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Size Effect on Axial Compressive Strength of Notched Concrete Specimens

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Abstract

In this study, size effect tests were conducted on axial compressive strength of concrete members. An experiment of Mode I failure, which is one of two representative compressive failure modes, was carried out by using dimensionally proportional cylindrical specimens (CS). An adequate notch length was taken from the experimental results obtained from the compressive strength experiment of various initial notch lengths. Utilizing the notch length, specimen sizes were then varied. In addition, new parameters for the modified size effect law (MSEL) were suggested using Levenberg-Marquardt's least square method (LSM).

The test results show that size effect was apparent for axial compressive strength of cracked specimens. Namely, the effect of initial notch length on axial compressive strength size effect was apparent.

Keywords: size effect, axial compressive strength, initial notch length, cylindrical specimen

1. Introduction

All materials have specific inherent material properties. For each material, the properties are considered unique when they are independent of specimen size and shape. For design purposes, the concrete compressive strength of standard cylinder (ϕ15x30 cm) is accepted as the most basic and important material property. However, the common notion that concrete compressive strength is a unique material property is erroneous since the compressive strength of concrete changes with size of specimens due to its fracture characteristics.

Size effect an compressive strength of concrete is not as distinct as in tensile, flexural, and shear strengths. However, researchers[1-4)] accept the conclusion that the ratio of the compressive failure stress to the compressive strength decreases as the specimen size increases. Recently, the studies[5-7)] on compressive loading based size effect became a focus of interest among researchers. The problem of selecting the correct compressive strength for structural concrete design is critical since the structural member size will be changed significantly based on the selection. Due to the size effect, strength of actual structural members will be different than those obtained from the laboratory-size specimens. There will also exist a considerable difference in strengths as can be seen in the laws,[1, 8-10)] which are related to the size effect. Therefore, when structures are designed it is more desirable to use the compressive strength of concrete appropriate to the size of structures obtained based on the size effect law (SEL) than to use the strength of standardized cylindrical specimens such as ϕ15x30 cm or ϕ10x20 cm.

Many pores and microcracks are present inside the concrete from the casting stage. Accordingly, it is necessary to apply fracture mechanics to the research when evaluating material characteristics of concrete. This is because the tensile and compressive failure occurs due to the pores and the cracks presented in concrete.

Various researches, which predicts the behavior and characteristics of concrete by applying fracture-based failure theory, have been performed, and the most representative of them is size effect. Size effect means that the strength of concrete changes with specimen size. This is a material characteristic which is present because concrete is a non-homogeneous and brittle material.

Although the failure mechanism and the size effect of tensile failure have been studied extensively, the behavior

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of compressive failure has not been sufficiently studied in comparison to the tensile failure mechanism. Concrete is a material normally used to withstand compressive force, thus research into compressive failure should be performed. For this reason, many studies on compressive failure of concrete are currently a focus of interest among researchers.

The purpose of this study is to investigate experimentally the size dependency on characteristics of compressive failure of concrete. Especially, this study only deals with axial compressive failure of the two representative compressive failure modes (i.e., axial compression and flexural compression). Also, an experimental method using cylindrical specimen with an initial notch a_o was suggested to establish a more systematic research method, which can simulate the axial compressive behavior of concrete.

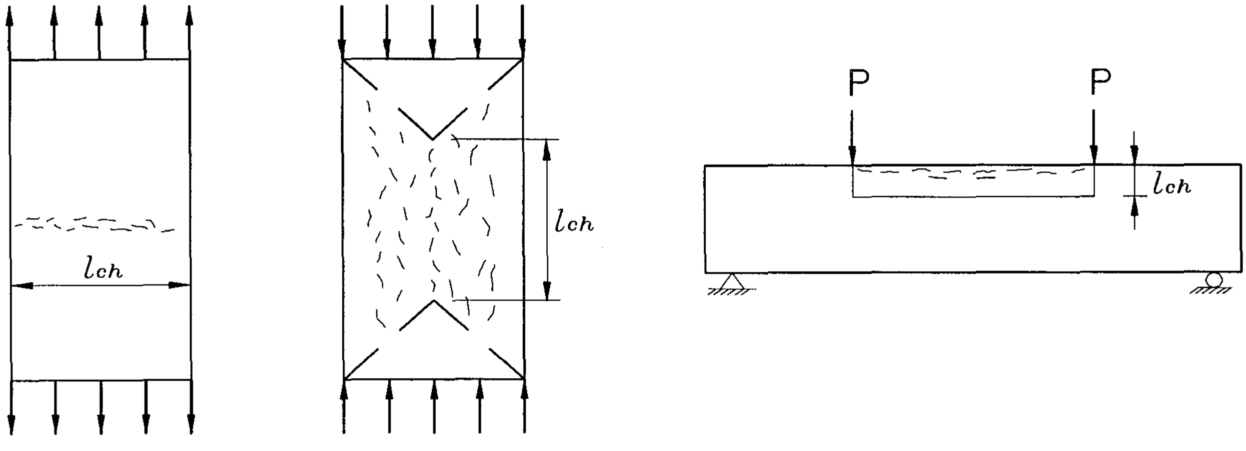
2. Compressive failure mechanism of concrete and modified size effect law (MSEL)

