◎ 논 문

Evaluation of the K-Epsilon-VV-F Turbulence Model for Natural Convection in a Rectangular Cavity

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직사각형 공동 내부 자연연대류 문제에 대한 k-epsilon-vv-f 난류모델의 평가

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The primary objective of the present study is evaluation of the $k-\varepsilon-\overline{vv}-f$ turbulence model for prediction of natural convection in a rectangular cavity. As a comparative study, the two-layer $k-\varepsilon$ model is also considered. Both models, with and without algebraic heat flux model, are applied to the analysis of natural convection in a rectangular cavity. The performances of turbulence models are investigated through comparison with available experimental data. The predicted results of vertical velocity component, turbulent heat fluxes, turbulent shear stress, local Nusselt number and wall shear stress are compared with experimental data. It is shown that, among the turbulence models considered in the present study, the $k-\varepsilon-\overline{vv}-f$ model with an algebraic heat flux model predicts best the vertical mean velocity and velocity fluctuation, and the inclusion of algebraic heat flux model slightly improves the accuracy of results.

Key Words : $k-\varepsilon-\overline{vv}-f$ Turbulence Model, Two-Layer Model, Natural Convection.

1. 서 론

The development of a better turbulence model has been a very important work for computational fluid dynamists during past three decades. There exist several turbulence models in the literatures. It is generally accepted that the second moment model performs better than the other simpler models, however, the implementation of the second moment model such as the Gibson and Launder model [1] in the commercial CFD code is not easy due to the existence of geometry dependent wall reflection terms and most commercial codes employ simpler second

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moment models [2]. It is quite questionable that such simpler second moment turbulence models, which do not consider the near wall anisotropy, work better than the existing other models for prediction of natural convection problems. When one considers the complexity of the second moment models and the difficulty of their implementation, the simpler models can be sought. Kenjeres [3] has shown that the inclusion of algebraic heat flux model to Launder and Sharma model [4] results in quite accurate results for the prediction of various natural convection flows. Durbin [5,6] developed a $k-\varepsilon-\overline{vv}-f$ model and showed that this model gives more accurate results for forced convection flows than the standard $k-\varepsilon$ turbulence models [7-9]. The primary objective of the present study is the evaluation of the Durbin's $k-\varepsilon-\overline{vv}-f$ model for the natural convection problem. It is also investigated that how the inclusion of the algebraic heat flux model [3] to the Durbin's $k-\varepsilon-\overline{vv}-f$ model improves the accuracy of the solution. As a comparative study, the two-layer $k-\varepsilon$ model developed by Chen and Patel [10] is also considered in the present study.

The turbulence models considered in the present study are implemented in the computer code specially designed for evaluation of turbulence models. The computer code employs the nonstaggered grid arrangement and the SIMPLE [11] algorithm for pressure-velocity coupling. The higher order bounded HLPA [12] scheme is used for treating the convection terms. The computer code is applied to the prediction of natural convection in a 5:1 rectangular cavity, experimentally studied by King [13]. The predicted results are compared with experimental data

2. MATHEMATICAL FORMULATION

The Reynolds averaged governing equations for mass conservation, momentum conservation, energy conservation and the transport equations for turbulent quantities in the two-layer model and $\overline{vv}-f$ model can be written as follows;

$$\frac{\partial}{\partial x_j}(\rho U_j)=0 \tag{1}$$

$$\frac{\partial}{\partial t}(\rho U_i)+\frac{\partial}{\partial x_j}(\rho U_j U_i)=-\frac{\partial p}{\partial x_i}+\frac{\partial}{\partial x_j}\left((\mu+\mu_T)\frac{\partial U_i}{\partial x_j}\right)-\rho g_i\beta(\Theta-\Theta_o) \tag{2}$$

$$\frac{\partial}{\partial t}(\rho\Theta)+\frac{\partial}{\partial x_j}(\rho U_j\Theta)=\frac{\partial}{\partial x_j}\left(\left(\frac{\mu}{\mathrm{Pr}}+\frac{\mu_T}{\mathrm{Pr}_\theta}\right)\frac{\partial\Theta}{\partial x_j}\right) \tag{3}$$

