C(T_1) = \alpha$ and $S_1(\alpha) = [T_1, \infty)$, which lead to the desired result. ■

6. Availability Priority Policy

In this section the problem is to determine the optimal policy T_{AP}^* so that the steady state availability is high enough and the long-run average cost rate is minimized.

In advance observe that $\lim_{T\to 0} A_\infty(T) = 0$, $\lim_{T\to\infty} A_\infty(T) = 0$ and there exists the unique T_A^* which maximizes $A_\infty(T)$.

Also observe that $A_\infty(T)$ strictly increases for $T \le T_A^*$ and strictly decreases for $T \ge T_A^*$. As in Section 5 let us define a set of $T > 0$,

$$S_2(\beta) \equiv \{T: A_\infty(T) \ge \beta\}, \qquad (7)$$

where $A_\infty(T)$ is given by (4) and β, $0 < \beta < A_\infty(T_A^*)$, is the predetermined lower bound of the steady state availability. Then the optimal policy T_{AP}^* is defined by the value which satisfies

$$C(T_{AP}^*) = \min_{T \in S_2(\beta)} C(T).$$

The property of the optimal policy T_{AP}^* is given as follows.

Theorem 4. Let T_C^* be the solution of the equation (3) and suppose that the lower bound β satisfies $0 < \beta < A_\infty(T_A^*)$. Then

(i) if $T_C^* < T_3$, then $T_{AP}^* = T_3$,

(ii) if $T_3 \le T_C^* \le T_4$, then $T_{AP}^* = T_C^*$, and

(iii) if $T_4 < T_C^*$, then $T_{AP}^* = T_4$,

where $T_3 < T_4$ are the values satisfying $A_\infty(T_3) = A_\infty(T_4) = \beta$.

proof.

Note that $\lim_{T \to 0} A_\infty(T) = 0$ and $\lim_{T \to \infty} A_\infty(T) = 0$ and observe that there exist two values satisfying

$$A_\infty(T_3) = A_\infty(T_4) = \beta, \; T_3 < T_4$$

and $S_2(\beta) = [T_3, T_4]$. Then by the property of $C(T)$, the results are obvious. ■

7. Numerical Example

In this section some numerical examples of the optimal policies for the maintenance model are presented. In the example, we assume that the failure rate of the system is given by

$\lambda(t) = \lambda \eta t^{\eta - 1}$, where $\lambda = 1.0$, $\eta = 2.0$. Suppose that the parameter values for repair times are given by $\nu_1 = 0.10$, $\nu_2 = 0.20$, and those for the cost rates are given by $C_1 = 0.10$, $C_2 = 2.00$. Furthermore, we assume that $r = 0.50$ and $a = 0.80$.

I. Optimal Policy for the Long-run Average Cost Rate

By Theorem 1, the optimal replacement policy T_C^* exists and is the unique solution of the equation (3). The solution of the equation (3) could not be obtained directly, hence a numerical study was performed using *Mathematica* software. The obtained solution is $T_C^* = 2.01526$ and, in this case, $C(T)$ is given by $C(2.01526) = -0.17032$.

II. Optimal Policy for the Steady State Availability

By Theorem 2, the optimal replacement policy T_A^* exists and is the unique solution of the equation (5), which is given by $T_A^* = 1.12805$. In this case, the steady state availability $A_\infty(T)$ is given by $A_\infty(1.12805) = 0.76329$.

III. Cost Priority Policy

Suppose that the predetermined upper bound α is given by $\alpha = 0.00$. Then observe that α satisfies $\alpha < C_1$. Therefore, we apply the first case of the Theorem 3. The two values T_1 and T_2 satisfying $C(T_1) = C(T_2) = 0.00$ are given by $T_1 = 0.89786$ and $T_2 = 3.80914$. In this case, observe that $T_A^* = 1.12805$ satisfies that $T_1 \le T_A^* \le T_2$. Therefore, by Theorem 3, the optimal policy T_{CP}^* is also given by $T_{CP}^* = 1.12805$, when the cost rate $C(T)$ is given by $C(1.12805) = -0.07378$.

IV. Availability Priority Policy

Suppose that the predetermined lower bound β is given by $\beta = 0.70$. Then the two values T_3 and T_4 satisfying $A_\infty(T_3) = A_\infty(T_4) = 0.70$ are given by $T_3 = 0.53623$ and $T_4 = 1.95135$. In this case, observe that $T_C^* = 2.01526$ satisfies $T_4 < T_C^*$. Therefore, by Theorem 4, the optimal policy T_{AP}^* is given by $T_{AP}^* = 1.95135$, when the cost rate $C(T)$ is given by $C(T_{AP}^*) = -0.16933$.

The graphs for the long-run average cost rate and the steady state availability are presented in Figure 1 and Figure 2, respectively.