There are various reasons for the presence of size effects. However, the most significant ones are limited ductility and the existence of microcracks. Even though there are no observable cracks when a concrete member is subjected to compressive force, stress is concentrated at the crack tip and microcracks occur in the longitudinal direction of member.$^{5, 11)}$ At a certain time, one or several main cracks are formed by the coalescence of these microcracks. The existing microcracks adjacent to the main crack close, the main crack grows rapidly due to the brittle characteristics of concrete, and finally propagates to failure. Occurrence and development of microcracks and the typical pattern of compressive failure are described in detail in reference 7. This phenomenon is more apparent as the specimen size increases. That is, the size effect is more apparent.

The size effect varies with the size of the microcrack zone due to the applied loading distribution. Before formation of a main crack, which leads to the failure of concrete if the region of microcracks is small, size effect increases. That is because the region of microcracks in compression is larger than in tension, size therefore effect decreases.

Compressive strength of concrete divides largely into axial compressive strength and flexural compressive strength. Presently, researchers$^{12\text{-}17)}$ have extensively studied size effect on compressive strength. Because the crack distribution of axial compressive specimens is wider than flexural compressive specimens and the strain gradients of the two cases are different, the size effect on axial compressive specimens is less. For axial and flexural compressive specimens, the shapes of strain gradients are rectangular and triangular distributions,$^{15\text{-}17)}$ respectively.

Thus, tensile and compressive failures show different characteristics. The typical patterns of failure on cylindrical specimens and beams are shown in Fig. 1. In case of (a) tensile failure, cracks are more concentrated than (b) axial compressive failure or (c) flexural compressive failure. In addition, the direction of the characteristic dimension l_{ch} of tensile failure is different from compressive failure. However, the compressive failure is also caused by the progression of splitting cracks due to localized tension effects. Thus, the tensile fracture-based concept can also be applied to compressive failure. Compressive failure depends on the shape of structure (slenderness, boundary conditions), size, energy release characteristics, and second-order geometric effects (internal buckling as described by Biot$^{18)}$). The occurrence and development of a band of microcracks can be characterized by two failure modes as follows:

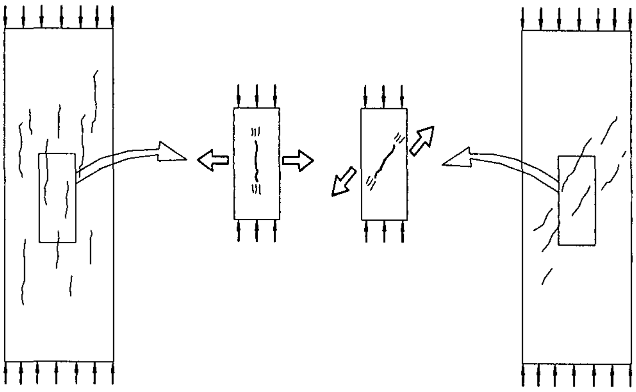


(a) tension (b) axial compression (c) flexural compression

Fig. 1 Tension failure and compression failure of concrete

Mode I: Axial propagation of a band of axial splitting microcracks that finally coalesce into one axial splitting crack (Fig. 2(a)).

Mode II: A transverse propagation of a band of axial splitting microcracks terminating with an inclined failure surface traditionally regarded as a shear failure (Fig. 2(b)). This case can happen when the aggregate is not distributed symmetrically within the specimen or the load is not applied exactly on the central line of the specimen.



