KCI CONRETE JOURNAL
Vol.14 No.1, March 2002, pp.36-42

Flexural Behaviors of Precast Prestressed Rectangular and Inverted-tee Concrete Beams for Buildings

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(This paper is English translation of the paper which has been awarded the 13th KCI paper of the year.)

Abstract

Flexural behaviors of the two typical precast beam sections (inverted tee and rectangular) for buildings were investigated and compared. The height of web in the inverted tee beam was generally less than half of beam depth to be adapted to that of the nib in the ends of double-tee where the total building height limited considerably. The inverted-tee beams were designed for a parking live load - 500kgf/m^2 and a market - 1,200kgf/m^2 from the currently used typical shape of a domestic building site in Korea. The area and bottom dimension of rectangular beams were the same as those of inverted tee beams. These two beams were also reinforced with a similar strength.

Following results were obtained from the studies above; 1) the rectangular beam is simpler in production, transportation, and erection, and more economic than the inverted tee beam in the construction test for these two beams with a same dimension and a similar strength, 2) all of the beams considered in the tests were generally failed in values close to those of the strength requirements in ACI Provisions. The ratios of test result to calculated value are averaged to 1.04. One rectangular and one inverted tee beams failed in a value only 2~3% larger than the estimated volue of the Strength Design Methool the results of the Strain Compatibility Method were slightly more accurate than those of the Strength Design Method, 4) the maximum deflections of all of the beams under the full service loads were less than those of the allowable limit in ACI Code Provisions. The rectangular beams experienced more deflection then inverted tee in the same loading condition and failed with more deflection, and 5) the rectangular and inverted tee beams showed good performances under the condition of service and ultimate loads. However, one inverted tee beams with 6m span developed an initial flexural crackings under 88% of the full service load even though they designed to satisfy the ACI tensile stress limit provisions.

***Keywords:** precast, concrete, prestressed, inverted-tee, rectangular, service, flexural, strength, PCI design*

1. Introduction

Two typical beam sections (inverte e and rectangular) are generally used for a architectural precast member in Korea. Structural Engineer may need some practical informations on them for a selection of slab system in design.

The original double tee slabs are generally supported on the inverted tee beams. The lower flange of inverted tee is provided to support the dapped end of double tees, and the additional web is helpful to resist the excessive loadings of flexural moment and shear forces. The re-entrained shape of inverted tee beam gives a fine view by conforming to a dapped end of double tee slab. The precast inverted tee beams need to support the self weight of beam, slab, and topping concrete and the composite section after the hardening of topping concrete for additional live load. Thus, the web of inverted tee is referred to resist a flexural load of self weight. The inverted tee beams are frequently used both for the bridge and building members. However, the height of web of inverted tee is more than 60% of total height of inverted tee in bridge structure as indicated by Mirza and Furlong,[1,2)] while the depth of web in building is less than 50% to conform to the dapped end of double tee.

The modified dapped ended double tees are developed for building members in Korean precast factory as shown in Fig. 1. This modified dapped ends(Fig. 2) are made by thickening the flange from original one(Fig. 3) and are supported by the rectangular section with a uniform load in the dapped ends rather than the inverted tee with a point load.

The continuous long line method is not available for a

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production of dapped ended double tee. Thus, the cost of production is similar because separated two end forms for one double tee are necessary for both of original and modified dapped ended double tees. The story height are reduced by deducting the dead space from double tee web. The height of thickened nib could be designed up to 1/3 of total height in a modified dapped end, while the height is 1/2 in the original one. Additional strong point is that no more forms are needed for casting a topping concrete in modified one.

In this study, flexural behaviors of the two typical architectural precast beam sections - inverted tee and rectangular - were examined and compared. The heights of web in inverted tee beams are generally less than half of beam depth in building structures to accomodate the nib of double-tee in which the total building height limited considerably. The inverted-tee beams were designed for a parking live load - 500kgf/m^2 and a market - 1,200kgf/m^2 from a currently used typical shape of a domestic building site in Korea.