$$\frac{\partial}{\partial t}(\rho k)+\frac{\partial}{\partial x_j}(\rho U_j k)=\frac{\partial}{\partial x_j}\left(\left(\mu+\frac{\mu_T}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right)+\rho(P+G)-\rho\varepsilon \tag{4}$$

$$\frac{\partial}{\partial t}(\rho\varepsilon)+\frac{\partial}{\partial x_j}(\rho U_j\varepsilon)=\frac{\partial}{\partial x_j}\left(\left(\mu+\frac{\mu_T}{\sigma_\varepsilon}\right)\frac{\partial\varepsilon}{\partial x_j}\right)+\frac{\rho C_{\varepsilon 1}(P+G)-\rho C_{\varepsilon 2}\varepsilon}{T} \tag{5}$$

$$\frac{\partial}{\partial t}(\rho\overline{\theta^2})+\frac{\partial}{\partial x_j}(\rho U_j\overline{\theta^2})=\frac{\partial}{\partial x_j}\left(\left(\frac{\mu}{\mathrm{Pr}}+\frac{\mu_T}{\sigma_\theta}\right)\frac{\partial\overline{\theta^2}}{\partial x_j}\right)+\rho\left(2P_\theta-R\frac{\varepsilon}{k}\overline{\theta^2}\right) \tag{6}$$

$$\frac{\partial}{\partial t}(\rho\overline{vv})+\frac{\partial}{\partial x_j}(\rho U_j\overline{vv})=\frac{\partial}{\partial x_j}\left(\left(\mu+\frac{\mu_T}{\sigma_k}\right)\frac{\partial\overline{vv}}{\partial x_j}\right)$$

$$+\rho\left(kf-\frac{\varepsilon}{k}\overline{vv}\right) \tag{7}$$

$$L^2\frac{\partial^2 f}{\partial x_j \partial x_j}-f=\left(\left(C_1-1\right)\frac{\left(\overline{vv}/k-2/3\right)}{T}-C_2\frac{\left(P+G\right)}{k}\right) \tag{8}$$

where

$$P=\frac{\mu_T}{\rho}\left(\frac{\partial U_j}{\partial x_i}+\frac{\partial U_i}{\partial x_j}\right)\frac{\partial U_j}{\partial x_i} \tag{9}$$

$$G=-\beta g_i\overline{\theta u_i} \tag{10}$$

$$P_\theta=-\overline{\theta u_i}\frac{\partial \Theta}{\partial x_i} \tag{11}$$

In the two-layer model, Eqs.(1)-(5) are solved with the time scale T in Eq.(5) given by $T=\frac{k}{\varepsilon}$. The near wall region is resolved by the one-equation model while the outer fully turbulent region is solved by the standard $k-\varepsilon$ model.

- Near wall region :

$$\varepsilon=\frac{k^{3/2}}{l_\varepsilon} \tag{12}$$

$$l_\mu=C_l n\left(1-\exp\left(-R_k/A_\mu\right)\right) \tag{13}$$

$$l_\varepsilon=C_l n\left(1-\exp\left(-R_k/A_\varepsilon\right)\right) \tag{14}$$

$$R_k=\frac{\rho n k^{1/2}}{\mu},\quad C_l=\kappa C_\mu^{-3/4},\quad A_\mu=70.,\quad A_\varepsilon=2C_l \tag{15}$$

- Outer fully turbulent region :

$$\mu_T=\rho C_\mu\frac{k^2}{\varepsilon} \tag{16}$$

The constants for the two-layer model are as follows;

$$C_\mu=0.09,\ \sigma_k=1,\ \sigma_\varepsilon=1.3,\ C_{\varepsilon 1}=1.44,\ C_{\varepsilon 2}=1.92 \tag{17}$$

In the $\overline{vv}-f$ model, two additional equations, Eq.(7)-Eq.(8), are solved, and the eddy viscosity, the time scale and the length scale in the governing equations are given by the following equations;

$$\mu_T=\rho C_\mu\overline{vv}T \tag{18}$$

$$T=\max\left(\frac{k}{\varepsilon},C_T\left(\frac{\nu}{\varepsilon}\right)^{1/2}\right),\quad \nu=\frac{\mu}{\rho} \tag{19}$$

$$L=C_L\max\left(\frac{k^{3/2}}{\varepsilon},C_\eta\left(\frac{\nu^3}{\varepsilon}\right)^{1/4}\right) \tag{20}$$

$$C_{\varepsilon 1}=C_{\varepsilon 1}^{'}\left(1.+0.045\left(\frac{k}{\overline{vv}}\right)^{1/2}\right) \tag{21}$$