(a) Failure by Mode I (b) Failure by Mode II

Fig. 2 Mode types of compression failure

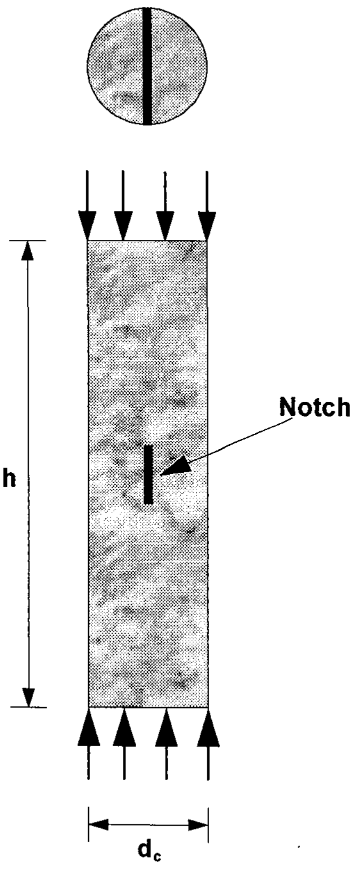


Fig. 3 Size and shape of specimens

Based on the experimental results of specimens subjected to axial compressive force, most of the specimens fail by Mode I when subjected to tensile stress normal to crack surface. Accordingly, in this study, the characteristics of Mode I failure were investigated.

After deriving the SEL by Bazant,[1] Kim et al.[9, 10] proposed the modified size effect law (MSEL) (Eq. (1)) by adding to the SEL the size independent strength $\sigma_0 (= \alpha f_t')$, which can predict the strength of concrete members with or without initial cracks and with similar or dissimilar cracks. This concept was also proposed by Bazant et al.[7, 19, 20] with a different approach.

$$\sigma_N(d) = \frac{B f_c'}{\sqrt{1 + \dfrac{d}{\lambda_o d_a}}} + \alpha f_c' \qquad (1)$$

where σ_N is nominal strength, f_c' is compressive strength of standard cylinder, d is characteristic dimension, d_a is maximum aggregate size, and B, λ_0, and α are empirical constants. As an application of this law, research has been performed on cylindrical specimens subjected to uniaxial compressive force[15] and C-shaped specimens subjected to flexural compressive force.[16, 17, 25] In Eq. (1), the width of crack band l_o is empirically known to be related to the maximum aggregate, e.g., $l_0 = \lambda_0 d_a$ in which λ_0 is an approximate constant with values between of 2.0 and 3.0.[15-17] In the regression analysis, this constant was chosen as 2.0 where l_o=2.0 × d_a=2.6 cm.

3. Test specimens and experimental program

In this paper, to simulate the compressive failure by axial direction cracks, size effect in compressive failure was evaluated using CS as shown in Fig. 3.

3.1 Mixture proportioning

High-strength concrete (HSC) shows more brittle behavior than low-strength concrete (LSC) or medium-strength concrete (MSC). This means that fracture process zone (FPZ) size of HSC is smaller and size effect is more apparent. Accordingly, the size of specimen can be reduced with a decrease of FPZ size. In addition, the capacity of the loading device and amount of concrete can be reduced and the handling of specimens would be easier.

The average concrete compressive strength was 50 MPa. The concrete mixture proportions are listed in Table 1. Type I Portland cement was employed in all mixtures. Crushed gravel was employed as the coarse aggregate and the maximum aggregate size d_a was 13 mm. In addition, a high-range water-reducing admixture and vibrator were used to improve workability and consolidation of concrete. All specimens were cast vertically on a level surface.

All specimens were removed from the molds after 24 hours. In addition, specimens were dry-cured under wet burlap/towel until testing.