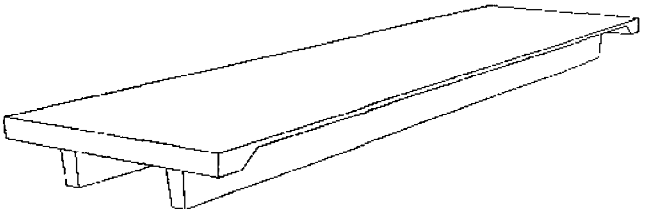

Fig. 1 Double-tee slab with modified dapped ends

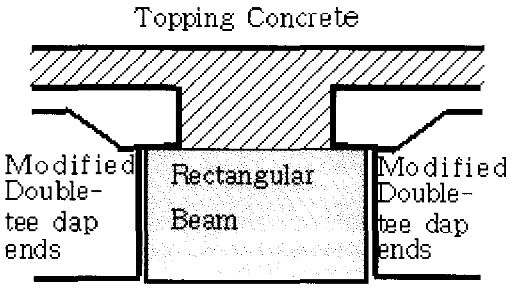

Fig. 2 Connection of rectangular beam and double-tee slab with modified dapped ends

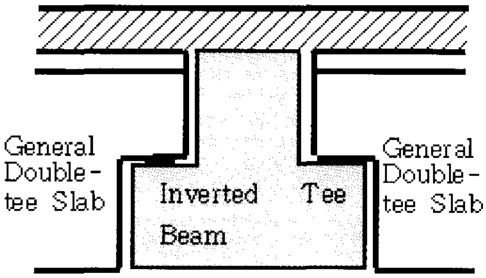

Fig. 3 Connection of inverted tee beam and double-tee slab with general dapped ends

The bottom dimension and area of rectangular beams are the same as those of inverted tee beams. These two beams are also reinforced for similar strength.

Eight flexural tests were performed on those beams. The rectangular beams have more merits than inverted tee beams from this experimental and construction tests on full scale specimens with a same dimension and similar strength in production, transportation, and erection. The merits of the rectangular beams are as follows;

1) The inverted tee beams are more difficult in production, transportation, and election than rectangular beams because of complex figure of section .
2) The inverted tee beams are more weak in the dapped ends than those of rectangular beams.
3) The inverted tee beams need more shear and torsional reinforcement than rectangular beams.
4) The inverted tee beams are more difficult in continuity of slab than rectangular beams.

2. Flexural member design

The flexural strength of prestressed members are obtained from the equations of equilibrium as that of a reinforced concrete. The tensile stress of strand at nominal strength of member, f_{ps} is determined when the maximum compressive strain of concrete (0.003) reaches at the top of section in the prestressed members. Thus, the stress, f_{ps} is determined from the linear strain in strain compatibility theory.

The design procedure of strain compatibility equation method are as follows; 1) assume neutral axis, 2) calculate strain of prestressed and non-prestressed reinforcement when concrete strain reach 0.003, 3) calculate the depth of equivalent rectangular stress block, and 4) check the force equilibrium at the section considered. The member strength can be calculated accurately in pursuit of the correct value of f_{ps}. By checking the equilibrium at the section from the stresses above, the flexural member strength is calculated reasonably but needs to calculated repeatedly until equilibrium.

The equation (1) is provided in the strength deign method[4] for simple estimation of flexural strength up to the members with supplementary tension and compression mild bar reinforcement.

$$f_{ps} = f_{pu}\left(1 - \frac{\gamma_p}{\beta_1}\left[\frac{\rho_p f_{pu}}{f_{ck}} + \frac{d}{d_p}(\omega - \omega')\right]\right) \qquad (1)$$

The amount and location of prestressing and mild steel in equation (1) are restricted as follows.

