■ 論 文 ■

Rolling Horizon Implementation for Real-Time Operation of Dynamic Traffic Assignment Model

동적통행배정모형의 실시간 교통상황 반영

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요 약

기 제안된 수리적 동적통행배정모형은 전체 시뮬레이션 기간동안 시간종속적 교통수요와 교통망의 교통상황이 이미 안정되어 있고 장래에도 예측가능 하다는 가정을 전제로 개발되었다. 이러한 가정은 실제 시시각각으로 변화하는 교통수요와 교통상황의 예측 불가능함 고려할 때 비현실적이라고 할 수 있다. 한편, Rolling Horizon Implementation(RHI)은 기종점간의 수요행렬(trip matrix)과 교통상황(traffic condition)이 단기간의 예측시간동안 현재의 예측정보를 기반으로 신뢰성 있게 모니터링 될 수 있고, 그 시점에서 보다 미래로 연장된 시간으로는 불확실성(uncertainty)의 증가를 고려한다는 가정을 전제로 제안되었다. 따라서, RHI개념과 부합되는 수리적 동적통행배정모형은 시뮬레이션 출발시점에 수요와 교통상황에 대한 확정적 정보가 이미 획득되어 있고, 그 기간이후의 정보에 대해서는 시간이 흐름에 따른 정보의 유용성을 근거로 각 운전자 그룹이 인지(perceived)하는 가로망의 통행비용(travel cost)을 최소화되도록 차량을 배정하는 것으로, 실시간적으로 인지된 교통수요와 교통망에 대한 정보를 통행배정초기에 입력변수로 사용하여 실시간 교통정보모형으로서 운영가능 하다는 장점을 제공한다.

본 연구는 수리적 동적통행배정모형이 RHI개념과 부합되어 교통상황과 수요변화를 실시간적으로 반영하여 운영되도록 모형의 기능을 확장하는 데 있다. 이를 위해, 다계층 이용자(multiple user classes) 동적통행배정모형을 변동등식(variational equality)이론에 근거한 모형식을 기반으로, 실시간 통행배정에서 발생하는 종점에 도착하지 못한 차량(unfinished trips)과 이들의 재배정(rerouting strategy) 문제를 인식하고, 이 차량들을 링크상의 교통량 전파조건(flow propagation constraint)을 토대로 다음 통행배정 시간대의 실시간 수요로서 반영할 수 있는 방안을 제시한다.

Ⅰ. Introduction

In the development of a deployable real-time information tool, it is essential for analytical DTA models to react with time varying traffic demand and network condition. However, the basic assumption for the analytical type of DTA models, mostly proposed in previous research, is that demand and network conditions are assumed to be known and unchanging during the whole planning time (Carey, 1992; Friesz, et al, 1989; Janson, 1991; Ran and Boyce, 1996). These types of DTA models may be of use for evaluating different strategies of off-line systems, or on-line systems on the condition that the time dependent demands and network conditions for the entire time horizon are constant for real-time traffic systems. However, the DTA models need to operationally reflect real-time variations in both OD demands and/or traffic conditions. These real-time variations can be modeled by introducing the rolling horizon implementation into DTA models(Peeta and Mamassani, 1995; Gartner and Stamatiadis, 1997; Ben-Akiva, 1997; Van Aerde, 1992).

The rolling horizon implementation recognizes that the prediction of origin-destination(OD) matrices and network conditions is usually more accurate in shorter period of time. Further into the whole horizon there exists a substantial uncertainty. Thus, rather than assuming that the time-dependent OD matrices and network conditions are known at the beginning of the horizon, it is more reasonable to assume that the required information will not be available until the time period rolls forward.

In the rolling horizon implementation, traffic assignment is implemented for each stage. Since the planning horizon is divided into several overlapping stages, there may be the unfinished trips problem, which are trips assigned in a certain stage that have not reached destinations during the same stage. Hence, the raised two issues in the rolling horizon framework are: 1) how to model the impact of unfinished trips that are still on the network at the end of previous stage in the assignment of current stage, and 2) how to load these trips on the network.

This paper focuses on enhancing the online functionality for real-time demand by coupling an analytical DTA model and the rolling horizon implementation. In the paper, the impact of unfinished trips is formulated as updated multi-class OD matrixes that are reconsidered as the realtime demands for the next stage. The rerouting strategy for the updated OD matrix for each class enables the DTA model to operationally model real-time variations of on-line OD demand and network conditions and thus more efficiently provide updated route information to travelers.

The paper is organized as follows. The next section presents the variational equality(VE) formulation of a multi-class analytical DTA model followed by a section that details the rolling horizon implementation. The treatment of unfinished trips at the end of each roll period is described with attention. A combined algorithm to solve this rolling horizon implementation with the multi-class DTA model is then presented. Two numerical examples of demand and network variations are presented and computational results are analyzed. Conclusions are presented in the last section.

Notation used in this paper is shown in ⟨Table 1⟩

In the following, superscript rs denotes origin-destination pair rs, subscript a denotes link a, subscript p denotes path p, and subscript m denotes traveler class m.

Ⅱ. Multi-Class Analytical Dta Model

The development of the Advanced Traveler Information System(ATIS) will make it possible that travelers are furnished with real-time(instantaneous and predictive) traffic information. The DTA(dynamic traffic assignment) models should be able to demonstrate the capability to differentiate travelers

〈Table 1〉 Notations

$a,\hat{a},\tilde{a}$	Link a, unused link $\hat{a}$, used link $\tilde{a}$
$p,\hat{p},\tilde{p}$	Route p, unused route $\hat{p}$, used route $\tilde{p}$
$A,\hat{A},\tilde{A}$	Link set A, unused link set $\hat{A}$, used link set $\tilde{A}$: ($A=\hat{A}\cup\tilde{A}$)
$t,\hat{t},\tilde{t}$	Continuous time t, unused time $\hat{t}$, used time $\tilde{t}$
$T,\hat{T},\tilde{T}$	Continuous time horizon set T, unused time horizon set $\hat{T}$, used time horizon set $\tilde{T}$:($T=\hat{T}\cup\tilde{T}$)
$K,\hat{K},\tilde{K}$	Discrete time horizon set K, unused time horizon set $\hat{K}$, used time horizon set $\tilde{K}$:($K=\hat{K}\cup\tilde{K}$)
n	Discrete departure time interval n
k	Discrete time interval k
rs	Origination and destination pair rs
$f^{rs}(t)$	Departure flow rate from origin r to destination s at time t
$u^{rs}_{ap}(t)$	Inflow rate on link a at time t
$v^{rs}_{ap}(t)$	Exit flow rate on link a at time t
$x^{rs}_{ap}(t)$	Number of vehicles on link a at time t
$E^{rs}(t)$	Cumulative number of vehicles arriving at destination s from origin r by time t
$p^{rs}_{ap}(k)$	Inflow rate on link a at the beginning of time interval k
$q^{rs}_{ap}(k)$	Exit flow rate on link a at the beginning of time interval k
$y^{rs}_{ap}(k)$	Number of vehicles on link a at the beginning of time interval k
$\overline{E}^{rs}(k)$	Cumulative number of vehicles arriving at destination s from origin r by the beginning of time interval k
$A(j)$ $B(j)$	Set of links whose tail node is j Set of links whose head node is j
$e^{rs}(t)$	Arrival flow rate from origin r toward destination s at time t
$\tau_a(t)$	Mean actual travel time over link a for flows entering link a at time t
$\eta^{rs}_p(t)$	Mean actual travel time for route p between (r, s) for flows departing origin r at time t
$\Omega^{rs}_a(k)$	Travel cost of link at the beginning of time interval k.
$\pi^{rs}(t)$	Minimal mean actual route travel time between (r, s) for flows departing origin r at time t

route choice behavior based on their realization of traffic conditions. The route choice behavior such as fixed route, stochastic dynamic user optimum (SDUO), and dynamic user optimum(DUO) should be considered as a whole.