The constants for the $\overline{vv}-f$ model used in the present study are as follows ;

$$C_\mu=0.22,\ \sigma_k=1,\ \sigma_\varepsilon=1.3,\ C_{\varepsilon 1}^{'}=1.4,\ C_{\varepsilon 2}=1.9 \tag{22}$$

$$C_1=1.4,\ C_2=0.3,\ C_T=6.,\ C_L=0.3,\ C_\eta=70 \tag{23}$$

In the present study, the algebraic flux model (AFM hereafter) developed by Kenjeres [3] has been also tested since this model has been very successful for prediction of various natural convection problems. In this model the

turbulent heat fluxes needed for calculating the buoyancy induced generation term (G in Eq.(10)) is algebraically calculated by the following equation;

$$\overline{\theta u_i} = -\frac{1}{C_{\theta 1}}\frac{k}{\varepsilon}\left(\overline{u_i u_k}\frac{\partial \Theta}{\partial x_k} + (1-C_{2\theta})\overline{\theta u_k}\frac{\partial U_i}{\partial x_k} + (1-C_{3\theta})\beta g_i \overline{\theta^2}\right) \tag{24}$$

where

$$C_{\theta 1} = 5.,\ C_{\theta 2} = 0.4,\ C_{\theta 3} = 0.4 \tag{25}$$

The temperature variance term ($\overline{\theta^2}$) in Eq.(24) is obtained by solving Eq.(6). The constants for Eq.(6) and Eq.(3) used in the present study are as follows;

$$\mathrm{Pr}_\theta = 0.9,\ \sigma_\theta = 0.9,\ R = 2. \tag{26}$$

The Reynolds stresses in Eq.(24) are calculated by the following simple gradient formulation.

$$\overline{u_i u_j} = \frac{2}{3}k\delta_{ij} - \nu_T\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) \tag{27}$$

In the present study, four turbulence models (two-layer model, two-layer model with AFM, $\overline{vv}-f$ model and $\overline{vv}-f$ model with AFM) are tested for calculation of turbulent natural convection in a rectangular cavity.

3. THE TEST PROBLEM

The test problem considered in the present study is a natural convection of air in a rectangular cavity with aspect ratio of 5:1. The height of cavity is $H = 2.5m$ and the width of cavity $L = 0.5m$ and the temperature difference between hot and cold wall is 45.8K. The Rayleigh number based on the height of cavity is $Ra = 4.5\times10^{10}$ and Prandtl number is $\mathrm{Pr} = 0.7$. King [13] has made extensive measurements for this problem and the experimental data are reported in Cheesewright et al. [14] and King [13]. These experimental data have been used extensively for evaluation of turbulence models [3, 15-17]. Within the present authors knowledge, the most successful computation for this problem so far is due to Kenjeres [3] using the Launder and Sharma model [4] with AFM and nobody has reported the computed results using the second moment turbulent models. Tieszen et al. [18] solved this problem to evaluate the Durbins $k-\varepsilon-\overline{vv}-f$ model for natural convection problem. As will be shown later, the predicted results by present authors using the same model are different from those reported in Tieszen et al. [18].

4. RESULTS AND DISCUSSION

As explained before, the primary objective of the present study is evaluation of Durbins $\overline{vv}-f$ model, which has been very successful for forced convection problems [7-9], for a natural convection problem. As a comparative study, the results by two-layer model are reported together. Calculations are performed using the 82×122 numerical grids. The computations by the two-layer model were always stable, but the solutions were a little affected by the location of the interfaces between the two regions where the $k-\varepsilon$ model and one equation model is used respectively. We gradually changed the

(a) (b)

Fig.1 Streamlines predicted by the two-layer model with AFM (a), and by the $\overline{vv}-f$ model with AFM (b)

locations of the interfaces until the solution is not further changed. We experienced the numerical stiffness problem when we used the $\overline{vv}-f$ model. The numerical stiffness problem occurred near the boundary, and we developed something like the source term linearization technique at the boundary to avoid such a problem. We also found that the initial conditions also affect the numerical stability. The results by the two-layer model are used for the initial conditions for the $\overline{vv}-f$ computations. The computations are continued until the maximum absolute sum of residual of all computed variables is less than 10^{-6}. This convergence criterion is sufficiently small to assure the convergence.

