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Hewitt Realcompactification and Basically Disconnected Cover*

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Abstract

We show that if the Stone-Čech compactification of ΛX and the minimal basically disconnected cover of βX are homeomorphic and every real $\sigma Z(X)^{\#}$-ultrafilter on X has the countable intersection property, then there is a covering map from $v(\Lambda X)$ to vX and every real $\sigma Z(X)^{\#}$-ultrafilter on X has the countable intersection property if and only if there is a homeomorphism from the Hewitt realcompactification of ΛX to the minimal basically disconnected space of vX.

0. Introduction

All spaces in this paper are assume to be Tychonoff and for a space X, let $(\beta X, \beta_X)$ ((vX, v_X), resp.) denotes the Stone-Čech compactification (Hewitt realcompactification, resp.) of X. For any regular space X, there is the absolute (EX, k_X) of X and if X is Tychonoff, then there is a homeomorphism $k: \beta(EX) \rightarrow E(\beta X)$. Moreover, for any space X, the following are equivalent :

(i) there is a homeomorphism $v(EX) \rightarrow E(vX)$,

(ii) if $\{A_n : n \in N\}$ is a decreasing sequence in $R(X)$ and $\bigcap\{A_n : n \in N\} = \phi$, then $\bigcap\{ \mathrm{cl}_{vX}(A_n) : n \in N\} = \phi$,

(iii) if $\{A_n : n \in N\}$ is a decreasing sequence in $R(X)$, then $\mathrm{cl}_{vX}(\bigcap\{A_n : n \in N\}) = \bigcap\{ \mathrm{cl}_{vX}(A_n) : n \in N\}$, and

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(iv) every stable R(X)-ultrafilter has the countable intersection prorerty [4].

For any Tychonoff space X, there is a minimal basically disconnected cover $(\Lambda X, \Lambda_X)$ [5] and if X is locally weakly Lindelöf, then ΛX are given by a filter space [2] and [4].

In this paper, we show that if the Stone-Čech compactification of ΛX and the minimal basically disconnsted cover of βX are homeomorphic, then ΛX is a filter space and that if every real $\sigma Z(X)^{\#}$-ultrafilter on X has the countable intersection property, then there is a covering map from $v(\Lambda X)$ to vX. Using this, we will show that if the Stone-Čech compactification of ΛX and the minimal basically disconnsted cover of βX are homeomorphic, then every real $\sigma Z(X)^{\#}$-ultrafilter on X has the countable intersection property if and only if there is a homeomorphism from the Hewitt realcompactification of ΛX to the minimal basically disconnected space of vX. For the terminology, we refer to [1] and [4].

1. Fixed $\sigma Z(X)^{\#}$-ultrafilter space

Recall that a subspace Y of a space X is said to be C^**-embedded in* X if for any bounded real-valued continuous map $f: Y \to R$, there is a bounded real-valued continuous map $g: X \to R$ with $g|_Y = f$ and that a space X is called ***basically disconnected*** if every cozero-set in X is C^*- embedded in X.

Definition 1.1. Let X be a space. Then a pair (Y, f) is called

(1) *a cover of* X if $f: Y \to X$ is a covering map,

(2) *a basically disconnected cover of* X if (Y, f) is a cover of X and Y is a basically disconnected space and,

(3) *a minimal basically disconnected cover of* X if (Y, f) is a cover of X and it is a basically disconnected cover of X and for any basically disconnected cover (Z, g) of X, there is a covering map $h: Z \to Y$ with $f \circ h = g$

For any space X, the collection $R(X)$ of all regular closed sets in X, when partially ordered by inclusion, becomes a complete Boolean algebra, in which the join, meet, and complementation operations are defined as follows:

If $A \in R(X)$ and $\{A_i : i \in I\} \subseteq R(X)$, then

$$\bigvee\{A_i : i \in I\} = \mathrm{cl}_X(\bigcup\{A_i : i \in I\}),$$

$$\bigwedge\{A_i : i \in I\} = \mathrm{cl}_X(\mathrm{int}_X(\bigcap\{A_i : i \in I\})), \text{ and}$$

$$A' = \mathrm{cl}_X(X - A)$$

and a sublattice of $R(X)$ is a subset of $R(X)$ that contains ϕ, X and is closed under finite joins and meets [4].