Travelers in this paper are classified into three classes : (1) predetermined or fixed routes, (2) stochastic dynamic user-optimal(SDUO), and (3) dynamic user-optimal (DUO).

Class 1 travelers are those who either do not have access to real-time traffic information hence continue their intended routes or those who refuse to change routing plans for whatever reasons. The route and departure time of this class are fixed. Thus, rerouting is not a concern for these travelers at any stage(refer to Figure 1 for the definition of stage). The background traffic at stage σ is represented by Class 1 travelers. It includes Class 1 travelers who have not reached their destinations at the beginning of stage σ and those who have scheduled to depart during stage σ.

Class 2 represents the travelers who determine their routes based on perceived shortest travel times. This class is to describe those who have partial information of network traffic conditions or those who obtain real-time traffic information but make the route choices based on individuals preference and/or experience combined with the real-time information they have received. Typically, the notion of perceived travel time in the static context is modeled through a stochastic generalization of Wardrops(1952) first principle(Daganzo and Sheffi, 1977; Sheffi and Powell, 1982). Ran and Boyce(1996) extended this notion to a dynamic context and established the principle of stochastic dynamic user-optimal(SDUO) as follows :

No traveler can improve his/her perceived actual travel cost by unilaterally changing the route.

It is noteworthy that Class 2 travelers can be divided into more sub-class groups based on individual drivers perception error on route travel time and risk-taking behavior in both static and dynamic transportation networks(Mirchandani and Soroush, 1987; Chen and Recker, 2000; Tatineni, et al, 1997; Boyce, et al, 1999).

Class 3 is for travelers who possess perfect(full) knowledge of dynamic traffic conditions and travel on the minimum travel time routes without any perception errors. The dynamic user-optimal(DUO) principle is the temporal generalization of Wardrops first principle which states(Ran and Boyce, 1996):

For each OD pair at each interval of time, if the actual travel times experienced by travelers departing at the same time are equal and minimal, the dynamic traffic flow over the network is in a travel-time based ideal dynamic user-optimal state.

The variational equality(VE) formulation and corresponding constraint sets of three class travelers route choice conditions is stated as follows(where * denotes the DUO state). Since the model is formulated using reduced sets of used links and used departure time intervals, compared to the previously proposed variational inequality(VI)(Ran and Boyce, 1996), inequality sign can be dropped, thus only equality sign is remained in the formulation. It enhances the proposed multi-class model to be able to applied for real-time distribution process of traffic information based on dramatically improved computational performance(Shin, et al, 2002).

$$\sum_{rs}\left\{\begin{array}{l}\sum_{\tilde a}\int_0^{\tilde T_{\tilde a}}\{\pi^{ri*}(\tilde t_{\tilde a})+\tau_{\tilde a}[\tilde t_{\tilde a}+\pi^{ri*}(\tilde t_{\tilde a})]-\pi^{rj*}(\tilde t_{\tilde a})\}d\tilde t_{\tilde a}+\\ \sum_{\tilde p}\int_0^{\tilde T_{\tilde p}}\left\{\left[f_{\tilde p 2}^{rs}(\tilde t_{\tilde p})-f_{\tilde p 2}^{rs}(\tilde t_{\tilde p})P_{\tilde p 2}^{rs}(\tilde t_{\tilde p})\right]\dfrac{\partial\eta_{\tilde p}^{rs}(\tilde t_{\tilde p})}{\partial f_{\tilde p 2}^{rs}(\tilde t_{\tilde p})}\right\}dt_{\tilde p}\end{array}\right\}=0$$

$$\forall \tilde t_{\tilde a}\in[0,+\infty], \tilde t_{\tilde p}\in[0,+\infty] \quad (1)$$

Subject to : Route flow assignment constraints for each class of travelers :

[Class 1 Travelers]

Fixed Routes : Each route $\tilde p$ is fixed, and

$$f_{\tilde p m}^{rs}(\tilde t_{\tilde p m})=u_{\tilde a\tilde p m}^{rs}(\tilde t_{\tilde p m})\ \forall r,s,\tilde p;\tilde a\in A(r);\tilde a\in\tilde p;m=1 \quad (2)$$

[Class 2 Travelers]

Stochastic Dynamic User-Optimal(SDUO) Routes :

$$f_{\tilde p m}^{rs}(\tilde t_{\tilde p m})=f_m^{rs}(\tilde t_{\tilde p m})P_{\tilde p}^{rs}(\tilde t_{\tilde p m})\ \forall r,s,\tilde p;m=2 \quad (3)$$

$$f_{\tilde p m}^{rs}(\tilde t_{\tilde p m})=u_{\tilde a\tilde p m}^{rs}(\tilde t_{\tilde a\tilde p m})\ \forall r,s,\tilde p;\tilde a\in A(r);\tilde a\in\tilde p;m=2 \quad (4)$$

[Class 3 Travelers]

Dynamic User-Optimal(DUO) Routes :

$$\sum_{\tilde p}\sum_{\tilde a\in A(r)}u_{\tilde a\tilde p m}^{rs}(\tilde t_{\tilde a\tilde p m})=f_m^{rs}(\tilde t_{\tilde a\tilde p m})\ \forall r,s;m=3 \quad (5)$$

Relationship between state and control variables :

$$\frac{dx_{\tilde a\tilde p m}^{rs}}{d\tilde t_{\tilde a\tilde p m}}=u_{\tilde a\tilde p m}^{rs}(\tilde t_{\tilde a\tilde p m})-v_{\tilde a\tilde p m}^{rs}(\tilde t_{\tilde a\tilde p m})\ \forall m,\tilde a,\tilde p,r,s \quad (6)$$

$$\frac{dE_{\tilde p m}^{rs}(\tilde t_{\tilde p m})}{d\tilde t_{\tilde p m}}=e_{\tilde p m}^{rs}(\tilde t_{\tilde p m})\ \forall\tilde p,m,r;s\neq r \quad (7)$$

Flow conservation constraints :

$$\sum_{\tilde a\in B(j)}v_{\tilde a\tilde p m}^{rs}(\tilde t_{\tilde a\tilde p m})=\sum_{\tilde a\in A(j)}u_{\tilde a\tilde p m}^{rs}(\tilde t_{\tilde a\tilde p m})\ \forall j,\tilde p,m,r,s;j\neq r,s \quad (8)$$

$$\sum_{\tilde{p}} \sum_{\tilde{a} \in B(s)} v^{rs}_{\tilde{a}\tilde{p}m}(\tilde{t}_{\tilde{a}\tilde{p}m}) = e^{rs}_{\tilde{p}m}(\tilde{t}_{\tilde{p}m}) \forall m, r, s; s \neq r \tag{9}$$