$$\left[\frac{\rho_p f_{pu}}{f_{ck}} + \frac{d}{d_p}(\omega - \omega')\right] \geq 0.17 \qquad (2)$$

in equations (1) and (2),

f_{pu} = ultimate strength of prestressing steel

f_{ps} = stress of prestressing steel at nominal strength of member

γ_p = factor for prestressing steel 0.28, for low relaxation.

d = effective depth of tensile mild steel

d_p = effective depth of prestressing steel

ρ_p = ratio of prestressed reinforcement

$$\omega = \rho\frac{f_y}{f_{ck}}, \quad \rho = \frac{A_s}{bd}$$

$$\omega' = \rho'\frac{f_y}{f_{ck}}, \quad \rho' = \frac{A'_s}{bd}$$

Thus, the nominal strength of prestressed members are calculated from equation (3) when non-prestressed reinforcement is included at the tensile side of members.

$$M_n = A_p f_{ps}\left(d_p - \frac{a}{2}\right) + A_s f_y\left(d_p - \frac{a}{2}\right) \qquad (3)$$

in equation (3),

$$a = \frac{A_p f_{ps} + A_s f_y}{0.85 f_{ck} b}$$

3. Specimens

3.1 Materials

3.1.1 Steel

Low relaxation, half inch diameter 270 Gr. seven wire strands are used. The strands are produced from K-company in Korea. Deformed mild bar is also used with the yield strength of 4,000kgf/cm^2. The stress-strain relationship of K-steel 0.5″-dia seven wire strands are shown in Fig. 5.

3.1.2 Concrete

The mix design of concrete is listed in Tables 1 and 2. The design strength of the concrete is 420kgf/cm^2

Table 1 Concrete mix design(1)

Design strength kgf/cm^2	Maximum size of aggregate (mm)	Slump (%)	Air content (%)	Fine aggregate ratio (%)	Water-cement ratio (%)	Unit fine aggregate kgf/cm^2
420	25	12	1.8	41	34	49

Table 2 Concrete mix design(2) (unit : kgf/m^3)

Unit water	Unit cement	Unit coarse aggregate	Unit fine aggregate	Unit add-mixture
165	485	1,011	708	4.9

3.2 Production of specimens

Two 8.4m long, full scale rectangular and two inverted tee beams are made from the span module 8.4 × 10m, live load 500kgf/m^2 and 1,200kgf/m^2(Fig. 4). The dimensions and details of reinforcement are shown in Figs. 6, 7, 8, and 9.

The bottom dimension of rectangular and inverted tee beams are the same as 60 cm. The section areas are also the same when compared to the flexural behavior of these two beams.