Fig.1 compares the streamlines predicted by the two-layer model with AFM and that by the $\overline{vv}-f$ model with AFM. There are not much differences observed between models with and without AFM, thus only the results by two models with AFM are presented here. Some differences are observed near the core, upper and lower regions, as shown in Fig.1. Fig.2 shows the isotherms predicted by the two models with AFM. There exist some differences between the two results. The isotherms predicted by the $\overline{vv}-f$ model with AFM are equally spaced, while those by the two-layer model are not equally spaced, indicating that the vertical centerline temperature distribution is not linear. The results by $\overline{vv}-f$ model with AFM are similar to those reported in Kenjeres [3] using the Launder and Sharma model [4] with AFM.

We now compare the predicted results with the measured data provided by Cheesewright et al. [14] for mean vertical velocity and velocity fluctuation at the mid-height (y/H=0.5) of the cavity. Fig.3 and Fig.4 show the comparisons of the predicted results with measured data for vertical velocity profiles at y/H=0.5. The experimental data are scanned from Cheesewright et al. [14] since no numeric data are tabulated in their paper. The scanned data may be slightly different from the actual measured data.

As shown in the figures, the $\overline{vv}-f$ model with AFM best predicts the vertical velocity component, while the two-layer model with or without AFM poorly predicts the vertical velocity component. It is interesting to note that the predicted velocity profile reported in Tieszen et al. [18] using the $\overline{vv}-f$ without

(a) (b)

Fig.2 Isotherms predicted by the two-layer model with AFM (a), and by the $\overline{vv}-f$ model with AFM. (b)

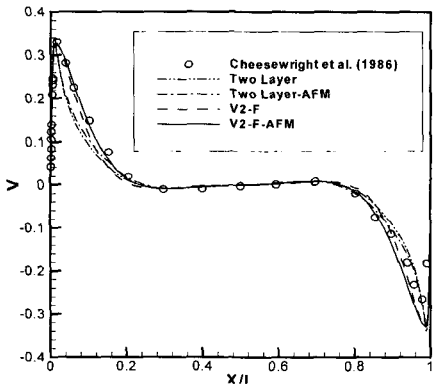

Fig.3 Mean vertical velocity profiles at y/H=0.5 predicted by four different turbulence models

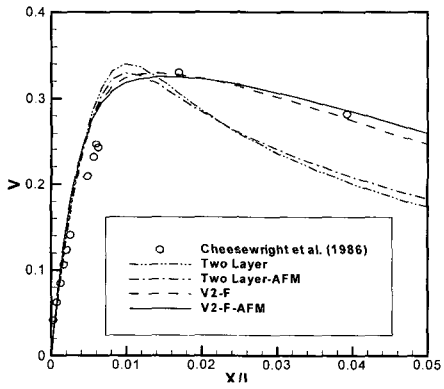

Fig.4 Mean vertical velocity profiles close to the hot wall at y/H=0.5 predicted by four different turbulence models

AFM is much different from our prediction using the same model. In our results, the $\overline{vv}-f$ model without AFM also predicts well the velocity profiles, while the predicted results given in Tieszen et al. [18] using the $\overline{vv}-f$ model without AFM show a sharp variation near the wall which is much different from experimental data. These figures also show that the incorporation of AFM does not much affect the prediction of velocity components. We note that the result by Kenjeres [3] using the Launder and Sharma model [4] with AFM is similar to the present prediction by the $\overline{vv}-f$ with AFM.

