

CAROD: Computer-Aided Reliable and Optimal Design as a concurrent system for real structures

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Abstract – Computer-Aided Reliable and Optimal Design (CAROD) system is an efficient tool defining the best compromise between cost and safety. Using the concurrent engineering concept, it can supply the designer with all numerical information in the design process. This system integrates several fields such as multidisciplinary optimization, reliability analysis, finite element analysis, geometrical modeling, sensitivity analysis and concurrent engineering. When integrating these disciplines, many difficulties are found such as model coupling and computational time. In this paper, we propose a new concurrent methodology satisfying the reliability requirement, allowing the coupling of different models and reducing the computational time. Two applications (rotating disk and hook structures) demonstrate that CAROD system can be a practical concurrent engineering application for designers.

Keywords: Reliability-Based Design Optimization, reliability analysis, CAD, concurrent engineering

1. Introduction

Several computer-aided tools have been introduced for engineers in order to satisfy different requirements not only to design (*i.e.* CAD) but also to the broader areas of manufacturing (CAM) and enterprise-wide issues (CAE) [10, 13]. Based on this philosophy, we provide an efficient computer-aided tool, called CAROD (Computer-Aided Reliable and Optimal Design), which integrates the reliability and the cost requirements into the design process. Not only a large amount of mathematical and numerical techniques are available in CAROD system but also the use of the concurrent engineering concept allows us to reduce the design phase and to satisfy cost and safety requirements. There are many perceptions about the nature of concurrent engineering. For example, [7] defined concurrent engineering as being “the process of forming and supporting multifunctional teams that set product and process parameters early in the design phase”. Therefore, concurrent engineering is designing for cost, safety, performance, manufacturability, assembly, availability, ... etc. Therefore, CAROD system integrates several fields such as multidisciplinary optimization, reliability analysis, finite element analysis, geometrical modeling, sensitivity analysis and concurrent engineering.

When integrating these disciplines, many difficulties can be found such as model coupling and computational time. Multidisciplinary optimization is a way of finding

the “best” solution, given an objective and a set of constraints, where the objective, the constraints and the variables come from the knowledge built in different disciplines. In the last decade, multi-objective optimization has gradually crept in engineering [17]. Topology optimization, shape optimization, sizing optimization and reliability-based design optimization are components of multidisciplinary optimization. The Reliability-Based Design Optimization (RBDO) is a multi-objective optimization because it extends optimization theory by allowing multiple objectives to be “optimized” simultaneously. The principle of this concept is to integrate the reliability analysis in the design optimization problem in order to minimize the cost and to maximize the reliability level. Since the introduction of reliability considerations in the shape optimization problems allows us to meet cost and safety requirements, we propose herein to integrate the reliability analysis in the topology optimization problem. This integration leads to a really optimal topology and design. In the following sections, we present the CAROD system and then we explain how each component is integrated in the process, a special attention is given to the RBDO components. Finally, two engineering applications are illustrated in order to show the performance of the proposed methodology.

2. CAROD system

In practical applications, the coupling between the reliability analyses and the optimization procedures leads to very high computing time and weak convergence stability. Thus, there is a strong motivation to develop efficient techniques with the aim of reducing the

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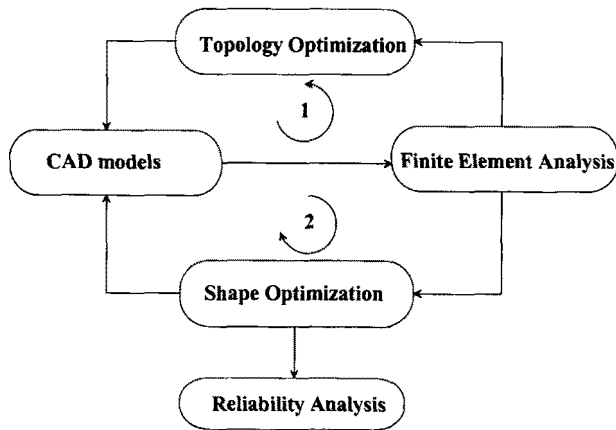


Fig. 1. Traditional DDO procedure (reliability analysis on the optimal solution).