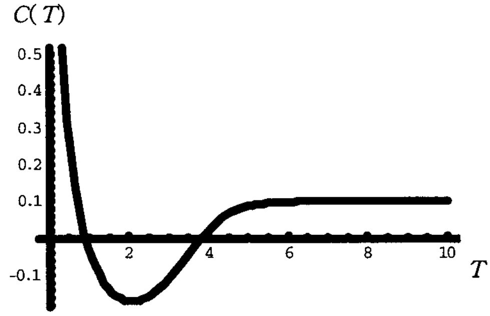

Figure 1. Long-run Average Cost Rate

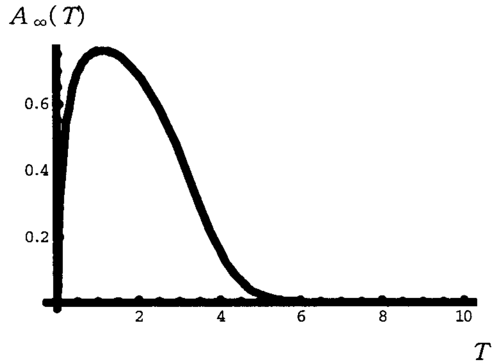

Figure 2. Steady State Availability

8. Concluding Remarks

In Section 3, it was assumed that $C_1 < C_2$. In the case of $C_1 \ge C_2$, it is easy to see that

$$\frac{\partial \Psi_1(T)}{\partial T} > 0, \ \forall T > 0.$$

Hence if

$\Psi_1(0) = \nu_1 \lambda(0)(C_1 - C_2)\nu_2 - (r + C_2)\nu_2 < 0,$

there exists the unique T_C^* which minimizes $C(T)$. Thus similar methods used in Section 5 and Section 6 can be applied.

Appendix

Proof of Lemma 1

Since $V_i = T + \sum_{j=1}^{N_i} Y_{i,j}$, $E(V_i)$ is given by

$$\begin{aligned} E(V_i) &= T + E\Big(\sum_{j=1}^{N_i'} Y_{i,j} \cdot I(N_i' \geq 1)\Big) \\ &\quad + E(Y_{i,N_i}) \\ &= T + E\Big(\sum_{j=1}^{N_i'} Y_{i,j} \cdot I(N_i' \geq 1)\Big) \\ &\quad + \nu_2 . \end{aligned}$$

In the above equation,

$$\begin{aligned} & E\Big(\sum_{j=1}^{N_i'} Y_{i,j} \cdot I(N_i' \geq 1)\Big) \\ &= E\Big(E\Big(\sum_{j=1}^{N_i'} Y_{i,j} \cdot I(N_i' \geq 1) \mid N_i'\Big)\Big) \\ &= \sum_{n=0}^{\infty} \frac{1 - (\frac{1}{a})^n}{1 - \frac{1}{a}} \nu_1 \cdot \frac{(\Lambda(T))^n \cdot \exp(-\Lambda(T))}{n!} \\ &= (\exp[(\tfrac{1}{a} - 1)\Lambda(T)] - 1)\frac{a}{1-a}\nu_1 , \end{aligned}$$

where $\Lambda(t) \equiv \int_0^t \lambda(u)du$.

Therefore,

$$E(V_i) = T + (\exp[(\tfrac{1}{a} - 1)\Lambda(T)] - 1) \cdot \frac{a}{1-a}\nu_1 + \nu_2 .$$

Thus, from (1) the long-run average cost per unit time $C(T)$ under policy T is given by

$$C(T) = \Big[(\exp[(\tfrac{1}{a} - 1)\Lambda(T)] - 1) \cdot \frac{a}{1-a}\nu_1 \cdot C_1 + \nu_2 \cdot C_2 - rT\Big] \Big/ \Big[T + (\exp[(\tfrac{1}{a} - 1)\Lambda(T)] - 1) \cdot \frac{a}{1-a}\nu_1 + \nu_2\Big].$$

Proof of Theorem 1

Since

$$C(T) = C_1 + [-(C_1 + r)T + (C_2 - C_1)\nu_2] / [T + (\exp[(\tfrac{1}{a} - 1) \cdot \Lambda(T)] - 1)\frac{a}{1-a}\nu_1 + \nu_2],$$

minimizing $C(T)$ is equivalent to minimizing

$$[-(C_1 + r)T + (C_2 - C_1)\nu_2] / [T + \Big(\exp[(\tfrac{1}{a} - 1)\Lambda(T)] - 1\Big)\frac{a}{1-a}\nu_1 + \nu_2]. \tag{8}$$

Differentiate (8) with respect to T and set it zero, which yields

$$\exp[(\tfrac{1}{a} - 1)\Lambda(T)] \cdot \{\nu_1 \lambda(T)[(C_1 + r)T + (C_1 - C_2)\nu_2] - (C_1 + r)\frac{a}{1-a}\nu_1\} + (C_1 + r)\frac{a}{1-a}\nu_1 - (r + C_2)\nu_2 = 0. \tag{9}$$

Let the left side in (9) be $\Psi_1(T)$ then note that $\Psi_1(0)<0$ and since

$$\frac{\partial \Psi_1(T)}{\partial T}=\left(\frac{1-a}{a}\right)\lambda(T)\cdot \exp\left[\left(\frac{1}{a}-1\right)\cdot \Lambda(T)\right]\cdot \{\nu_1\lambda(T)[(C_1+r)T+(C_1-C_2)\nu_2]\} + \exp\left[\left(\frac{1}{a}-1\right)\Lambda(T)\right]\cdot \{\nu_1\lambda'(T)[(C_1+r)T+(C_1-C_2)\nu_2]\},$$