Flow propagation constraint :

$$\begin{aligned} x^{rs}_{\tilde{a}\tilde{p}}(\tilde{t}_{\tilde{a}}) = \sum_{\tilde{b} \in \tilde{p}} \{ & x^{rs}_{\tilde{b}\tilde{a}}[\tilde{t}_{\tilde{a}} + \tau_{\tilde{a}}(\tilde{t}_{\tilde{a}})] - x^{rs}_{\tilde{b}\tilde{p}}(\tilde{t}_{\tilde{a}})\} \\ & \{E^{rs}_{\tilde{p}}[\tilde{t}_{\tilde{a}} + \tau_{\tilde{a}}(\tilde{t}_{\tilde{a}})] - E^{rs}_{\tilde{p}}(\tilde{t}_{\tilde{a}})\}, \\ & \forall \tilde{a} \in B(j), \tilde{p}, r, s, j \neq r \end{aligned} \tag{10}$$

Definitional constraints :

$$\begin{aligned} \sum_{rspm} u^{rs}_{\tilde{a}\tilde{p}m}(\tilde{t}_{\tilde{a}\tilde{p}m}) = u_a(\tilde{t}_{\tilde{a}}), \sum_{rspm} v^{rs}_{\tilde{a}\tilde{p}m}(\tilde{t}_{\tilde{a}}) \\ = v_a(t), \sum_{rspm} x^{rs}_{\tilde{a}\tilde{p}m}(\tilde{t}_{\tilde{a}}) = x_a(\tilde{t}_{\tilde{a}}), \forall \tilde{a} \end{aligned} \tag{11}$$

Nonnegativity conditions :

$$\begin{aligned} x^{rs}_{\tilde{a}\tilde{p}m}(\tilde{t}_{\tilde{a}\tilde{p}m}) \geq 0, u^{rs}_{\tilde{a}\tilde{p}m}(\tilde{t}_{\tilde{a}\tilde{p}m}) > 0, v^{rs}_{\tilde{a}\tilde{p}m}(\tilde{t}_{\tilde{a}\tilde{p}m}) \\ \geq 0, \forall m, \tilde{a}, \tilde{p}, r, s \end{aligned} \tag{12}$$

$$e^{rs}_{\tilde{p}m}(\tilde{t}_{\tilde{p}m}) \geq 0, E^{rs}_{\tilde{p}m}(\tilde{t}_{\tilde{p}m}) \geq 0, \forall \tilde{p}, m, r, s \tag{13}$$

Boundary conditions :

$$E^{rs}_{\tilde{p}m}(1) = 0, \forall \tilde{p}, m, r, s \tag{14}$$

$$x^{rs}_{\tilde{a}\tilde{p}m}(1) = 0, \forall \tilde{a}, \tilde{p}, m, r, s \tag{15}$$

The first two terms of Equation(1) represent the route choice conditions of Class 3 travelers and the third term describes the route choice condition of Class 2 travelers. Since Class 1 travelers follow fixed or predetermined routes, there is no need to express them in Equation(1). Instead, they are embedded in the constraint sets. For each class of travelers, the constraints presented in (2)-(15), including flow conservation and propagation constraints, are applicable. These constraints are used to generate used-path and used-link flows when used-route departure flows and time intervals are determined. The proposed VE problem can be solved by extending the diagonalization algorithm which is based on a combination of relaxation procedure, Frank-Wolfe algorithm(Frank and Wolfe, 1956), and Method of Successive Averages(Sheffi and Powell, 1982)(Ran, et al,1996; Shin, 2001).

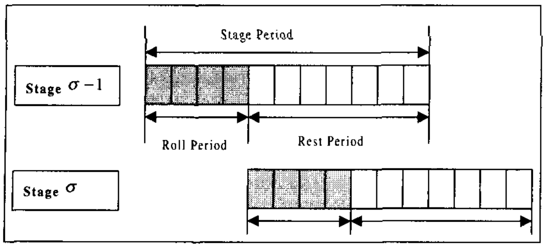
Ⅲ. Rolling Horizon Implementation

1. State-of-the-art

The initial concept of rolling horizon process was utilized for production inventory control(Wagner, 1977). It had been applied in the areas of online demand responsive traffic signal control for transportation engineering problems(Gartner, 1982, 1983). Recently, it has been developed for traffic and control assignment problems of simulation type of DTA models(Peeta and Mahmassani, 1995; Ben-Akiva, et al, 1997; Van Aerde, 1994; Gartner and Stamatiadis, 1997).

In reality, the travel demand(in the form of OD matrices) and network conditions are changing with time. Rather than assuming time-dependent OD matrices and network conditions are known at the beginning of the horizon, it is more reasonable to assume that deterministic information of OD and traffic conditions for a shorter period are available from the Traffic Management Center(TMC), whereas information beyond this short period will not be available until the next time period, that is, until time rolls forward. The rolling horizon implementation is based on this assumption of data availability and provides a practical method to address the real-time traffic assignment problem.

The basic idea of the rolling horizon implementation is to exploit currently obtainable information for short-term forecasts with degree of reliability, and survey and/or historical data for mid-term



〈Figure 1〉 The Rolling Horizon Approach

forecasts with some degree of dependability to solve an online and/or real-time based problem, while maintaining the efficiency of the model in order for determining sound control strategies. For the purpose of online/real-time functionality, short-term(roll period) usually means a segment of an hour, about 5 to 10 minutes; mid-term(rest period) means about 15 to 30 minutes into the future.

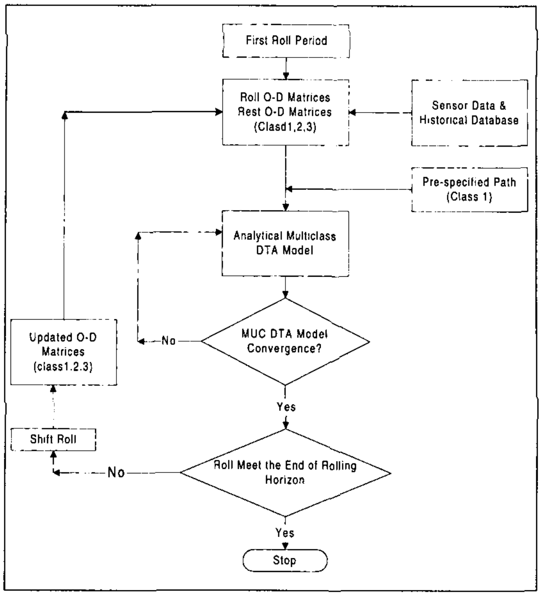
Since more accurate dynamic OD matrices and network conditions are available for a short period only, the model in the rolling horizon implementation is implemented in every roll period as shown in 〈Figure 1〉. In each roll period, the DTA model requires knowledge of the dynamic OD matrices and network conditions for the rest of the assignment horizon because this information is expected to influence the assignment result of the current roll period. The time horizon is then rolled forward by a length equal to the roll period and the above process is repeated.

Implementation Procedure of Rolling Horzion Process

The procedure for rolling horizon implementation with DTA models is depicted in 〈Figure 2〉 and described as follows:

[Step 1]

Before the start of the planning horizon, obtain the OD matrices for the first roll period and the forecast matrices for the rest of the first stage. The latter includes the information of historical routes for Class 2 and 3 travelers in each assignment time step of the first stage, and the routes to be assigned to Class 1 travelers.