A lattice L is called *σ-complete* if every countable subset of L has join and meet. For a subset M of a complete Boolean algebra L, σM denotes the smallest σ-complete Boolean subalgebra of L containing M. For any space X, $Z(X)$ denotes the set of all zero-sets and let $Z(X)^{\#} = \{\mathrm{cl}_X(\mathrm{int}_X(A)) : A \in Z(X)\}$. For a space X and a zero-set Z in X, there is a zero-set A in βX with $A \cap X = Z$. It is well-known that for any covering map $f : Y \to X$, the map $\psi : R(Y) \to R(X)$, defined by $\psi(A) = f(A)$, is a Boolean isomorphism and that for any extension Y of a space X, the map $\phi : R(Y) \to R(X)$, defined by $\phi(A) = A \cap X$, is a Boolean isomorphism. Hence, for any space X, the isomorphism $\phi : R(\beta X) \to R(X)$ induces Boolean isomorphisms $\sigma Z(\beta X)^{\#} \to \sigma Z(X)^{\#}$ and $\sigma Z(\upsilon X)^{\#} \to \sigma Z(X)^{\#}$.

For any space X, $(\Lambda X, \Lambda_X)$ $((\Lambda(\beta X), \Lambda_\beta)$, resp.) denotes the minimal basically disconnected cover of X (βX, resp.). Vermeer showed that for a compact space X, ΛX is given by the Stone-space $S(\sigma Z(X)^{\#})$ of $\sigma Z(X)^{\#}$ and $\Lambda_X(\alpha) = \bigcap \alpha$ [5].

Recall that a space X is called *weakly Lindelöf* if every open cover of X has a countable subfamily that is dense in X and that a space X is called *locally weakly Lindelöf* if every element of X has a weakly Lindelöf neighborhood. In [2] and [4], it is shown that for any locally weakly Lindelöf space X, ΛX is given by the filter space $\{\alpha : \alpha$ is a fixed $\sigma Z(X)^{\#}$-ultrafilter$\}$ and $\Lambda_X(\alpha) = \bigcap \alpha$.

For a space X, there is the Stone extension $\Lambda^\beta : \beta(\Lambda X) \to \beta X$ of $\beta_X \circ \Lambda_X$. Since $\beta(\Lambda X)$ and βX are compact, Λ^β is a covering map and since $\beta(\Lambda X)$ is basically disconnected [5], there is a covering map $h_X : \beta(\Lambda X) \to \Lambda(\beta X)$ $\Lambda^\beta = \Lambda_\beta \circ h_X$. If h_X is a homeomorphism, then we write $\beta(\Lambda X) = \Lambda(\beta X)$ and in case, we will identify $(\beta(\Lambda X), \Lambda^\beta)$ and $(\Lambda(\beta X), \Lambda_\beta)$. In [2], it is shown that if X is a weakly Lindelöf space, then $\beta(\Lambda X) = \Lambda(\beta X)$.

Proposition 1.2. Suppose that X is a space and $\beta(\Lambda X) = \Lambda(\beta X)$. Then ΛX is given by the filter space $\{\alpha : \alpha$ is a fixed $\sigma Z(X)^{\#}-$ ultrafilter $\}$.

Proof. Since the diagram

$$\begin{array}{ccc} \Lambda_{\beta}^{-1}(X) & \xrightarrow{\Lambda_{\beta_X}} & X \\ j \downarrow & & \downarrow \beta_X \\ \beta(\Lambda X) & \xrightarrow[\Lambda_{\beta}]{} & \beta X \end{array}$$

is a pullback in the category **Top**, there is a continuous map $h_X^{\cdot} : \Lambda X \rightarrow \Lambda_{\beta}^{-1}(X)$ such that $\Lambda_{\beta_X} \circ h_X^{\cdot} = \Lambda_X$ and $j \circ h_X^{\cdot} = h_X \circ \beta_{\Lambda X}$, where j is the inclusion map and Λ_{β_X} is the restriction and corestriction of Λ_{β} with respect to $\Lambda_{\beta}^{-1}(X)$ and X, respectively. Take any $x \in \Lambda_{\beta}^{-1}(X)$. Then there is $y \in \beta(\Lambda X)$ with $h_X(y) = x$ and $\Lambda_{\beta}(x) = \Lambda_{\beta_X}(x) \in X$. Since Λ_X is a covering maps, $y \in \Lambda X$. Hence $h_X^{\cdot}$ is onto. Since $\Lambda_{\beta_X} \circ h_X^{\cdot} = \Lambda_X$ and Λ_X is perfect, $h_X^{\cdot}$ is a perfect map [4]. Since h_X is $1-1$, $h_X^{\cdot}$ is a homeomorphism. Hence $(\Lambda_{\beta}^{-1}(X), \Lambda_{\beta_X})$ is the minimal basically disconnected cover of X. Thus $\Lambda_{\beta}^{-1}(X)$ is the fixed $\sigma Z(X)^{\#}$-ultrafilter $\{\alpha : \alpha$ is a fixed $\sigma Z(X)^{\#}-$ ultrafilter $\}$.