Table 1 Concrete mixture proportions

w/c, %	s/a, %	Unit weight, kg/m^3				S.P.**,%
		W	C	S	G*	
35	40	175	500	671	1027	2

* maximum aggregate size of 13 mm
** super-plasticizer (ratio of cement weight)

Table 2 Dimension of specimens

Specimen No.	Specimen size, $d_c \times h$ (cm)	Initial notch length (cm)	Number of specimens
S-N0	5×37.5	0	4
S-N6		6	5
S-N12		12	5
M-N0	10×75	0	3
M-N12		12	3
M-N24		24	3
L-N0	20×150	0	2
L-N12		12	3
L-N24		24	3
L-N48		48	3

3.2 Details of test specimens

The dimensions, shape, and notch locations of CS used in the experiments are shown in Fig. 3. In this study, experiments were performed on strength variance with an initial notch length a_o and size effect with specimen size. Notch length to maximum aggregate size ratios of 0.5:1, 1:1, 2:1, 3:1, 4:1, and 5:1 in ϕ 10x75 cm cylindrical specimens are used to study the effect of initial notch length on strength variation. To perform size effect tests, an adequate initial notch length was selected from the strength variance test. Specimen size and initial notch length are listed in Table 2. In the size effect test, geometrically similar specimens, and at least three specimens of each size were tested. Namely, *a* size ratio of 1:2:4 was used. In the nomenclature of CS, S, M, and L represent the size of the specimens with S being the smallest and increasing accordingly. N0, N6, N12, N24, and N48 are serial numbers of specimens with an initial notch length for each specimen size. In case of size S, 4 or 5 specimens, instead of 3 specimens, were selected because more data scattering was expected. For concrete beams loaded in compression, the crack pattern for compression and the selection method of the specimen for modeling of compressive failure are shown in Fig. 4.

3.3 Test procedure

The axial compressive load, shown in Fig. 3, was applied using a universal testing machine (UTM) with a capacity of

Table 3 Experimental results on initial notch length (unit: MPa)

a_o	$0.5d_a$	$1.0d_a$	$2.0d_a$	$3.0d_a$	$4.0d_a$	$5.0d_a$	f_c'
f_c	58.27 57.98	57.00 59.35	58.76 56.21	58.27	51.70	59.35	59.84
	53.66	55.03	55.43	56.02 58.08	54.25	49.84 54.25	59.45

Table 4 Ultimate load obtained from experiments

P_u (kN)										f_c'	f_{ct}
S (d_c= 5 cm)			M (d_c = 10 cm)			L (d_c = 20 cm)					
N0	N6	N12	N0	N12	N24	N0	N12	N24	N48	MPa	
111			383			1491				49.1	5.1
96			353			1432				47.6	4.8
103 94			353							47.6	5.2
94		96		373			1452 1383			51.9	5.5
	99 95	99		373			1364	1216		50.2	4.8
	94 97 110			373				1266 1275		49.7	4.8
		110			373				1118 1403	47.8	5.0
		96 92			343 353				981	52.9	4.8

2,500 kN using a displacement control method. Control velocity for CS was adjusted to fail within 10 minutes. This procedure continued until the specimen failed.

4. Analysis of test results

4.1 Test results

Table 3 and Table 4 tabulate the experimental data of ultimate strength f_c with an initial notch length and P_u, f_c' and f_{ct} with specimen size d_c, respectively. The experimental data of f_c' and f_{ct} were obtained from the ϕ 10×20 cm cylinders where concrete from the same batch was used to cast CS for size effect tests.

4.2 Strength variance with an initial notch length

Strength variance with an initial notch length is shown in Fig. 4. From this figure, it can be seen that when initial notch length is more than four times maximum aggregate size, the strength of concrete appears to decrease. In this case, it is important to know that when the loading is applied, crack growth continues at the crack tip and subsequently it propagates to failure when sufficient main cracks have occurred inside the concrete. When the notch length to maximum aggregate size ratio is smaller than 4.0, however, additional load is required for sufficient development of the crack. Subsequently, failure occurs due to the progression of crack.

The lateral displacement at the middle part of the initial notch obtained using LVDT is given in Fig. 5. Normal and parallel direction displacements to the initial notch are shown in Fig. 5(a) and (b), respectively.

This testing was carried out to determine the adequate notch length, which can result in failure by Mode I. When a notch length to maximum aggregate size ratio is 4.0, the normal direction displacement to the initial notch is greater than the parallel direction displacement. Based on the foregoing discussion, it is apparent that the Mode I failure can be predicted by using this specimen and experimental procedure. For this reason, in the following "Size effect with specimen size" section, notch length to maximum aggregate size ratios of 9.2 and 18.5 (i.e., 12.0 cm and 24.0 cm) in ϕ 10×75 cm cylindrical specimens were used.