〈Figure 2〉 Rolling Horizon Implementation with DTA Model

[Step 2]

Perform a complete multi-class dynamic traffic assignment to convergence(with the current stage OD matrices). If the end of the planning horizon is reached, stop; otherwise, go to Step 3.

[Step 3]

Moving to the next stage, shift the current stage and the shifted length equals the roll period. The next stage now becomes the current stage.

[Step 4]

Update routes, identification numbers, and positions for all travelers assigned in previous stage(s) that have not reached their destinations at the end of the stage.

[Step 5]

Before the start of the current stage, update the OD for current stage based on the OD matrices of the current roll period, and forecast OD for the rest of the current stage. The newly forecast OD matrices include travelers left from the previous stage(s) who have not reached their destinations and still remain in the network. Go to Step 2.

2. Treatment for Unfinished Trips

Three types of OD matrices are required in the rolling horizon implementation : (1) roll OD matrices, (2) rest OD matrices, and (3) updated OD matrices. Roll OD matrices can be estimated from detector data or collected through other means in each roll period. gRest OD matrices are obtained by utilizing the estimation technique based on historical OD data and/or survey data during the rest periods. Updated OD matrices are calculated from those trips that have not reached their destinations at the end of the stage.

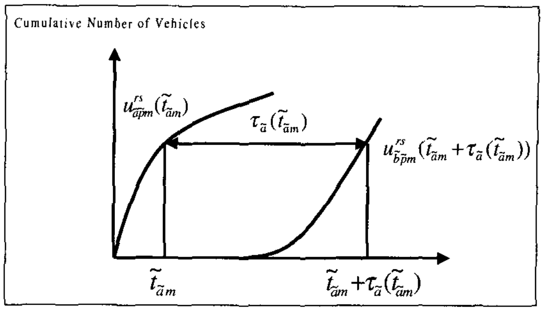
If the impact of unfinished trips is negligible or can be included in roll OD and rest OD matrices, then there is no complication involving the unfinished trips. While those unfinished trips are not accounted for by roll OD and rest OD matrices, they keep contributing congestion effects to the evolving traffic. Thus, congestion is underestimated if unfinished trips are ignored in the solution process. The treatment of unfinished trips in each roll period is thus crucial, and complicates the rolling horizon implementation.

- **Update OD for Class 1 Travelers**

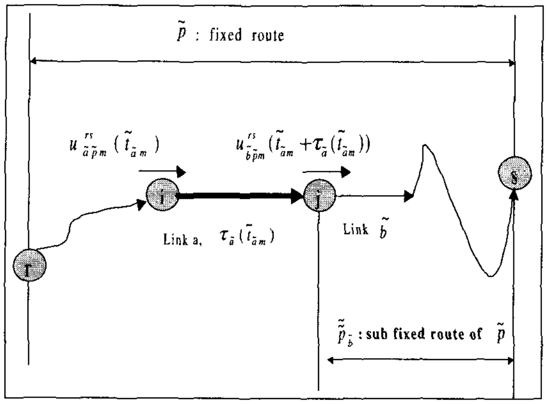
Let $u^{rs}_{\tilde{a}\tilde{p}m}(\tilde{t}_{\tilde{a}m})$ denote the inflow of Class m (m=1) to used link $\tilde{a}$ of a fixed route $\tilde{p}$ connecting origin r to destination s at time $\tilde{t}_{\tilde{a}m}$. As shown in ⟨Figure 3⟩, Equation(16) is the flow propagation constraint expressed solely by inflow, $u^{rs}_{\tilde{a}\tilde{p}m}(\tilde{t}_{\tilde{a}m})$ and $u^{rs}_{\tilde{b}\tilde{p}m}(\tilde{t}_{\tilde{a}m}+\tau_{\tilde{a}}(\tilde{t}_{\tilde{a}m}))$ where link $\tilde{b}$ is a link subsequent to link $\tilde{a}$ in path $\tilde{p}$.

$$u^{rs}_{\tilde{a}\tilde{p}m}(\tilde{t}_{\tilde{a}m}) = u^{rs}_{\tilde{b}\tilde{p}m}(\tilde{t}_{\tilde{a}m}+\tau_{\tilde{a}}(\tilde{t}_{\tilde{a}m})) \tag{16}$$

If the end of the stage falls between time $\tilde{t}_{\tilde{a}m}$ and $\tilde{t}_{\tilde{a}m}+\tau_{\tilde{a}}(\tilde{t}_{\tilde{a}m})$, then the link flow at the end of the stage is included in updated OD matrices.



⟨Figure 3⟩ Flow Propagation on Link $\tilde{a}$ for Class 1 Travelers



⟨Figure 4⟩ Flow Propagation of Class 1 Travelers on Link $\tilde{a}$ $\tilde{p}$ over Route

Let $OD^{js}_{\tilde{p}m}(\tilde{t}_{\tilde{a}m})$ denote the updated OD matrices for Class m (m=1) travelers using route $\tilde{p}$ from origin j to destination s at time $\tilde{t}_{\tilde{a}m}$. Equation(17) shows that unfinished trips of Class 1 traveling over link $\tilde{a}$ at the end of stage can be expressed an updated OD matrix for the next stage.

$$OD^{js}_{\tilde{p}m}(\tilde{t}_{\tilde{a}m}+\tau_{\tilde{a}}(\tilde{t}_{\tilde{a}m})) = u^{rs}_{\tilde{a}\tilde{p}m}(\tilde{t}_{\tilde{a}m}) \tag{17}$$

Assume link $\tilde{a}$ to be any link on fixed route $\tilde{p}$ that consists of plural links, the route used by trips in the updated OD matrices is not anymore route $\tilde{p}$ but a sub-route of $\tilde{p}$. Denote $\tilde{\tilde{p}}_{\tilde{b}}$ as the sub-route the first link of which is link $\tilde{b}$. As shown in ⟨Figure 4⟩, sequence of links of sub-route $\tilde{\tilde{p}}_{\tilde{b}}$ is the same as remaining links on route $\tilde{p}$ from the start link of sub-route $\tilde{\tilde{p}}_{\tilde{b}}$. Then $OD^{js\tilde{p}_{\tilde{b}}}_{\tilde{\tilde{p}}_{\tilde{b}}m}(\tilde{t}_{\tilde{a}m}+\tau_{\tilde{a}}(\tilde{t}_{\tilde{a}m}))$ in Equation(18) denotes the updated OD matrices of Class m (m=1) travelers using sub-route $\tilde{\tilde{p}}_{\tilde{b}}$ from origin j at time $\tilde{t}_{\tilde{a}m}+\tau_{\tilde{a}}(\tilde{t}_{\tilde{a}m})$ where node j is the end node of link $\tilde{a}$.

$$OD^{js}_{\tilde{\tilde{p}}_{\tilde{b}}m}(\tilde{t}_{\tilde{a}m}+\tau_{\tilde{a}}(\tilde{t}_{\tilde{a}m}))=u^{rs}_{\tilde{a}\tilde{p}m}(\tilde{t}_{\tilde{a}m}) \quad (18)$$

At the beginning of the next stage period, we have an updated OD matrix for Class 1 with a newly generated fixed $\tilde{\tilde{p}}_{\tilde{b}}$ and an origin j. It follows that

$$OD^{js}_{\tilde{\tilde{p}}_{\tilde{b}}m}(\tilde{t}_{\tilde{a}m}+\tau_{\tilde{a}}(\tilde{t}_{\tilde{a}m})-stage\ period)=u^{\tilde{p}_{\tilde{b}}}_{\tilde{a}m}(\tilde{t}_{\tilde{a}m}) \quad (19)$$

This updated OD matrix will be assigned on the sub fixed route $\tilde{\tilde{p}}_{\tilde{b}}$ at the beginning of the next stage.