Propostion 1.3. Let X be a space. Suppose that ΛX is given by the fixed $\sigma Z(X)^{\#}$ -ultrafilter space. Then for any decreasing sequence $(A_n)_n$ in $\sigma Z(X)^{\#}$, $\Lambda_X(\bigcap \{A_n^* : n \in N\}) = \bigcap \{A_n : n \in N\}$, where $A_n^* = \{\alpha : \alpha$ *is a* fixed $\sigma Z(X)^{\#}$ $-$ ultrafilter and $A_n \in \alpha\}$.

Proof. Take any $A \in \sigma Z(X)^{\#}$ and $\alpha \in A_n^*$. Then $\Lambda_X(A^*) \subseteq A$.

Take any $x \in A$. Let $\alpha_x = \{B \in \sigma Z(X)^{\#} : x \in \mathrm{int}_X(B)\}$. Then $\alpha_x \bigcup \{A\}$ has the finite meet property and hence there is a $\sigma Z(X)^{\#}$-ultrafilter α containing $\alpha_x \bigcup \{A\}$.

Since α_x is a local base at x in X, $\Lambda_X(\alpha)=\bigcap\alpha=x$ and so $A\subseteq\Lambda_X(A)$. Thus $\Lambda_X(\bigcap\{A^*_n: n\in N\})\subseteq\bigcap\{A_n: n\in N\}$. Take any $y\in\bigcap\{A_n: n\in N\}$, then $\alpha_y\bigcup\{A_n: n\in N\}$ has the finite meet property and hence it is contained in a $\sigma Z(X)^\#$–ultrafilter η and so $\eta\in\bigcap\{A^*_n: n\in N\}$ and $\Lambda_X(\eta)=y$.

2. Hewitt realcompactification and minimal basically disconnected cover

In the following, we may assume that every space has the property $\Lambda(\beta X)=\beta(\Lambda X)$. For any space X, let $\upsilon: \Lambda X\rightarrow\upsilon(\Lambda X)$ be the Hewitt realcompactification of ΛX and $(\Lambda(\upsilon X),\Lambda_\upsilon)$ the minimal basically disconnected cover of υX. Since υX is realcompact, there is a continuous map $r_X: \upsilon(\Lambda X)\rightarrow\upsilon X$ such that $\upsilon_X\circ\Lambda_X=r_X\circ\upsilon_\Lambda$ [4]. If there is a homeomorphism $k: \upsilon(\Lambda X)\rightarrow\Lambda(\upsilon X)$ such that $\Lambda_\upsilon\circ k=r_X$, then we write $\Lambda(\upsilon X)=\upsilon(\Lambda X)$ and in case, we will identify $(\upsilon(\Lambda X),r_X)$ and $(\Lambda(\upsilon X),\Lambda_\upsilon)$. Recall that a covering map $f:Y\rightarrow X$ is called *$\sigma Z^\#$– irreducible* if $\{f(A): A\in\sigma Z(Y)^\#\}=\sigma Z(X)^\#$ and that a subspace D of a space X is *$\sigma Z^\#$– embedded* if for any $B\in\sigma Z(D)^\#$, there is S $B\in\sigma Z(X)^\#$ such that S $\bigcap D=B$. For any compact space X, Λ_X is *$\sigma Z^\#$–irreducible* [3] and every dense *C^*–embedded* subspace of a space is *$\sigma Z^\#$–embedded.*

We will give some characterizations of a space X for which $\Lambda(\upsilon X)=\upsilon(\Lambda X)$.

Definition 2.1. Let X be a space. A $\sigma Z(X)^\#$- ultrafilter α is called *real* if $\bigcap\{\mathrm{cl}_{\beta X}(A): A\in\alpha\}\in\upsilon X$.