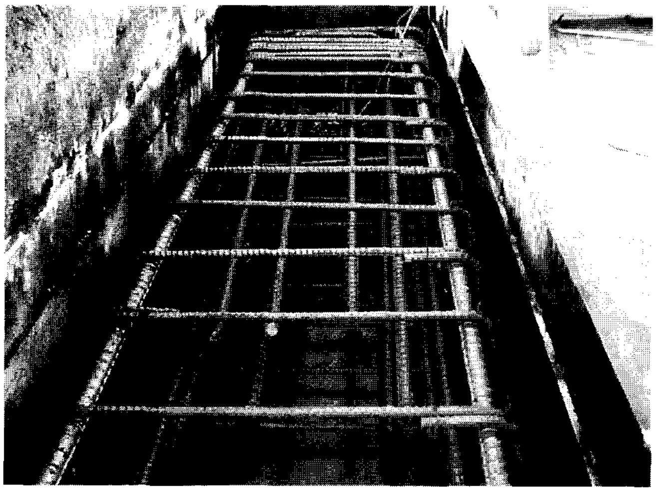

Fig. 4 Reinforcement for specimen RB500

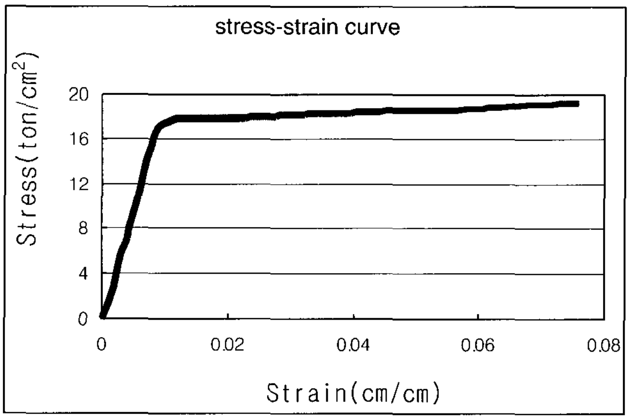

Fig. 5 Stress-strain curves for domestic K-steel 0.5”-dia seven wire strand

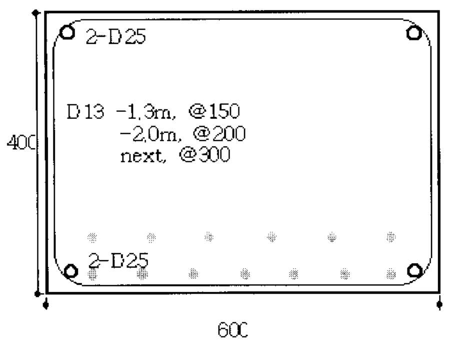

Fig. 6 Section of specimen RB500

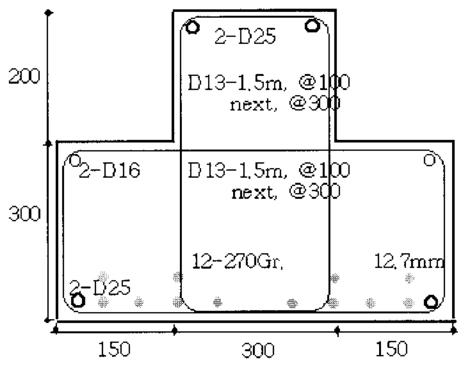

Fig. 7 Section of specimen IB500

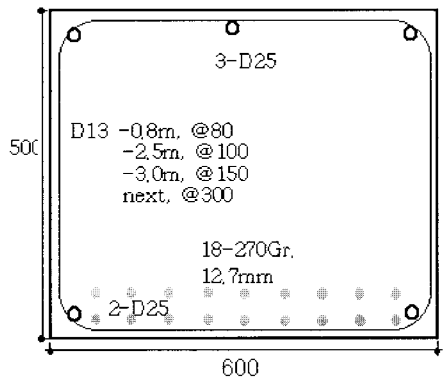

Fig. 8 Section of specimen RB1200

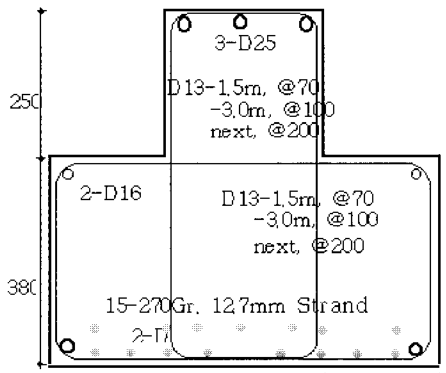

Fig. 9 Section of specimen IB1200

4. Experiments

4.1 Test plan

Flexural tests were performed on two full scale rectangular and two inverted tee beams with span 8m as shown in Fig 10. Two flexural tests are performed on the single specimen by spans with 6m and 4m. The installation of specimen is shown in Fig. 10. Two point loads with a space of 1.2m are applied by considering the leg space of 1.2m in a double tee.

4.2 Design of specimens

The symbol of specimens are shown above. The steel stress, f_{ps} at failure and nominal member strength are shown in Table 3 according to the ultimate strength design and the strain compatibility analysis. The results of strain compatibility analysis give lower steel stress, f_{ps} but higher nominal strength than those of the ultimate strength design.