Table 5 Results obtained from MSEL and LSM

Specimen series	B	$\lambda_a d_a$	α
N6:N12:N24	0.92	2.6	0.48
N12:N24:N48	0.90	2.6	0.44
N0:N0:N0	0.52	2.6	0.80
N12:N12:N12	0.72	2.6	0.63

4.3 Size effect with specimen size

Ultimate loads corresponding to size of specimens with an initial notch are given in Table 4. To differentiate size

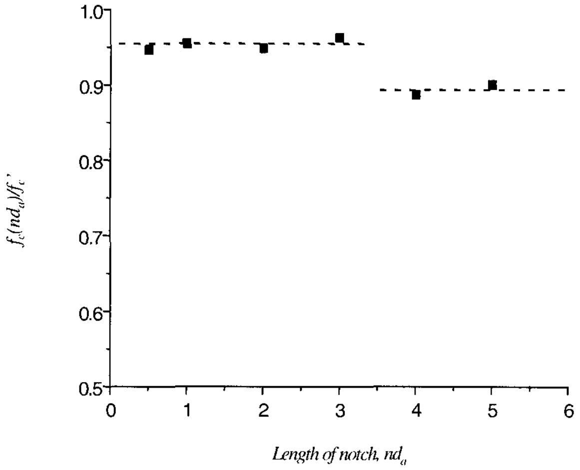
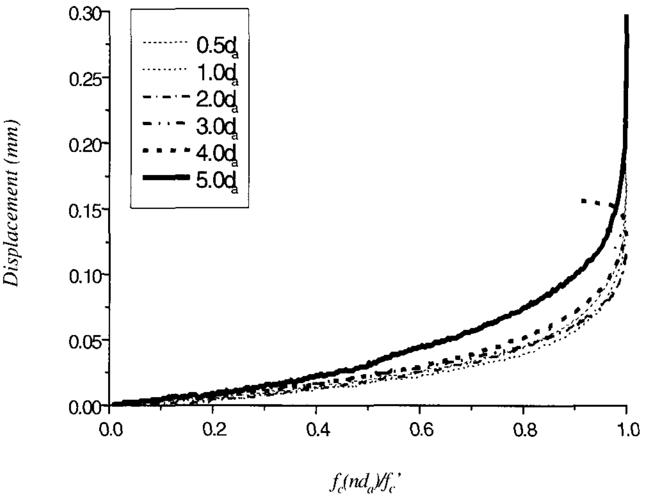
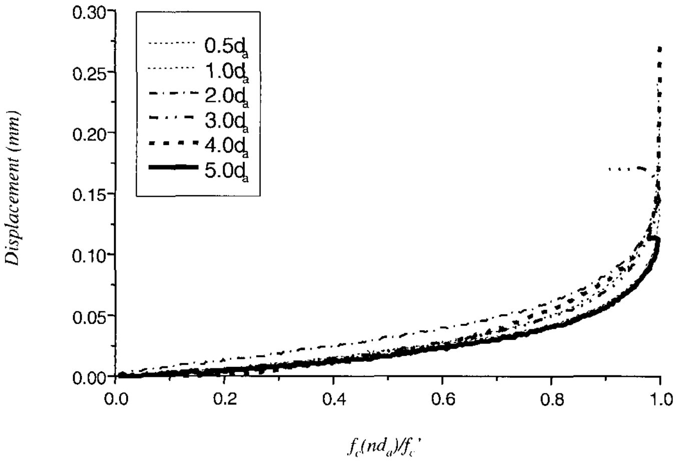