Theorem 2.2. Let X be a space. Then we have the following:

(a) Suppose that every $\sigma Z(X)^\#$-ultrafilter has the countable intersection property and $\Lambda(\beta X)=\beta(\Lambda X)$. Then r_X is a covering map.

(b) The following are equivalent:

(1) $\Lambda(\upsilon X)=\upsilon(\Lambda X)$,

(2) if $\{A_n: n\in N\}$ is a decreasing sequence in $\sigma Z(X)^\#$ with $\bigcap\{A_n: n\in N\}=\phi$,

then $\bigcap\{\ \mathrm{cl}_{\upsilon X}(A_n) : n\in N\ \}=\phi$,

(3) if $\{A_n : n\in N\}$ is a decreasing sequence in $\sigma Z(X)^\#$, then $\mathrm{cl}_{\upsilon X}(\bigcap\{A_n: n\in N\}) =\bigcap\{\ \mathrm{cl}_{\upsilon X}(A_n) : n\in N\}$, and

(4) every real $\sigma Z(X)^\#$-ultrafilter has the countable intersection prorerty.

Proof. (a) Let $j_1 : \upsilon(\Lambda X) \to \beta(\Lambda X)$ and $j_2 : \upsilon X \to \beta X$ be inclusion maps. The following diagram commutes.

$$\begin{array}{ccc} \upsilon(\Lambda X) & \xrightarrow{\ r_X\ } & \upsilon X \\ {\scriptstyle j_1}\downarrow & & \downarrow{\scriptstyle j_2} \\ \beta(\Lambda X) & \xrightarrow[\Lambda_\beta \circ h_X]{} & \beta X \end{array}$$

Since $j_2\circ r_X \circ \upsilon_\Lambda = j_2 . \upsilon_X \circ \Lambda_X = \Lambda_\beta \circ h_X \circ j_1 \circ \upsilon_\Lambda$ and υ_Λ is dense, $j_2\circ m_X = \Lambda_\beta\circ h_X \circ j_1$. Let $p\in \upsilon X$ and $\alpha\in \Lambda^{-1}{}_\beta(p)$. Suppose that $\alpha\notin \upsilon(\Lambda X)$. Then there is a sequence $\{Z_n: n\in N\}\in \sigma Z(\beta(\Lambda X))^\#$ such that for any $n\in N$, $\alpha\in \mathrm{int}_{\beta(\Lambda X)}(Z_n)$ and $(\bigcap\{Z_n: n\in N\})\cap \Lambda X=\phi$ [4]. Since Λ_β is $\sigma Z^\#$-irreducible, $\Lambda_\beta(Z_n)\in \sigma Z(\beta X)^\#$. Hence $\alpha_X=\{U\cap X : U\in\alpha\}$ is a $\sigma Z(X)^\#$-ultrafilter. Let $n\in N$. Since $\alpha\in \mathrm{int}_{\beta(\Lambda X)}(Z_n)$ and $\{A^* : A\in \sigma Z(\beta X)^\#\}$ is a base for $\beta(\Lambda X)$, there is $A\in \sigma Z(\beta X)^\#$ with $\alpha\in A^*\subseteq Z_n$ and hence $\Lambda_\beta(\alpha)\in\Lambda_\beta(A^*)= A\subseteq \Lambda_\beta(Z_n)$. So $\Lambda_\beta(Z_n)\in\alpha$. Hence for any $n\in N$, $\Lambda_\beta(Z_n)\cap X\in\alpha_X$. Since $p\in \upsilon X$, α_X is real and so $\bigcap\{\Lambda_\beta(Z_n)\cap X : n\in N\}\neq\phi$. Pick $x\in\bigcap\{\Lambda_\beta(Z_n)\cap X : n\in N\}$. Let $n\in N$. Then $\Lambda_\beta^{-1}(x)\cap Z_n\neq\phi$. Since $\Lambda_\beta^{-1}(x)=\Lambda_X^{-1}(x)$, $\Lambda_X^{-1}(x)\cap Z_n\neq\phi$. Since $\Lambda_X{}^{-1}(x)$ is compact and $\bigcap\{\Lambda_X{}^{-1}(x)\cap Z_n : n\in N\}$ is a decreasing family of closed sets in $\Lambda_X{}^{-1}(x)$ with the finite intersection property, $\bigcap\{\Lambda_X{}^{-1}(x)\cap Z_n : n\in N\}\neq\phi$ and hence $(\bigcap\{Z_n: n\in N\})\cap\Lambda X\neq\phi$. This is a contradiction. Hence $\alpha\in\upsilon(\Lambda X)$. Thus r_X is onto. Since j_1 and j_2 are dense and $\beta(\Lambda X)$ and βX are compact, r_X is a covering map [4].