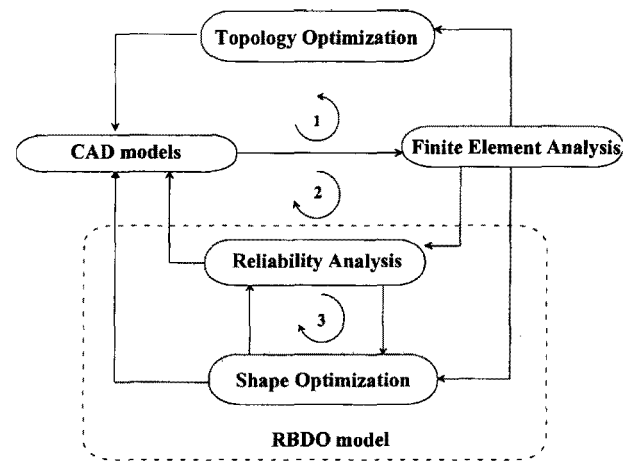


Fig. 2. Sequential optimization process including RBDO model.

computational time. The traditional Deterministic Design Optimization(DDO) process, which is illustrated in Fig. 1, can be realized in two loops. The first loop contains three steps:

- (1) describe the structural geometry by using suitable CAD models,
- (2) mesh and evaluate the design model using Finite Element Analysis(FEA).
- (3) apply the Topology Optimization in order to eliminate the unnecessary parts in the structure.

The initial geometry may be a pre-defined shape or a bulk domain. These steps will be repeated until satisfying the constraints and optimizing the cost function. The second loop is to optimize a given topology (geometry) of the structure. This loop contains three steps:

- (1) describe the geometry,
- (2) mesh and evaluate the design model,
- (3) optimize the structure shape by minimizing an objective function such as cost or volume.

Next, when obtaining the optimal design, we analyze the reliability level of the optimal solution. However, the reliability at the deterministic optimum may be low and needs to be improved. The traditional DDO procedure can then be improved by integrating the RBDO model, which reduced the structure volume in uncritical regions and can satisfy the required reliability level. The sequential optimization process integrated with RBDO model is illustrated in Fig. 2. The classical solution of the RBDO is carried out by alternating reliability and optimization iterations in two different variable spaces (*i.e.* design and random spaces). For simple shapes, this loop essentially contains two sub-problems:

- (1) optimize an objective function such as cost or volume.

- (2) compute the system reliability by a particular optimization procedure (Fig. 2).

In the RBDO model, additional variables are introduced to take account for design uncertainties and randomness. Hence, a lot of numerical calculations are required in the space of random variables in order to evaluate the system reliability. Furthermore, the optimization process itself is executed in the space of design variables, which are deterministic. Consequently, in order to search for the optimal system, the design variables are repeatedly changed, and each set of design variables corresponds to a new random variable space, which then needs to be manipulated to evaluate the system reliability at that point. Furthermore, for complex shapes, at each iteration of reliability analysis and shape optimization process, CAD and FEA models are required. As too many searches are needed in the above two spaces, the computational time for such an optimization is a great problem. The coupling scheme between geometrical modeling, mechanical modeling, reliability analysis and optimization methods represents a real problem. Because of these two difficulties, only few researchers integrate the RBDO model within the design process, especially for large-scale problems. In the present work, CAROD system is introduced to overcome the above difficulties (*i.e.* model coupling and computational time) for the design of complex engineering problems. By using the concurrent engineering concepts, the designer can obtain the necessary information about the structure during the optimization process and so, the design phase is largely reduced. CAROD system is defined by the following three layers (Fig. 3):

- Layer 1** RBDO model: for explicit mechanical models (shape optimization and reliability analysis),
- Layer 2** Integrated RBDO with CAD and FEA: for implicit problems
- Layer 3** RBTO model: (topology optimization, reliability analysis, FEA and CAD description)

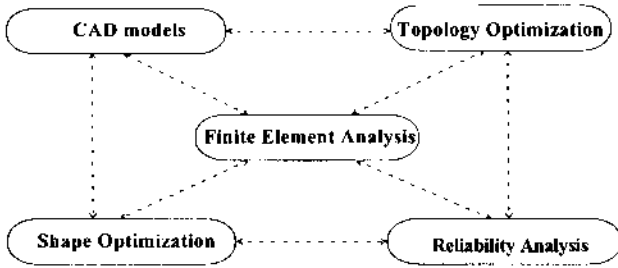


Fig. 3. CAROD system.