- **Update OD for Class 2 and 3 Travelers**

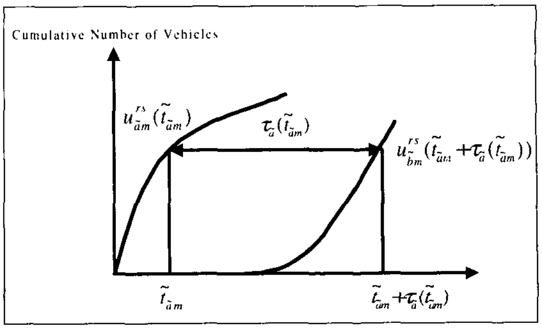
⟨Figure 5⟩ depicts Equation(20) where inflows of a link $\tilde{a}$ enter into subsequent links $\tilde{b}$ after link travel time in flow propagation.

$$u^{rs}_{\tilde{a}m}(\tilde{t}_{\tilde{a}m})=\sum_{\tilde{b}}u^{rs}_{\tilde{a}m}(\tilde{t}_{\tilde{a}m}+\tau_{\tilde{a}}(\tilde{t}_{\tilde{a}m})) \quad (20)$$

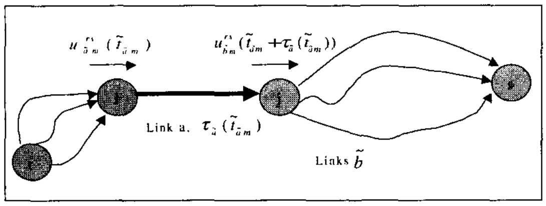
If the end of the stage falls between time $\tilde{t}_{\tilde{a}m}$ and $\tilde{t}_{\tilde{a}m}+\tau_{\tilde{a}}(\tilde{t}_{\tilde{a}m})$, then the inflows of link $u^{rs}_{\tilde{a}m}(\tilde{t}_{\tilde{a}m})$ can be considered as updated OD matrices at the end of the stage. Let $OD^{js}_{\tilde{a}m}(\tilde{t}_{\tilde{a}m})$ denote the updated OD matrices for Class m (m=2 or 3) from intermediate node j to destination s at time $\tilde{t}_{\tilde{a}m}$. As shown in ⟨Figure 6⟩, considering $u^{rs}_{\tilde{a}m}(\tilde{t}_{\tilde{a}m})$ can not reach to the destination s travelling over link $\tilde{a}$, theirs impact toward traffic networks needs to be reappeared as updated OD matrices for the next stage with a new origin j. It is given by Equation(21).

$$OD^{js}_{m}(\tilde{t}_{\tilde{a}m}+\tau_{\tilde{a}}(\tilde{t}_{\tilde{a}m}))=\sum_{r}u^{rs}_{\tilde{a}m}(\tilde{t}_{\tilde{a}m}) \quad (21)$$

At the beginning of the next stage, the updated



⟨Figure 5⟩ Flow Propagation on Link $\tilde{a}$ for Class 2 and 3 Travelers



⟨Figure 6⟩ Flow Propagation of Class 2 & 3 on Link a with Path Enumeration

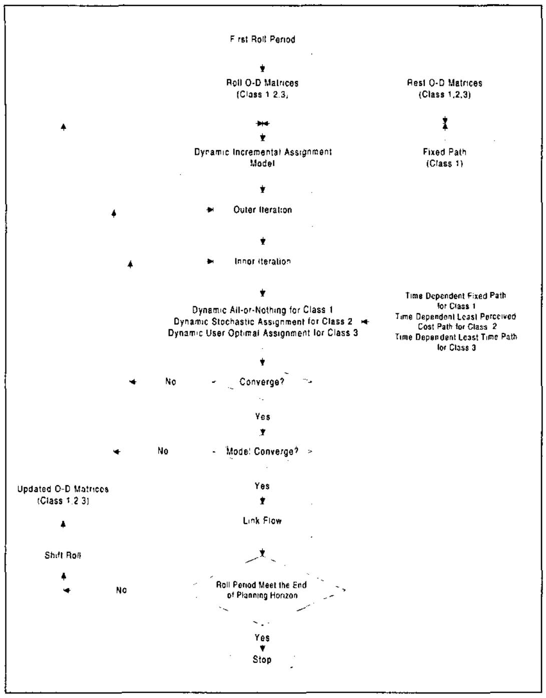
OD matrices for Class 2 and 3 becomes:

$$OD^{js}_{m}(\tilde{t}_{\tilde{a}m}+\tau_{\tilde{a}}(\tilde{t}_{\tilde{a}m})-stage\ period)=\sum_{r}u^{rs}_{\tilde{a}m}(\tilde{t}_{\tilde{a}m}) \quad (22)$$

These updated OD matrices are assigned in the next stage based on the rerouting strategy of each class, SDUO for Class 2 and DUO for Class 3.

3. Solution Algorithm

⟨Figure 7⟩ illustrates a combined algorithm embedded with the rolling horizon mechanism. The outer iteration covers the real time rolling horizon implementation, while the inner iteration solves the multi-class DTA problem. OD matrices are updated at each roll period. In the inner iteration, the link cost is computed at each time interval with newly assigned link flows. The assignment is performed for each class of travelers based on the underlying route choice criteria. The solution algorithm stops at the end of the planning horizon.



〈Figure 7〉 Flowchart of the Combined Solution Algorithm

Ⅳ. Computational Experiments

The basic feature of the rolling horizon implementation is to consider variations of demands and network conditions real-time. This section provides two separate computational studies; 1) demand variations and 2) network variations.

1. Demand Variations

Demand variations in the rolling horizon framework are described by three OD matrixes; 1) roll OD matrix, 2) rest OD matrix, and 3) updated OD matrix. This section provides case studies on how to deal with these OD matrixes.

A hypothetical network that contains 7 nodes and 8 links(〈Figure 8〉) is constructed for the computational studies. Each link in this network is assumed as a one-lane freeway link of 1.0 mile. Other assumptions of this network include:

- Five 20-second time intervals for each roll period.
- Three 20-second time intervals for each rest period.
- Eight 20-second time intervals in each stage.
- A modified Greenshield formula below is used for the link travel time function.