Fig. 4 Relationship between $f_c(nd_a)/f_c'$ and initial notch length



(a) Normal direction to initial notch



(b) Parallel direction to initial notch

Fig. 5 Lateral displacement length

effect between the cases with and without an initial notch, tests on specimens without initial notches were also conducted and experimental results are included in Table 4. To confirm whether or not a size effect is present, MSEL is used, and LSM regression analyses[21, 22] are carried out for maximum stress value obtained from tests. The results are given in Table 5. Fig. 6 shows $f_c(d)/f_c'$ as a function of the diameter *d*. For comparison, all results are shown together in Fig. 7. In this figure, the data points represent the mean value of experimental results. As shown in Fig. 6, the results indicate a strong size effect. In Fig. 7, it is found that size effect between specimens with an initial notch and without initial notches (N0:N0:N0) show a considerable difference. Specimen N12:N12:N12 with constant initial notch length regardless of specimen size shows that notch length to cylinder height ratio increases with decrease of cylinder diameter and vice versa. Accordingly, the compressive strength at failure is smaller as the specimen diameter decreases. In addition, strength reduction level decreases for increasing diameter. Specimen N12:N24:N48, with the largest notch length, would show more apparent reduction phenomenon than other specimens, which have smaller initial notch length than this specimen. Kim et al.,[17] Markeset et al.,[23] and Jansen and Shah,[24] however, showed experimentally that the strength reduction is independent of the specimen size when the specimen height/diameter or length/depth is greater than a constant value (i.e., 2.0~2.5