Fig.5 shows comparison of predicted results with the measured data for the vertical velocity fluctuation at y/H=0.5. As shown in the figure, the $\overline{vv}-f$ model with AFM predicts best the velocity fluctuation and the agreement with the measured data is pretty good except for the center region. We can observe that the two-layer models with or without AFM severely underpredict the velocity fluctuation. We can also observe that the incorporation of

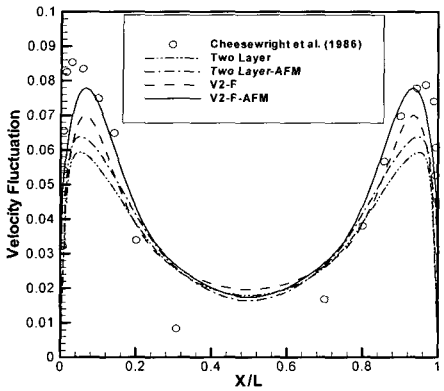

Fig.5 Vertical velocity fluctuation profiles at y/H=0.5 predicted by four different turbulence models

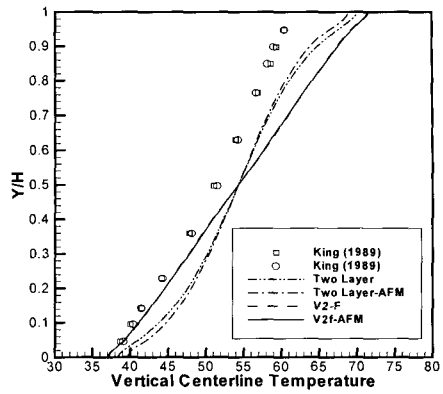

Fig.6 Vertical centerline temperature profiles at x/L=0.5 predicted by four different turbulence models

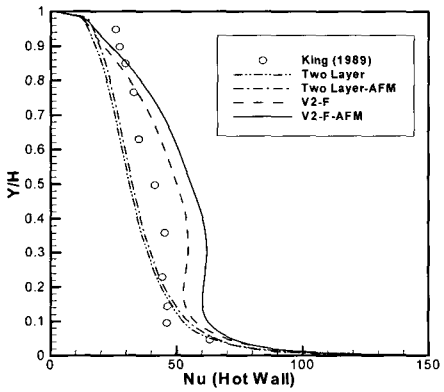

Fig.7 Profiles of Nusselt number at hot wall predicted by four different turbulence models

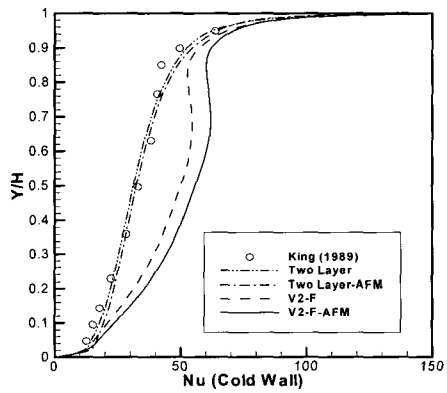

Fig.8 Profiles of Nusselt number at cold wall predicted by four different turbulence models.

AFM improves the prediction of velocity fluctuation.

Fig.6 shows the comparison of the predicted vertical centerline temperature profiles at x/L=0.5 with the measured data. We first note that the measured data of vertical centerline temperature do not show the linear variation, and Cheesewright et al. [14] explain this phenomenon is due to the insufficient insulation of the side walls. The heat loss from the side walls causes the reduction of temperature, and the centerline temperature deviates from the linear variation at the upper region of the cavity.

The predicted results by the $\overline{vv}-f$ model clearly exhibit the linear variation while the predictions by the two-layer model do not show such a trend. The differences between measured data with predictions by the $\overline{vv}-f$ model are believed to be due to the experimental difficulties