If $k \le k_j$ ' $\tau_a(t) = \dfrac{L_a}{u_{min} + (u_{max} - u_{min})(1 - \dfrac{k}{k_j})}$ (23)

If $k > k_j$ ' $\tau_a(t) = \dfrac{L_a}{u_{min}}$ (24)

where
u : speed,
umin : minimum speed at jam density,
umax : free flow speed,
k : density,
kj : jam density,
La : length of link a,

- Free flow speed of 60 mph.
- No occurrence of incident or accident.
- Perceived travel time function of Class 2 travelers is modeled by a truncated Normal distribution derived from the Monte-Carlo simulation method (Sheffi and Powell, 1982):

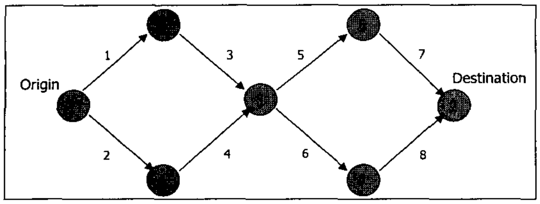
$$T_a(t) = Z\sqrt{\beta t^0} + \tau_a(t) \quad (25)$$

where $Z = \dfrac{N-\mu}{\sigma}$, $Z \sim (0,1)$, $N \sim (\mu, \sigma^2)$, $\beta = 0.5$

- Diagonalization technique is applied for solving the proposed multi-class DTA model(Ran, et al, 1996; Shin, 2002).

The two fixed routes for Class 1 travelers are shown in 〈Table 2〉

The OD matrices for the roll period and the rest time period are presented in 〈Table 3〉 and 〈Table 4〉.



〈Figure 8〉 Test Network

〈Table 2〉 Fixed Routes

Fixed Route	Path(Link Number Sequence)
1	1357
2	1368

〈Table 3〉 O-D Matrices for Roll Period

Roll Period	k	Sum of OD Flow	Class 1			Class 2			Class 3		
			O	D	Flow	O	D	Flow	O	D	Flow
1	1	10	1	2	2	1	2	4	1	2	4
	2	10	1	2	2	1	2	4	1	2	4
	3	10	1	2	2	1	2	4	1	2	4
	4	10	1	2	2	1	2	4	1	2	4
	5	10	1	2	2	1	2	4	1	2	4
2	1	8	1	2	0	1	2	4	1	2	4
	2	8	1	2	0	1	2	4	1	2	4
	3	8	1	2	0	1	2	4	1	2	4
	4	8	1	2	0	1	2	4	1	2	4
	5	8	1	2	0	1	2	4	1	2	4

Notes) P : path, O : origin, D : destination

〈Table 4〉 O-D Matrices for Rest Period

Rest Period	k	Sum of OD Flow	Class 1			Class 2			Class 3		
			O	D	Flow	O	D	Flow	O	D	Flow
1	6	5	1	1	1	1	2	2	1	2	2
	7	5	1	1	1	1	2	2	1	2	2
	8	6	1	2	2	1	2	2	1	2	2
2	6	4	1	1	0	1	2	2	1	2	2
	7	5	1	1	1	1	2	2	1	2	2
	8	5	1	2	1	1	2	2	1	2	2

Notes) P : path, O : origin, D : destination

1) Results for the First Stage

The result of multi-class DTA model for the first stage is summarized at 〈Table 5〉. Since there is only one OD pair represented for the roll period and the rest period, the travel time on used routes between origin and destination should be equal. This implies that Class 3 travelers receive

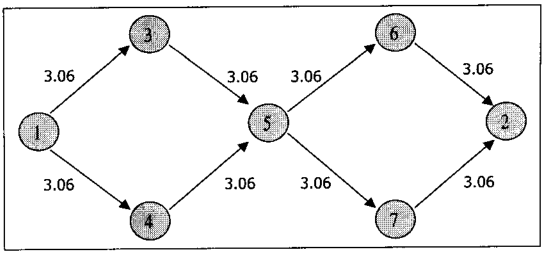
〈Table 5〉 DTA Model Result For The Roll Period 1 (continue)

Link No. (i,j)	Time Int. (k)	Link Flow($x_{am}(t)$)				Link Travel Cost
		Sum	m=1	m=2	m=3	
1 (1,3)	1	5.02	2.00	2.11	0.90	3.06
	2	10.03	4.00	4.30	1.73	3.12
	3	15.01	6.00	6.31	2.70	3.18
	4	15.05	6.00	6.39	2.66	3.18
	5	15.05	6.00	6.08	2.97	3.18
	6	12.54	5.00	5.05	2.49	3.15
	7	9.99	4.00	3.88	2.11	3.12
	8	8.06	4.00	2.94	1.12	3.10
	9	5.58	3.00	1.96	0.62	3.07
	10	3.07	2.00	0.93	0.14	3.04
2 (1,4)	1	4.98	0.00	1.89	3.10	3.06
	2	9.97	0.00	3.70	6.27	3.12
	3	14.99	0.00	5.69	9.30	3.18
	4	14.95	0.00	5.61	9.34	3.18
	5	14.95	0.00	5.92	9.03	3.18
	6	12.46	0.00	4.95	7.51	3.15
	7	10.01	0.00	4.12	5.89	3.12
	8	7.94	0.00	3.06	4.88	3.09
	9	5.42	0.00	2.04	3.38	3.06
	10	2.93	0.00	1.07	1.86	3.03
3 (3,5)	4	5.02	2.00	2.11	0.90	3.06
	5	10.03	4.00	4.30	1.73	3.12
	6	15.01	6.00	6.31	2.70	3.18
	7	15.05	6.00	6.39	2.66	3.18
	8	15.05	6.00	6.08	2.97	3.18
	9	12.54	5.00	5.05	2.49	3.15
	10	9.99	4.00	3.88	2.11	3.12
	11	8.06	4.00	2.94	1.12	3.10
	12	5.58	3.00	1.96	0.62	3.07
	13	3.07	2.00	0.93	0.14	3.04
4 (4,5)	4	4.98	0.00	1.89	3.10	3.06
	5	9.97	0.00	3.70	6.27	3.12
	6	14.99	0.00	5.69	9.30	3.18
	7	14.95	0.00	5.61	9.34	3.18
	8	14.95	0.00	5.92	9.03	3.18
	9	12.46	0.00	4.95	7.51	3.15
	10	10.01	0.00	4.12	5.89	3.12
	11	7.94	0.00	3.06	4.88	3.09
	12	5.42	0.00	2.04	3.38	3.06
	13	2.93	0.00	1.07	1.86	3.03

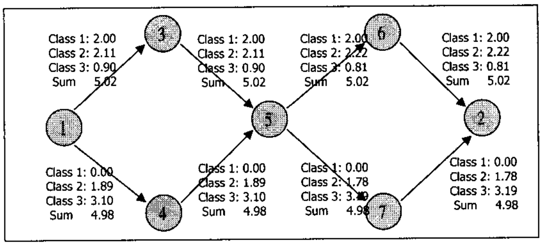
〈Table 5〉 DTA Model Result For The Roll Period 1