The deficiency for required tensile stress at the top of the member from the camber from the initial prestressing

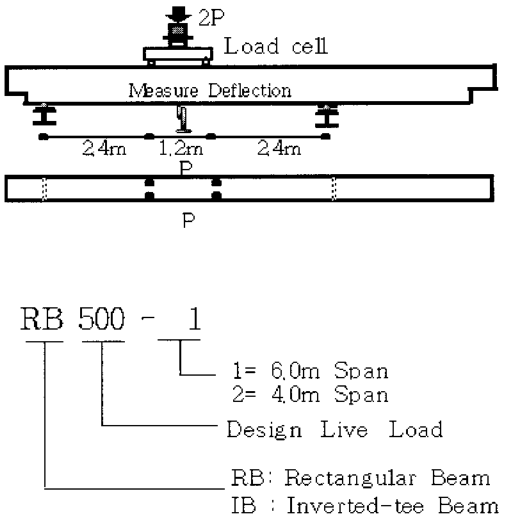

Fig. 10 Specimen setting (6m-span)

Table 3 Comparison of ultimate stress of strand and nominal flexural strength

Name of specimens	Ultimate strength design		Strain compatibility equation	
	f_{ps} (kgf/cm^2)	Mn (tonf · m)	f_{ps} (kgf/cm^2)	Mn (tonf · m)
RB500	17,079.4	70.81	16,974.3	73.72
RB1200	17,063.0	121.10	16,993.7	126.84
IB500	16,547.2	74.44	16,242.6	78.97
IB1200	16,757.6	122.38	16,423.2	131.92

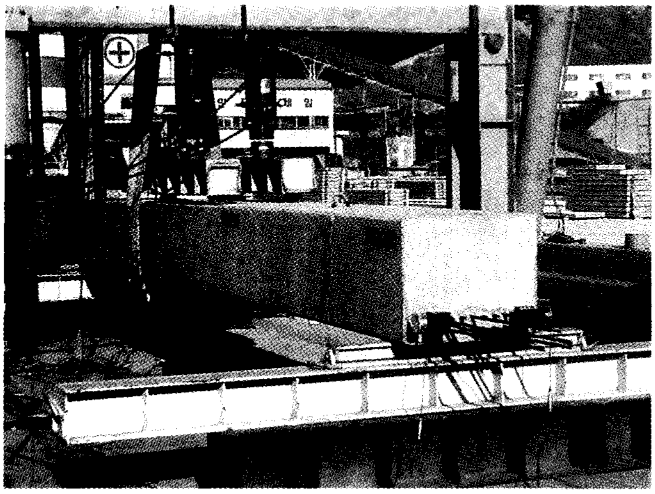

Fig. 11 View of experiment 1 (RB1200)

step are properly added with a mild steel in detail. The punching shear failures, the shear friction failures between web and flange, and the splitting failure of flange and web in inverted tee beams are prevented by adopting ample flange area and hanger reinforcement.

4.3 Anaysis on test results

The results of flexural test are listed in Table 4. The behaviors of two type specimens in tests are as follows.

4.3.1 Rectangular beams

Similar failure behaviors are shown for test span of 6.0m and 4.0m. Initial flexural cracks are shown 95% of the service loading for RB500-1 and 96% of the service loading for RB1200-1. Initial flexural cracking is always