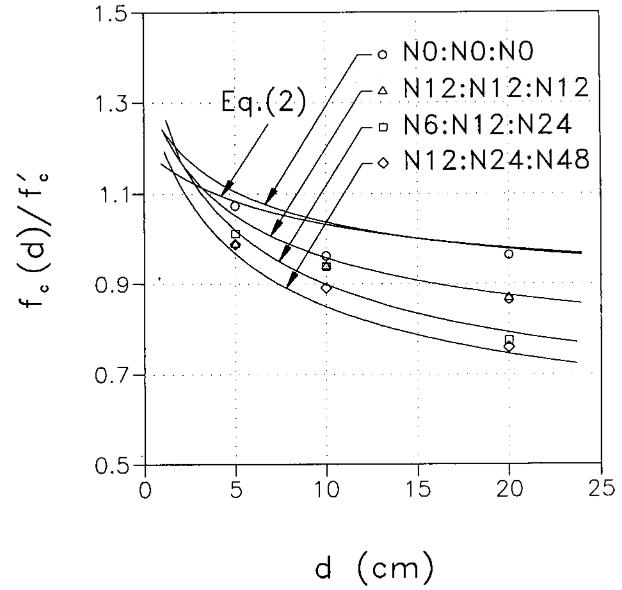


Fig. 7 Comparison of model equations obtained from MSEL and experimental results

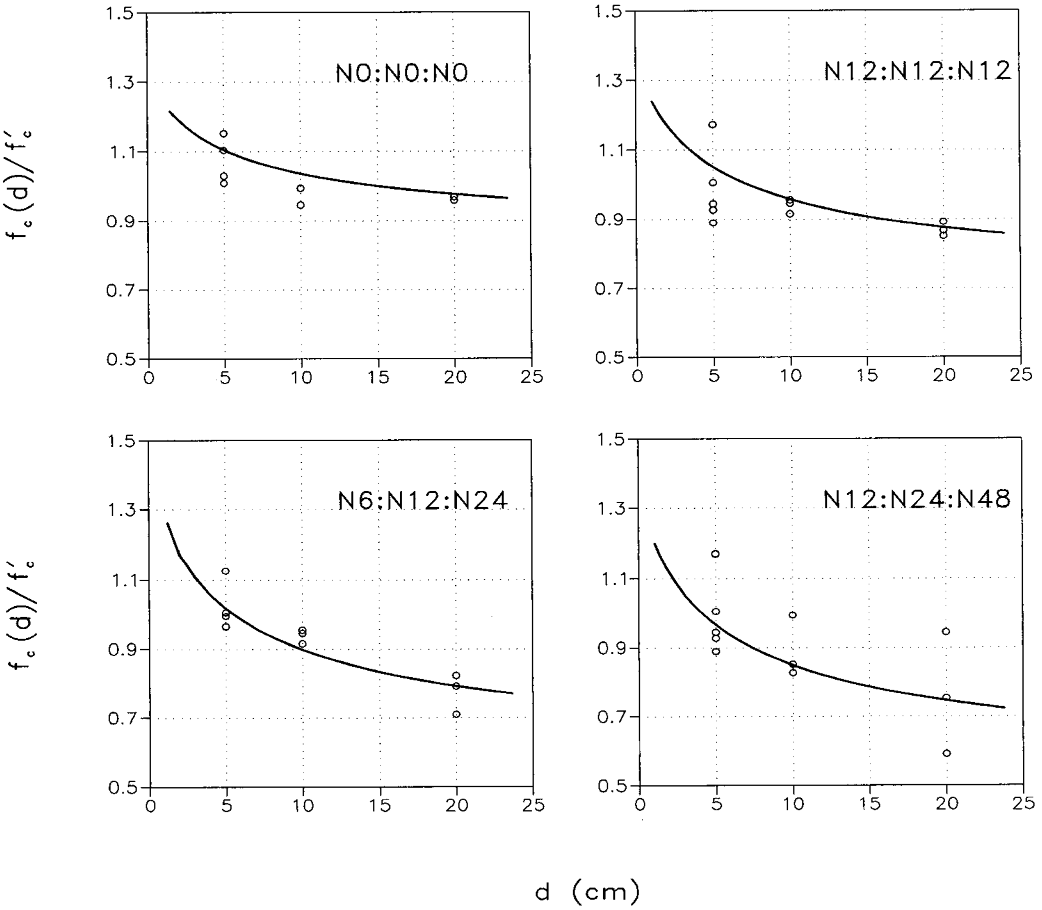


Fig. 6 Relationship between $f_c(d)/f_c'$ and specimen diameter d

and 3.0 for cylinder and C-shaped specimens, respectively). In this study, it was concluded that compressive strength decreases to a certain level with an increase of notch length. However, beyond that level it is nearly consistent with the literature. Namely, the comparison of $f_c(d)/f_c'$ for specimens N12:N24:N48 and N6:N12:N24 shows a similar difference.

When test results on specimen N0:N0:N0 without initial notches are compared with Eq. (2),[15] it can be seen that compressive strengths of specimens with a diameter greater than 10.0 cm show similar values as shown in Fig. 7. When the diameter is approximately 5.0 cm, however, the results show a difference since maximum aggregate size is different. Specifically, Eq. (2) was obtained from tests with maximum aggregate size of about 25 mm, however, in this study the aggregate size of 13 mm was used.

$$f_c(h,d_c) = \frac{0.4f_c'}{\sqrt{1+\frac{(h-d_c)}{5}}} + 0.8f_c' \qquad (2)$$

where compressive strength of general cylinder $f_c(h,d_c)$ and f_c' are in MPa, and the height of cylinder h and diameter of cylinder d_c are in cm.

5. Application of size effect on axial compressive strength

A number of research activities on size effect of axial compressive strength of concrete have been pursued. In this study, experimental evaluations using CS having an initial notch were performed. The axial compressive strength was obtained from standard concrete cylinders without notches. However, size effect was apparent not only in standard cylindrical specimens but also specimens used in this study.

The axial compressive strength of concrete is the most important material property used in designing structural members based on strength criteria. Thus, the size effect on compressive strength could introduce a major problem. In future studies, a criteria for adequate specimen size based on the size effect law is required. It could be of considerable help to designers and contractors to suggest adequate specimen sizes corresponding to structure and member size to obtain compressive strength of concrete.

Cylinders are used in the United States, Korea, Canada, France, Australia, etc. whereas cubes are the standard shapes in the United Kingdom, Germany, and many other European countries. There are several countries where tests are made on both cylinders and cubes. In further experiments, it may be necessary to suggest a more general model equation to be more commonly applicable to both cylindrical specimens and cubes.

When LSC is used, the FPZ takes place in a larger zone. In this case, size effect decreases since the failure occurs under the imperfect growth of FPZ. In standard concrete cylinder specimens generally used for obtaining compressive strength, the reason size effect is occasionally not found is because the specimen size is very small and the failure occurs when the FPZ does not grow sufficiently. To obtain more accurate results, experiments should be performed after adjustment of specimen size and identification of a minimum FPZ.

6. Conclusion

Research activities on size effect of axial compressive strength of concrete carried out previously were mainly on cylindrical specimens without notches. However, to more systematically study size effect, an experimental evaluation was performed using CS with a notch. The following conclusions were drawn based on the findings of this study:

1) Size effect for the axial compressive strength in CS with an initial notch is distinct. Namely, it can be concluded that size effect is present for tests depending upon the specimen size and notch length.
2) For CS, if the notch length to maximum aggregate size ratio is greater than 4.0, stable failure takes place and the failure by Mode I can be derived.
3) The results suggest that the current strength criteria based design practice should be revised. In designing structures it is more desirable to use the compressive strength of concrete obtained not from standardized specimens such as ϕ15×30 cm or ϕ10×20 cm cylinders but from adequately sized specimens based on the size effect law.

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