Fig.7 and Fig.8 show the comparisons of the predicted results with the measured data for Nusselt number at the hot wall and cold wall reported in King [13]. The heat transfer

coefficient reported in King [13] is based on the centerline temperature as follows;

$$h_H(\Theta_{Hot}-\Theta_\infty)=-k\left.\frac{\partial\Theta}{\partial x}\right|_{Hot\ Wall} \quad (28)$$

where h_H is the heat transfer coefficient at hot wall, k is the thermal conductivity and Θ_∞ is the temperature at the centerline ($x/L=0.5$). The Nussselt number given in Fig.7 is based on the temperature difference between hot and cold wall. Thus, some manipulations are made using the experimental data of centerline temperature given in Fig.6. As explained before, the measured data of centerline temperature do not exhibit the linear variation due to insufficient insulation and this may affect the heat transfer coefficient. The agreement between measured data with predicted results is not good as shown in Fig. 7 and Fig. 8. The $\overline{vv}-f$ model properly predicts the transition phenomenon at lower portion of hot wall and upper portion of cold wall. It is noted that the two-layer model do not predicts the transition phenomenon. It may be due to the fact that the length scales for the two-layer model near the wall are artificially specified by Eqs.(13)-(14). It is interesting to note that the two-layer model predicts the Nusselt number very closely with the measured data. However, it is noted that the predictions by Kenjeres [3] using the Launder and Sharma model [4] with AFM show the same trend as the present predictions by the $\overline{vv}-f$ model. But, the predictions by Kenjeres [3] agree slightly better with the measured data than the present predictions by the $\overline{vv}-f$ model.

Fig.9 and Fig.10 show the comparisons of the predicted results with the measured data for the wall shear stress at the hot wall and cold wall reported in King [13]. We observe that the $\overline{vv}-f$ model severely overpredicts the wall shear stress at the lower portion of the hot wall and at the upper portion of cold wall. It is quite a surprising result when we observe that the $\overline{vv}-f$ model predicts well the mean vertical velocity as shown in Fig.3 and Fig.4. It is noted that the measured data deviate severely from the symmetry at hot and cold walls, while the predicted results slightly deviate the symmetry nature. Thus, it is quite questionable that the experimental data reported in King [13] is reliable for validation of turbulence models. It is of interest to see that the predictions by the two-layer model, which show poor predictions for vertical velocity in Fig.3 and Fig.4, agree better with the measured data than the predictions by the $\overline{vv}-f$ model. The predictions by Kenjeres [3] show a similar trend as the results by the present the $\overline{vv}-f$ model.

Fig.11 and Fig.12 show the profiles of the predicted turbulent heat fluxes $\overline{\theta u}, \overline{\theta v}$ at the mid-plane (y/H=0.5) of the cavity. It is noted that these turbulent heat fluxes are obtained algebraically using Eq.(24), not from the solution of the governing equations for these quantities. The predicted turbulent heat fluxes $\overline{\theta u}$ and $\overline{\theta v}$ by the $\overline{vv}-f$ model generally follow the trends of measuring data, however, severely overpredict them in the region close to the wall, indicating that the assumptions employed for derivation of Eq.(24) is not valid in these regions. It is noted that the turbulent heat fluxes obtained by Eq.(24) affect the buoyancy induced generation term (G) only for the present two-layer and $\overline{vv}-f$ models with AFM. The magnitude of overprediction is more

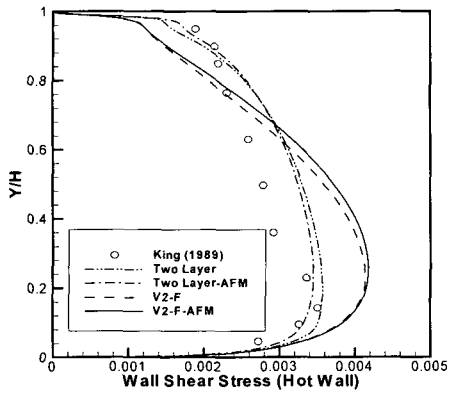

Fig.9 Profiles of wall shear stress at hot wall predicted by four different turbulence models

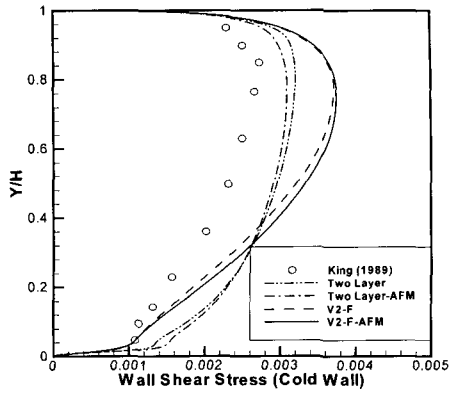

Fig.10 Profiles of wall shear stress at cold wall predicted by four different turbulence models