Table 4 Flexural strength of specimens

Name of specimens	f_{ck} (kgf/cm²)	Full service loading condition					Failing loading condition				
		① Design moment (tf · m)	② Cracking moment Mcr (tf · m)	③ Test gracking strength (tf · m)	③ Test/ ② Theory	③ Test/ ① Design	④ Theory (tf · m)	⑤ Theory (tf · m)	⑥ Test (tf · m)	⑥ Test/ ④ Theory	⑥ Test/ ⑤ Theory
RB500-1	442.7	41	32.29	38.4	1.19	0.95	70.81	73.72	76.49	1.08	1.04
RB500-2	463.8	41	32.29	41.33	1.28	1.01	70.81	73.72	78.87	1.11	1.07
RB1200-1	512.6	67.7	58.9	64.8	1.1	0.96	121.1	126.8	117.7	0.97	0.93
RB1200-2	486.7	67.7	58.9	79.75	1.35	1.18	121.1	126.8	121.9	1.00	0.96
Average	476.5	54.35	45.59	56.07	1.23	1.03	95.95	100.26	98.74	1.04	1.0
IB500-1	430.2	41	37.77	36	0.95	0.88	74.44	78.97	75.74	1.02	0.96
IB500-2	483.8	41	37.77	52.2	1.38	1.27	74.44	78.97	72.98	0.98	0.93
IB1200-1	475.6	67.7	61.54	67.2	1.09	0.99	122.4	131.9	127.6	1.04	0.97
IB1200-2	468.3	67.7	61.54	97.55	1.58	1.44	122.4	131.9	135.6	1.11	1.03
Average	464.48	54.35	49.65	63.23	1.25	1.15	98.42	105.43	102.98	1.04	0.97

* ④ Calculated norminal strength by ultimate strength design methods
⑤ Calculated norminal strength by strain compatibility methods

Table 5 Deflection of rectangular beams

(unit : mm)

Name of specimens	Theoretical camber at center of 8.4m beam	Measured camber at center of 8.4m beam	Measured camber for real beam(6-4m) ①	Measured deflection with full service loading ②	Measured deflection ②-①	Maximum allowable deflection, L/360	Measured deflection at failure
RB500-1	19.08	24	19.6	15.2	-4.4	16.7	55.4
RB500-2	19.08	22	9.7	11.63	1.93	11.1	36.9
RB1200-1	20.16	18	14.7	16.15	1.45	16.7	43.4
RB1200-2	20.16	23	10.2	14.39	4.19	11.1	34.4

Table 6 Deflection of inverted tee beams

(unit : mm)

Name of specimens	Theoretical camber at center of 8.4m beam	Measured camber at center of 8.4m beam	Measured camber for real beam(6-4m) ①	Measured deflection with full service loading ②	Measured deflection ②-①	Maximum allowable deflection, L/360	Measured deflection at failure
IB500-1	14.2	21	17.2	13.7	-3.5	16.7	41.6
IB500-2	14.2	19	8.4	9.9	1.5	11.1	16.8
IB1200-1	12.29	17	13.9	11.84	-2.06	16.7	38.5
IB1200-2	12.29	15	6.7	12.56	5.86	11.1	22.9

developed from the center of below loading point. New additional flexural crackings are developed continuously from the loading side to the ends as loads increased. They are normal to a bottom line up to the neutral axis, and then inclined to loading points over the neutral axis. However, the width of crackings do not develop more than 1mm up to the failure. These crack were developed widely all over the span when beam failed. The beams failed in tensile failure with a compression crushing at the top of the beams(Fig. 11).

The ratios of test result to calculated strength by strength design method are in the range of 0.97~1.11 and averaged to 1.04. The ratios of test to calculated by the strain compatibility analysis are 0.93-1.07 and averaged 1.00. Thus, the results of strain compatibility show a little bit closer value to the experimental test results.

The cambers of calculated and measured at the center of the span for rectangular section are listed in table 5. Measured camber is a little bit higher than calculated one which based on the elastic theory. The cambers at the center of span for span with 6m and 4m are calculated by a theory of interpolation from the camber at the center of 8m beam.

The actual deflection of beams are obtained by deducting camber with a loading of self weight of specimen. All of the specimens are within the allowable limit of strength design provisions. particularly, RB500-1 specimen show very small initial flexural cracking when this deflection is negative.

4.3.2 Inverted tee beams

All of the specimens except IB500-1 are within the limit of intial cracking provisions for full service loading in the Strength design method. The specimen, IB500-1 shows the first flexural cracking at the load level of 88% of the full service loading. All of the specimens show the first flexural crackings from the bottom of the flange below the loading points, and create new flexural crackings by a space of 20cm as loads increased.