The integration of these layers constitutes the CAROD system, which aims to design structures that should be economic and reliable by using the concurrent engineering concepts (Fig. 3). According to the concurrent engineering definition, CAROD is getting the right models together at the right time to solve design problems.

In order to implement CAROD, we have to relate all modules presented in Fig. 3. For example, when using ANSYS software, which contains topology and shape optimization modules with geometric modeler and FEA, we have to implement a reliability algorithm and relate it with the other modules. In the next sections, we present the integration of each of these CAROD components. A rotating disk example shows the advantage of CAROD system with respect to the traditional DDO procedure. A hook structure demonstrates the efficiency of CAROD system with respect to the sequential optimization process including RBDO model.

3. Reliability Analysis

The design of structures and the prediction of their good functioning lead to the verification of a certain number of rules resulting from the knowledge of physical and mechanical experience of designers and constructors. These rules traduce the necessity to limit the loading effects such as stresses and displacements. Each rule represents an elementary event and the occurrence of several events leads to a failure scenario. The objective is then to evaluate the failure probability corresponding to the occurrence of critical failure modes.

3.1. Importance of safety criteria

In deterministic structural optimization, the designer aims to reduce the construction cost without caring about the effects of uncertainties concerning materials, geometry and loading. In this way, the resulting optimal configuration may present a lower reliability level and then, leads to higher failure rate. The equilibrium between the cost minimization and the reliability maximization is a great challenge for the designer. In general design problems, we distinguish between two kinds of variables:

- (1) the design variables $\{x\}$ which are deterministic variables to be defined in order to optimize the design. They represent the control parameters of

the mechanical system (e.g. dimensions, materials, loads) and of the probabilistic model (e.g. means and standard-deviations of the random variables),

- (2) the random variables $\{y\}$ which represent the structural uncertainties, identified by probabilistic distributions. These variables can be geometrical dimensions, material characteristics or applied external loading.

3.2. Failure probability

In addition to the vector of deterministic variables $\{x\}$ to be used in the system design and optimization, the uncertainties are modeled by a vector of stochastic physical variables affecting the failure scenario. The knowledge of these variables is not, at best, more than statistical information and we admit a representation in the form of random variables. For a given design rule, the basic random variables are defined by their joint probability distribution associated with some expected parameters; the vector of random variables is noted herein $\{Y\}$ whose realizations are written $\{y\}$. The safety is the state in which the structure is able to fulfill all the functioning requirements (e.g. strength and serviceability) for which it is designed. To evaluate the failure probability with respect to a chosen failure scenario, a limit state function $G(\{x\}, \{y\})$ is defined by the condition of good functioning of the structure. In Fig. 4, the limit between the state of failure $G(\{x\}, \{y\}) < 0$ and the state of safety $G(\{x\}, \{y\}) > 0$ is known as the **limit state surface** $G(\{x\}, \{y\}) = 0$. The failure probability is then calculated by:

$$P_f = \Pr[G(\{x\}, \{y\}) \leq 0] = \int_{G(\{x\}, \{y\}) \leq 0} f_Y(\{y\}) dy_1 \dots dy_n \quad (1)$$

where P_f is the failure probability, $f_{\{Y\}}(\{y\})$ is the joint density function of the random variables $\{Y\}$ and $P_r[\cdot]$ is the probability operator. The evaluation of integral (1) is not easy, because it represents a very small quantity and all the necessary information for the joint density function are not available. For these reasons, the *First and the Second Order Reliability Methods FORM/SORM* [2] have been developed. They are based on the reliability index concept, followed by an estimation of the failure probability. The invariant reliability index β was introduced by [4], who proposed to work in the space of standard independent Gaussian variables instead

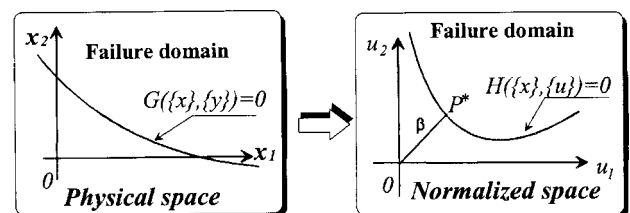


Fig. 4. Physical and normalized spaces.