5 (5,6)	7	5.02	2.00	2.22	0.81	3.06
	8	10.01	4.00	3.99	2.02	3.12
	9	15.02	6.00	5.83	3.19	3.18
	10	15.02	6.00	5.71	3.31	3.18
	11	15.04	6.00	5.81	3.24	3.18
	12	12.39	4.00	4.96	3.43	3.15
	13	9.93	2.00	3.72	4.21	3.12
	14	8.20	2.00	2.91	3.29	3.10
	15	5.84	2.00	1.91	1.92	3.07
	16	3.28	2.00	1.05	0.22	3.04
6 (5,7)	7	4.98	0.00	1.78	3.19	3.06
	8	9.99	0.00	4.01	5.98	3.12
	9	14.98	0.00	6.17	8.81	3.18
	10	14.98	0.00	6.29	8.69	3.18
	11	14.96	0.00	6.19	8.76	3.18
	12	12.61	1.00	5.04	6.57	3.15
	13	10.07	2.00	4.28	3.79	3.12
	14	7.80	2.00	3.09	2.71	3.09
	15	5.16	1.00	2.09	2.08	3.06
	16	2.72	0.00	0.95	1.78	3.03
7 (6,2)	10	5.02	2.00	2.22	0.81	3.06
	11	10.01	4.00	3.99	2.02	3.12
	12	15.02	6.00	5.83	3.19	3.18
	13	15.02	6.00	5.71	3.31	3.18
	14	15.04	6.00	5.81	3.24	3.18
	15	12.39	4.00	4.96	3.43	3.15
	16	9.93	2.00	3.72	4.21	3.12
	17	8.20	2.00	2.91	3.29	3.10
	18	5.84	2.00	1.91	1.92	3.07
	19	3.28	2.00	1.05	0.22	3.04
8 (7,2)	10	4.98	0.00	1.78	3.19	3.06
	11	9.99	0.00	4.01	5.98	3.12
	12	14.98	0.00	6.17	8.81	3.18
	13	14.98	0.00	6.29	8.69	3.18
	14	14.96	0.00	6.19	8.76	3.18
	15	12.61	1.00	5.04	6.57	3.15
	16	10.07	2.00	4.28	3.79	3.12
	17	7.80	2.00	3.09	2.71	3.09
	18	5.16	1.00	2.09	2.08	3.06
	19	2.72	0.00	0.95	1.78	3.03

Notes) m : Class, 1 : fixed route, 2 : unguided travelers, 3 : guided travelers)
(i,j) : (start node, end node) of link
Time interval 8 : the end of stage



〈Figure 9〉 Actual Link Travel Times Experienced by the First Departed Platoon



〈Figure 10〉 Link Flows for Each Class When the First Platoon Entering Each Link

correct traffic information, which compensates for the biased link flows generated by Class 1 and Class 2 travelers and bring the network to an equilibrium state. This can be verified from the fact that the travel times of used paths for the first departed platoon are equal(〈Figure 9〉) and Class 3 travelers are guided to make the path travel time equal(〈Figure 10〉). From 〈Table 5〉, at time interval 8 of the end of stage, the flows of 6 links can be updated as Updated OD matrices.

2) Update OD Matrix

The updated OD matrices result for each class is summarized in 〈Table 6〉. The updated OD result of rolling horizon implementation can be evaluated from the following aspect: At the end of stage (time interval 8), the flows on each link should be equal to the sum of each updated OD matrix whose origin is the end node of that link. This fact is verified from the combined results of 〈Table 5〉 and 〈Table 6〉.

[For Class 1]

$$X_{a1}(\text{roll period} + \text{rest period}) = \sum_{\Gamma}\sum_{p} OD^{jp}_{m=1}(\Gamma) \qquad (26)$$

where,

$\Gamma = roll\ period + rest\ period - \tau_a(t)$

j is the end node of link a

$X_{11}(8) = 1.00+1.00+2.00=4.00$

$X_{31}(8) = 2.00+2.00+2.00=6.00$

$X_{51}(8) = 2.00+2.00=4.00$

[For Class 2]

$$\sum_{(i,j)\in a} X_{a2}(\text{roll period} + \text{rest period}) = \sum_{\Gamma}\sum_{j}\sum_{s} OD^{js}_{m=2}(\Gamma) \quad \forall j,s \qquad (27)$$

where,

$\Gamma = roll\ period + rest\ period - \tau_a(t)$

$X_{12}(8) = 0.98+1.03+0.93=2.94$

$X_{22}(8) = 1.02+0.97+1.07=3.06$

$X_{32}(8) + X_{42}(8) = 4.00+4.00+4.00=12.00$

(node conservation : the end node of two links is node 5)

$X_{52}(8) = 2.22+1.77=3.99$

$X_{62}(8) = 1.78+2.23=4.01$

[For Class 3]

$$\sum_{(i,j)\in a} X_{a3}(\text{roll period} + \text{rest period}) = \sum_{\Gamma}\sum_{j}\sum_{s} OD^{js}_{m=3}(\Gamma) \quad \forall j,s \qquad (28)$$

where, $\Gamma = roll\ period + rest\ period - \tau_a(t)$

$X_{13}(8) = 0.50+0.48+0.14=1.12$

$X_{23}(8) = 1.50+1.52+1.86=4.88$

$X_{33}(8) + X_{43}(8) = 4.00+4.00+4.00=12.00$

(node conservation : the end node of two links is node 5)

$X_{53}(8) = 0.81+1.21=2.02$

$X_{63}(8) = 3.19+2.79=5.98$

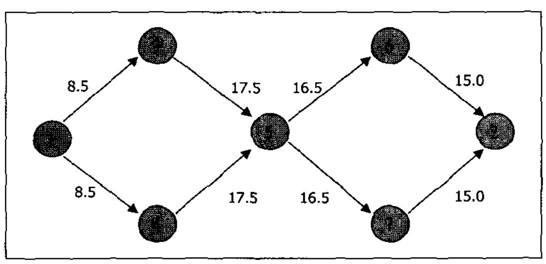
〈Table 6〉 Updated O-D Matrices of Each Class

Γ	Class 1			Class 2			Class 3		
	O	D	OD flows	O	D	OD flows	O	D	OD flows
4	3	1	1.00	3	2	0.98	3	2	0.50
	5	2	2.00	4	2	1.02	4	2	1.50
	-	-	-	5	2	4.00	5	2	4.00
5	3	1	1.00	3	2	1.03	3	2	0.48
	5	2	2.00	4	2	0.97	4	2	1.52
	6	2	2.00	5	2	4.00	5	2	4.00
	-	-	-	6	2	2.22	6	2	0.81
	-	-	-	7	2	1.78	7	2	3.19
6	3	2	2.00	3	2	0.93	3	2	0.14
	5	2	2.00	4	2	1.07	4	2	1.86
	6	2	2.00	5	2	4.00	5	2	4.00
	-	-	-	6	2	1.77	6	2	1.21
	-	-	-	7	2	2.23	7	2	2.79

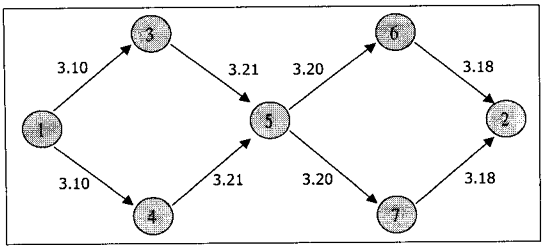
Notes) Γ : time interval, P : path, O : origin, D : destination

3) Results of Next Stage

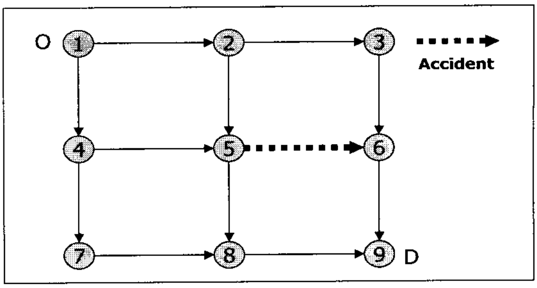
It is not straightforward to evaluate the results of stage 2 because it involves the flow of different classes of travelers that originate in different stage periods. Therefore, 100% of Class 3 travelers are configured and the rolling horizon implementation is processed again to obtain of results of stage 2. The results are evaluated by checking flows and actual travel times at any time interval (time interval 7 is used here). According to 〈Figures 11 and 12〉, all routes connected OD pair (1,2) have the same flow values and travel time which shows that the results of rolling horizon are reasonable as the time rolls forward.