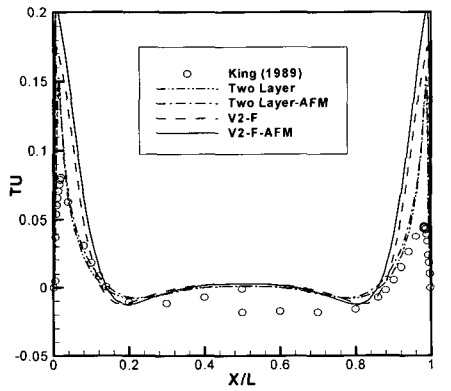

Fig.11 Turbulent heat flux $\overline{\theta u}$ profiles at y/H=0.5 predicted by four different turbulence models

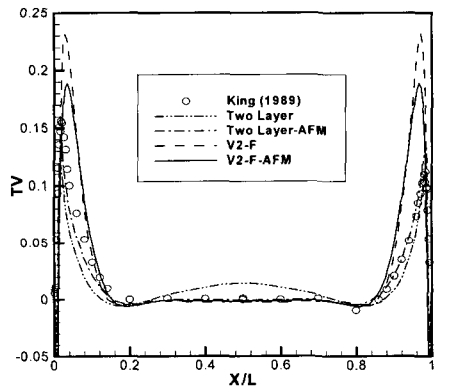

Fig.12 Turbulent heat flux $\overline{\theta v}$ profiles at y/H=0.5 predicted by four different turbulence models

severe for the predictions by the $\overline{vv}-f$ model than those by the two-layer model.

Fig.13 shows the profiles of the predicted Reynolds shear stress $\overline{uv}$ at the mid-plane (y/H=0.5) of the cavity. The predicted results show the symmetric nature, while the experimental data show asymmetric nature. The $\overline{vv}-f$ model predicts well the $\overline{uv}$ near the hot wall, however, it overpredicts it near the cold wall. The magnitudes of $\overline{uv}$ predicted by the two-layer model are smaller than those by the $\overline{vv}-f$ model and both predictions follow the trend of measured data.

5. CONCLUSIONS

The $\overline{vv}-f$ model and two-layer $k-\varepsilon$ model, both with and without algebraic heat flux model are tested for natural convection in a rectangular cavity. The primary emphasis of the present study is placed on evaluation of

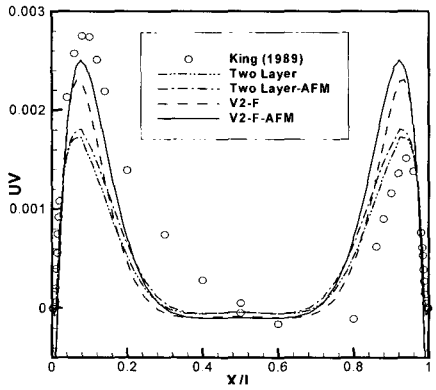

Fig.13 Turbulent shear stress $\overline{uv}$ profiles at y/H=0.5 predicted by four different turbulence models

the $\overline{vv}-f$ model for natural convection problem. The performances of turbulence models are investigated through comparison with available experimental data. The $\overline{vv}-f$ model with AFM best predicts the vertical velocity component and velocity fluctuation, and the predicted results fairly well agree with the measured data. However, it rather poorly predicts the local Nusselt number and the wall shear stress on the hot and cold walls when its results are compared with experimental data although it properly predicts the transition phenomenon. It also overpredicts the turbulent heat fluxes near the walls. It is also questioned whether the experimental data used in the present study for evaluation of turbulence models are reliable since the experimental data, except for the vertical velocity component and its fluctuation, do not show the required natures such as the symmetry nature or the linear variation of vertical centerline temperature. Thus, it is very difficult to draw a complete conclusion, but the $\overline{vv}-f$ model predicts nearly the same level of accuracy as the earlier computation by Kenjeres [3] using the Launder and Sharma model [4] with the algebraic heat flux model. It is also shown that the incorporation of the algebraic heat flux model slightly improves the accuracy of solutions.

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