(b) (1)⇒(2) Suppose that there is a sequence $\{A_n : n \in N\}$ in $\sigma Z(X)^\#$ such that $\bigcap\{\mathrm{cl}_{\upsilon X}(A_n) : n \in N\} \neq \phi$. Since $\beta(\upsilon(\Lambda X)) = \beta(\Lambda X) = \Lambda(\beta X) = \Lambda(\beta(\upsilon X))$, $\Lambda(\upsilon X)$ is given by the filter space $\{\alpha : \alpha$ is a fixed $\sigma Z(\upsilon X)^\#-$ ultrafilter $\}$ and $\mathrm{cl}_{\upsilon X}(A_n) \in \sigma Z(\upsilon X)^\#$ for all $n \in N$, by Proposition 1.3, $\Lambda_\upsilon(\bigcap\{(\mathrm{cl}_{\upsilon X}(A_n))^* : n \in N\}) = \bigcap\{\mathrm{cl}_{\upsilon X}(A_n) : n \in N\}$. Since $\bigcap\{\mathrm{cl}_{\upsilon X}(A_n) : n \in N\} /= \phi$, $\bigcap\{(\mathrm{cl}_{\upsilon X}(A_n))^* : n \in N\} \neq \phi$. Note that for any $n \in N$, $(\mathrm{cl}_{\upsilon X}(A_n))^* = \mathrm{cl}_{\Lambda(\upsilon X)}(\Lambda_\upsilon^{-1}(\mathrm{int}_{\upsilon X}(\mathrm{cl}_{\upsilon X}(A_n))))$. Let $t \in \bigcap\{\mathrm{cl}_{\Lambda(\upsilon X)}(\Lambda_\upsilon^{-1}(\mathrm{int}_{\upsilon X}(\mathrm{cl}_{\upsilon X}(A_n)))) : n \in N\}$. Then there is the $\sigma Z(X)^\#$-ultrafilter α such that $t \in \bigcap\{\mathrm{cl}_{\Lambda(\upsilon X)}(A) : A \in \alpha\}$ [4]. Since $t \in \upsilon(\Lambda X)$, the $\sigma Z(X)^\#$-ultrafilter α has the countable intersection property. Let $n \in N$. Then there is $B_n \in \sigma Z(\upsilon X)^\#$ such that $B_n \cap X = A_n$. Since $\Lambda_X(\mathrm{cl}_{\Lambda(\upsilon X)}(\Lambda_\upsilon^{-1}(\mathrm{int}_{\upsilon X}(B_n))) \cap \Lambda X) = \Lambda_X(\mathrm{cl}_{\Lambda X}(\Lambda_X^{-1}(\mathrm{int}_X(A_n))))$, $\mathrm{cl}_{\Lambda(\upsilon X)}(\Lambda_\upsilon^{-1}(\mathrm{int}_{\upsilon X}(B_n))) = \mathrm{cl}_{\Lambda(\upsilon X)}(\Lambda_\upsilon^{-1}(\mathrm{int}_{\upsilon X}(\mathrm{cl}_{\upsilon X}(A_n)))) = \mathrm{cl}_{\Lambda(\upsilon X)}(\mathrm{cl}_{\Lambda X}(\Lambda_X^{-1}(\mathrm{int}_X(A_n))))$. Thus $t \in \mathrm{cl}_{\Lambda(\upsilon X)}(\mathrm{cl}_{\Lambda X}(\Lambda_X^{-1}(\mathrm{int}_X(A_n))))$. Since $\mathrm{cl}_{\Lambda(\upsilon X)}(\Lambda_\upsilon^{-1}(\mathrm{int}_{\upsilon X}(B_n))) \in \sigma Z(\Lambda(\upsilon X))^\#$ and $\Lambda(\upsilon X)$ is basically disconnected, $\mathrm{cl}_{\Lambda(\upsilon X)}(\Lambda_\upsilon^{-1}(\mathrm{int}_{\upsilon X}(B_n))) \in B(\Lambda(\upsilon X))$. Hence $\mathrm{cl}_{\Lambda X}(\Lambda_X^{-1}(\mathrm{int}_X(A_n))) \in \alpha$ and so $\bigcap\{\mathrm{cl}_{\Lambda X}(\Lambda_X^{-1}(\mathrm{int}_X(A_n))) : n \in N\} = \bigcap\{A_n : n \in N\} \neq \phi$.