〈Figure 11〉 Link Flows at Time Interval 7 in Roll Period 2



〈Figure 12〉 Actual Link Travel Times at Time Interval 7 in Roll Period 2



〈Figure 13〉 An Employed Network

2. Network Variations

Substantial network variations are usually generated by traffic accidents, incidents, events or construction impacts on traffic networks. These can cause frequently severe traffic congestion locally and area-wide. In the rolling horizon framework, when traffic variations occur, corresponding information is reflected as network inputs from successive stages, and it lasts until the impact of the variations disappears.

When traffic congestion happens from abnormal traffic variations, the best solution in terms of both travelers and network performance is to guide as many as travelers possible to circumvent the congested areas. In case of guided travelers, since they can be informed of network variations, they can avoid the congested area. In the case of the unguided traveler, however, because they have no reliable information sources, they can more easily be exposed to the continuing traffic congestion. This can result in additional traffic congestion and deterioration of network performance.

This study provides three scenarios to demonstrate how unguided travelers contribute to additional traffic congestion and the impact of providing information to them to circumvent the congested areas.

The employed hypothetical network has 9 nodes and 12 links(〈Figure 13〉) with one-lane freeway links of 1.0 mile each. It is assumed that an accident occurs on link 5->6 causing a reduction of 90% of link capacity, and the accident information is reported starting from the stage 2. The horizon time consists of six stages. Each stage consists of ten time intervals of a roll period and no rest period. Among the six roll periods, only the first three roll periods have demands, 80 vehicles departing from node 1 to 9 at each time interval.

1) Travelers Guidance

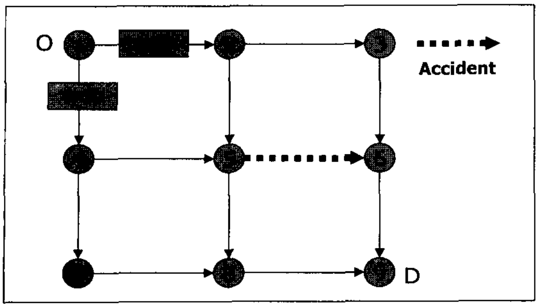
〈Table 7〉 shows when more travelers have perfect traffic information(Class 3), network travel time decreases and average speed increases. When all of them receive perfect traffic information, network travel time is reduced by 11.3% and average speed is increased by 28% compared with the case when all of them do not receive any traffic information.

〈Table 7〉 Travelers Guidance and Network Variation Information

Scenarios	Travelers		Network Travel Time (hours)	Average Speed (mph)
	Class 2 (%)	Class 3 (%)		
No Information	100	0	84.4	20.3
Partial Information	50	50	74.9	22.1
Full Information	0	100	58.6	26.0

2) Number of Traffic Information Devices

〈Figure 14〉 depicts the location of two VMS signs. 〈Table 8〉 shows that as more unguided travelers see the VMS signs, congestion is reduced. When all OD flows see the VMS1 and VMS2 signs, network travel time is reduced by 15% and average speed is increased by 16.2% compared with the case of no VMS information.



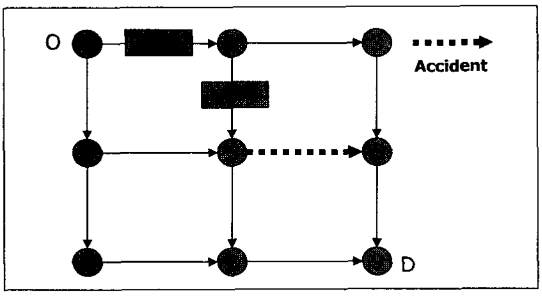
〈Figure 14〉 Location of VMS Sign1 and Sign2

〈Table 8〉 Rolling Horizon Implementation and Number of Variable Message Sign

Scenarios	Travelers		Network Travel Time (hours)	Average Speed (mph)
	Class 2 (%)	Class 3 (%)		
No VMS Sign	100	0	84.4	20.3
VMS1	100	0	76.5	22.0
VMS1 and VMS2	100	0	71.7	23.6

3) Location of Traffic Information Devices

〈Figure 15〉 illustrates two locations of VMS signs, VMS1 and VMS3. Since all unguided travelers depart from origin 1 to destination 9, it can be assumed that more travelers can see the VMS1 sign than VMS3. 〈Table 9〉 shows that as more travelers see the VMS sign, network travel time is reduced and average speed is increased. For VMS1, network travel time is reduced by 9.3% and average speed is increased by 7.7%. However, in case of VMS3, network travel time is reduced by 5.0% and average speed is increased by 4.4%.



〈Figure 15〉 Locations of VMS Sign1 and Sign3

〈Table 9〉 Rolling Horizon Implementation and Location of Variable Message Sign

Scenarios	Demands		Total Travel Time (hours)	Average Speed (mph)
	Class 2 (%)	Class 3 (%)		
No VMS Sign	100	0	84.4	20.3
VMS1	100	0	76.5	22.0
VMS3	100	0	80.2	21.2

V. Concluding Remarks

In this paper, the rolling horizon implementation and its solution process have been proposed for the multi-class analytical dynamic traffic assignment model. Since it is assumed that the travel demand and network condition are not predictable, the rolling horizon process makes possible the analysis of more realistic network scenarios using an analytical DTA model.

Two raised issues of rolling horizon implementation are 1) how to deal with unfinished trips, trips which do not reach their destination by the end of the stage, and 2) how reroute these unfinished trips on the network. The unfinished trips were formulated as updated OD matrixes to keep the impact of unfinished trips. Fixed route travelers are assumed to continue to use the same route, sub-fixed route, while unguided and guided route travelers are rerouted based on travelers route choice principles, SDUO for unguided and DUO for guided travelers.

Two experimental studies have been prepared to verify the effectiveness of the rolling horizon implementation in the DTA system and how it can be applied for real traffic situations.

In the first study, three sets of results are have been demonstrated from the computational experiments. First, the Class 3 travelers receive correct traffic information which can compensate for the biased link flows caused by Class 1 and Class 2 travelers that further bring the network to an equilibrium state. Second, the impact of unfinished trips, which have not reached their destinations at the end of the stage, can be considered as the updated demand at the next stage period. Third, as the time rolls forward, the equilibrium state will remain valid if all travelers in the network are assumed as Class 3.

In the second study, it has been demonstrated that when abnormal network variations occur, the rolling horizon implementation could reduce the level of additional traffic congestion through traffic information or other information devices. The result showed that rolling horizon implementation could be an effective way to model the impact of providing traffic information to travelers. In this study, the number of informed travelers, the number of the Variable Message Sign(VMS), and the location of the VMS are considered as variables for the case study.

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