The moment-deflection curve in Fig 14 shows linear at first and deviate to non-linear when the number and width of crackings increase. No particular differences have been detected in the specimens of span with 6.0m and span with 4.0m. The flexural cracking developed from the side of bottom flange are folded with the cracking developed from the top of the web. All of the specimens are failed in tensile flexural failure and accompanied with concrete compression crushing crackings at the ultimate stage as shown in Fig. 13.

The ratios of test result to calculated strength by the strength design method are in the range of 0.98~1.11 and averaged to 1.04. The ratios of test result to calculated strength by the strain compatibility analysis are in the range of 0.93~1.03 and averaged to 0.97. Thus, the result of strain compatibility shows a little closer value to the experimental test results in inverted tee beams.

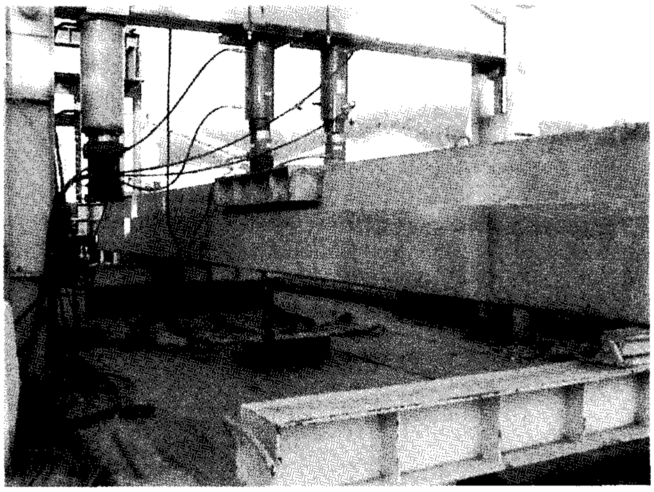

Fig. 12 View of experiment 2 (IB1200)

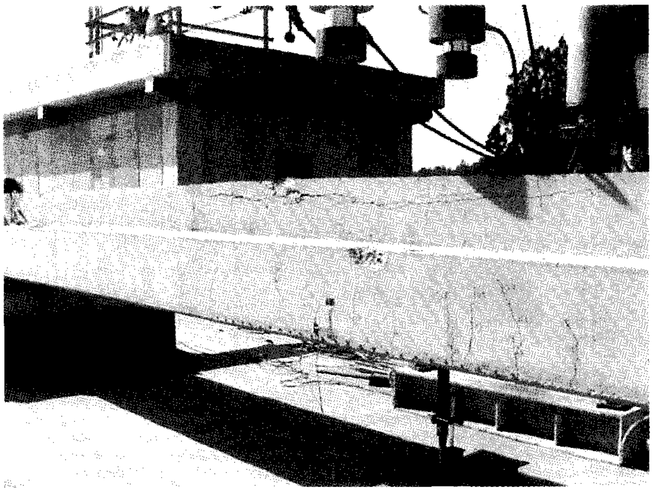

Fig. 13 Example of flexural failure with local compression crackings (IB500-1)

The calculated camber and measured camber at the center of the span for inverted tee section are listed in table 6. Measured cambers are a little higher than calculated value of the elastic theory as in the case of rectangular section. The camber at the point where measuring point is estimated from the camber at the center of 8m span to that of 6m and 4m span by interpolation.

All of the specimens show that the actual deflection of beams by deducting camber with a loading of self weight of specimen are within the allowable limit of strength design provisions. Particularly, IB500-1 and IB1200-1 specimen show very small initial flexural cracking when this deflection is negative.