$\Psi_1(T)$ strictly decreases for $T\le (C_2-C_1)\nu_2/(C_1+r)$ and strictly increases for $T>(C_2-C_1)\nu_2/(C_1+r)$. Also observe that $\lim_{T\to\infty}\Psi_1(T)=\infty$. These imply that the equation (9) has the unique solution. Therefore there exists the unique T_C^* which minimizes $C(T)$ and T_C^*, $T_C^*>(C_2-C_1)\nu_2/(C_1+r)$, is the solution of the equation (9).

Proof of Lemma 2

First of all, we define some notations.

$F_{(r)j}(x)$: the conditional distribution function of $X_{i,j}$ given $N_i=r, i=1,2,\cdots, j=1,2,\cdots,r, \ r=2,3,\cdots$

$\overline{F}_{(r)j}(x)$: $1-F_{(r)j}(x)$

$V_{i,j}$: $\sum_{m=1}^{i}(X_{i,m}+Y_{i,m}),\ \ i=1,2,\cdots, j=1,2,\cdots,N_i-1$

$F_{(r)V_{i,j}}(t)$: the conditional distribution function of $V_{i,j}$ given $N_i=r,\ \ i=1,2,\cdots, j=1,2,\cdots,r-1, r=2,3,\cdots$

$\overline{F}_{(r)j+1|V_{i,j}=s}(t)$: the conditional survivor function of $X_{i,j+1}$ given $V_{i,j}=s$ and $N_i=r$, that is, $P\{X_{i,j+1}\ge t|\ V_{i,j}=s,\ N_i=r\}$, $i=1,2,\cdots,\ \ j=1,2,\cdots,r-1, r=2,3,\cdots$

$H(t)$: the distribution of V_i

$H^{(n)}(t)$: the n-fold convolution of $H(t)$

$M_H(t)$: $\sum_{n=1}^{\infty}H^{(n)}(t)$

$A_0(t)$: $P\{X(t)=1,\ t\le S_1\}$

$$I_T(t)\equiv\begin{cases}1 & \text{if } t\le T\\ 0 & \text{otherwise}\end{cases}$$

Then, now, it can be shown that

$$A_0(t)=\exp(-\Lambda(T))\cdot I_T(t)+\sum_{r=2}^{\infty}\frac{(\Lambda(T))^{r-1}\cdot\exp(-\Lambda(T))}{(r-1)!}\times\left[\overline{F}_{(r)1}(t)+\sum_{j=1}^{r-1}\int_0^t\overline{F}_{(r)j+1|V_{1,j}=s}(t-s)\,dF_{(r)V_{1,j}}(s)\right].$$

Furthermore, by the definition of $A(t)$, we can see that the following equation holds;

$$A(t)=A_0(t)+\int_0^t A(t-x)dH(x),\ \ t\ge 0,$$

which is a renewal equation. Then the function $A(t)$ satisfies(see Theorem 4.1 of Karlin and Taylor(1975))

$$A(t)=A_0(t)+\int_0^t A_0(t-u)dM_H(u).$$

Note that

$$\lim_{t\to\infty} A_0(t) \le \lim_{t\to\infty} P\{S_1 \ge t\} = 0,$$

and by the Key Renewal Theorem,

$$\lim_{t\to\infty} A(t) = \lim_{t\to\infty}\int_0^t A_0(t-u)dM_H(u) = \frac{1}{E(V_i)}\int_0^\infty A_0(t)dt.$$

On the other hand, it can also be shown that

$$\int_0^\infty A_0(t)dt = T.$$

Therefore, the steady state availability of the system under policy T, which is denoted by $A_\infty(T)$, exists and is given by

$$A_\infty(T) = T/[\,T+(\exp[(\frac{1}{a}-1)\Lambda(T)] -1)\frac{a}{1-a}\nu_1+\nu_2].$$

Proof of Theorem 2

Differentiate (4) with respect to T and set it zero, which yields

$$\exp[(\frac{1}{a}-1)\Lambda(T)]\cdot\{T\lambda(T)\nu_1 - \frac{a}{1-a}\nu_1\}+\frac{a}{1-a}\nu_1-\nu_2=0. \qquad (10)$$

Let the left side in (10) be $\Psi_2(T)$ then note that $\Psi_2(T)$ strictly increases as T increases since

$$\frac{\partial\Psi_2(T)}{\partial T}=\nu_1 T\cdot\exp[(\frac{1}{a}-1)\Lambda(T)]\cdot\{(\frac{1}{a}-1)\lambda^2(T)+\lambda'(T)\}>0,\quad \forall T>0,$$

and $\Psi_2(0)=-\nu_2<0$, $\lim_{T\to\infty}\Psi_2(T)=\infty$.

Therefore there exists the unique T_A^* which maximizes $A_\infty(T)$ and T_A^* is the solution of the equation (10).

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