(2)⇒(3) Suppose that $p \notin \mathrm{cl}_{\upsilon X}(\bigcap\{A_n : n \in N\})$. Then there is $B \in \sigma Z(\upsilon X)^\#$ such that $p \in \mathrm{int}_{\upsilon X}(B)$ and $B \cap (\bigcap\{A_n : n \in N\}) = \phi$. Since $\{C \wedge A_n : n \in N\}$ is a decreasing sequence in $\sigma Z(X)^\#$ with empty intersection, $\bigcap\{\mathrm{cl}_{\upsilon X}(C \wedge A_n) : n \in N\} = \phi$. Suppose that $p \in \bigcap\{\mathrm{cl}_{\upsilon X}(A_n) : n \in N\}$. Let W be a neighborhood of p in υX and $n \in N$. Then $\mathrm{int}_{\upsilon X}(W) \cap \mathrm{int}_{\upsilon X}(B) \cap A_n \neq \phi$. Since $C \wedge A_n = \mathrm{cl}_X(\mathrm{int}_X(C \cap A_n)) = \mathrm{cl}_X(\mathrm{int}_X(C) \cap \mathrm{int}_X(A_n)) \supseteq \mathrm{int}_X(C) \cap A_n = \mathrm{int}_X(B \cap X) \cap A_n \supseteq \mathrm{int}\, \upsilon X(B) \cap A_n$, $(C \wedge A_n) \cap W \supseteq \mathrm{int}_{\upsilon X}(B) \cap A_n \cap W \neq \phi$. Hence $p \in \bigcap\{\mathrm{cl}_{\upsilon X}(C \wedge A_n) : n \in N\}$ and so $p \in \bigcap\{cl_{\upsilon_x}(A_n) : n \in N\}$.

(3) ⇒ (4) Let α be a real $\sigma Z(\upsilon X)^\#$-ultrafilter and $\{B_n : n \in N\} \subseteq \alpha$. For any $n \in N$, let $A_n = \wedge\{B_i : 1 \leq i \leq n\}$. Then $\{A_n : n \in N\}$ is a decreasing sequence in

$\sigma Z(X)^{\#}$. Since α is real, there exist a $p \in \upsilon X$ such that $p \in \bigcap \{ \mathrm{cl}_{\upsilon X}(A_n) : n \in N \}$. By the hypothesis, $p \in \mathrm{cl}_{\upsilon X}(\bigcap \{A_n : n \in N\})$. Hence $\bigcap\{A_n : n \in N\} \neq \phi$ and so $\bigcap\{B_i : i \in N\} \neq \phi$. Thus α has the countable intersection property.

(4) $\Rightarrow$ (1) By Propersition 1.2 and (a) in this theorem, r_X is covering and so there is a covering map $t: \upsilon(\Lambda X) \rightarrow \Lambda(\upsilon X)$ with $r_X = \Lambda_\upsilon \circ t$. Suppose that $x \neq y$ in $\upsilon(\Lambda X)$. Then there are $A, B \in \sigma Z(\upsilon(\Lambda X))^{\#}$ such that $x \in A$, $y \in B$ and $A \cap B = \phi$. Since $\Lambda(\upsilon X)$ is dense C^*-embedded in $\Lambda(\beta X) = \beta(\Lambda X)$, $\Lambda(\upsilon X)$ is $\sigma Z(X)^{\#}$-embedded and so $t \circ \Lambda_\upsilon$ is $\sigma Z^{\#}$-irreducible [3]. Hence t is $\sigma Z^{\#}$-irreducible. Since $A \wedge B = \phi$ and t is a covering map, $t(A) \wedge t(B) = \phi$. Since t is $\sigma Z^{\#}$-irreducible, $t(A)$, $t(B) \in \sigma Z(\Lambda(\upsilon X))^{\#} = B(\Lambda(\upsilon X))$ and so $t(A) \cap t(B) = \phi$. Hence $t(x) \neq t(y)$ and so t is $1-1$. Thus t is a homeomorphism.

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