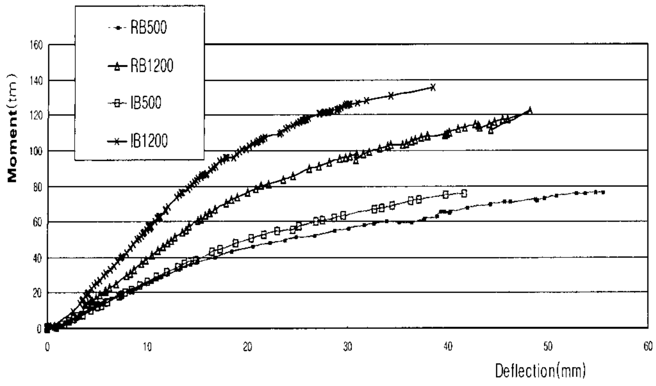

Fig. 14 Moment-deflection curve

4.4 Moment-deflection relationship

The moment-deflection curves are shown in Fig. 14 for specimens with a span 6.0m. The ultimate failure loadings are similar in rectangular and inverted tee beams according to their design live load either 500kgf/m^2 or 1200kgf/m^2.

However, the rectangular beams experienced more deflection than that of inverted tee with the same loading condition and failed with more deflection. The modulus of elasticity of inverted tee is larger than that of rectangular section, thus it experience less deflection than rectangular, on the other hand, inverted tee has less energy consumption ability than that of rectangular as shown in Fig. 14.

The rectangular shows 2mm more deflection under the full service loading and 12.56mm more in ultimate as shown in Table 5 and 6 and in Fig. 14.

5. Conclusions

Intensive flexural tests are performed on the two typical precast beam sections for buildings - inverted tee and rectangular. The inverted-tee beams were designed for a parking live load - 500kgf/m^2 and a market - 1,200kgf/m^2 from a currently used typical shape of a domestic building site in Korea. The bottom dimension and area of rectangular beams were the same as those of inverted tee beams, when comparing the flexural behaviors of two beams. These two beams were also reinforced with a similar strength. Following conclusions are obtained from the study above;

1) The rectangular beam is simpler in production, transportation, and erection, and more economic than the inverted tee beam in a construction test for these two beams with a same dimension and a similar strength.
2) All of the beams considered in the tests are generally failed in close values to those of the strength requirements in ACI Provisions. The ratios of test result to calculated value is averaged to 1.04. One rectangular and one inverted tee beams failed in less than estimation of the strength design method within the range of 2 to 3%.
3) The results of the strain compatibility method is slightly more accurate than those of strength design method.
4) The maximum deflections of all of the beams under the full service loads are less than those of the allowable limit in ACI Code Provisions. The rectangular beams experienced more deflection then inverted tee with a same loading condition and failed with more deflection.
5) The rectangular and inverted tee beams show good performances under the service and ultimate load conditions. However, one inverted tee beams with 6m span develop initial flexural crackings under 88% of the full service load even though they designed to satisfy the ACI tensile stress limit provisions.

ACKNOWLEDGEMENT

This work is supported by the research program of Dongguk University.

REFERENCES

1. Mirza, S. A. and Furlong, R. W., "Design of Reinforced and Prestressed Concrete Inverted T Beams for Bridge Structures," *PCI Journal,* Vol. 30, No. 4, July-Aug. 1985, pp. 112-136.
2.Mirza, S. A. and Furlong, R. W., "Strength Criteria for Concrete Inverted T-Girders," *ASCE Journal,* Vol. 109, No. 8, Aug. 1983, pp. 1836-1853.
3. Yu, S. Y., *"Development of Prestressed Precast Double-tee Slab System for Domestic Large Span Building,"* Report for Kistep, 2000. 11. 2.
4. ACI Committee 318, "Building Code Requirements for Reinforced Concrete, ACI 318-95," *American Concrete Institute,* Detroit, Michigan, 1995.
5. *PCI Design Hand Book - Precast and Prestressed Concrete,* fifth edition, Prestressed Concrete Institute, Chicago, Illinois, 1999.
6. Yu, S. Y. and Yoo J. C., "Evaluation on Flexural Behavior of Double-tee slabs with the Least Depth from Optimization Process,"*Journal of the Korea Concrete Institute,* Vol. 11, No. 3, June 1999.