of the space of physical variables. The transformation from the physical variables $\{y\}$ to the normalized variables $\{u\}$ is given by:

$$\{u\} = T(\{x\}, \{y\}) \quad \text{and} \quad \{y\} = T^{-1}(\{x\}, \{u\}) \quad (2)$$

This operator $T(\cdot)$ is called the *probabilistic transformation*. In this standard space, the limit state function takes the form:

$$H(\{x\}, \{u\}) \equiv G(\{x\}, \{y\}) = 0 \quad (3)$$

In the *FORM* approximation, the failure probability is simply evaluated by:

$$P_f \approx \Phi(-\beta) \quad (4)$$

where $\Phi(\cdot)$ is the standard Gaussian cumulated function. For practical engineering, equation (4) gives sufficiently accurate estimation of the failure probability.

3.3. Reliability evaluation

For a given failure scenario, the reliability index β is evaluated by solving a constrained optimization problem (Fig. 5).

The calculation of the reliability index can be realized by:

$$\beta = \min(\sqrt{\{u\}^T \{u\}}) \quad \text{subject to} \quad G(\{x\}, \{y\}) \leq 0 \quad (5)$$

The solution of this problem is called the *design point* P^* , as illustrated in Fig. 4. When the mechanical model is defined by numerical methods, such as the finite element analysis, the evaluation of the reliability implies a special coupling procedure between both reliability and mechanical models [11].

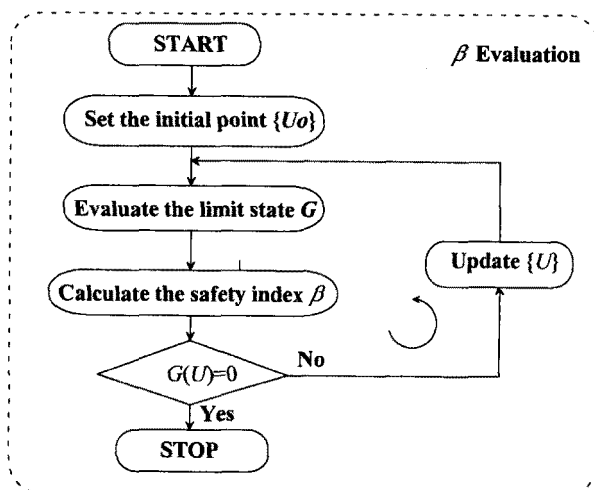


Fig. 5. Calculation of safety index.

4. RBDO Models

For deterministic optimization, many efficient numerical methods have been developed and applied to different kinds of structures. But for RBDO problems, the coupling between the mechanical modeling, the reliability analysis and the optimization methods represents a very complex task and leads to very high computational time. The major difficulty lies in the evaluation of system reliability, which is carried out by a particular optimization procedure [3]. Efforts were directed towards developing efficient techniques [1, 16] and general proposed programs to integrate the reliability analysis for given uncertain information. These programs and procedures compute the reliability index of a structure for the defined failure modes, but do not provide an optimum set of the design parameters, in order to improve the reliability of the structure. An enormous amount of computer time is also involved in the whole design process. In this section, we present the sequential (or the classical) RBDO procedure and the proposed concurrent approach, which is based on the simultaneous solution of the reliability and optimization problems.

4.1. Sequential approach

The sequential RBDO algorithm which is illustrated in Fig. 6, is calculated by nesting the two following sub-problems:

1 - optimization problem under deterministic and reliability constraints:

$$\begin{aligned} \min & \quad : f(\{x\}) \\ \text{subject to} & \quad : g_k(\{x\}) \leq 0 \\ \text{and} & \quad : \beta(\{x\}, \{u\}) \geq \beta_t \end{aligned} \quad (6)$$

where $f(\{x\})$ is the objective function, $g_k(\{x\})=0$ are the associated deterministic constraints, $\beta(\{x\}, \{u\})$ is the reliability index of the structure and β_t is the target reliability.

2 - calculation of the reliability index $\beta(\{x\}, \{u\})$:

$$\begin{aligned} \min & \quad : d(\{u\}) \\ \text{subject to} & \quad : H(\{x\}, \{u\}) \leq 0 \end{aligned} \quad (7)$$

where $d(\{u\})$ is the distance in the normalized random space and $H(\{x\}, \{u\})$ is the limit state function as shown in section 3.

The constrained minimization of the objective function $f(\{x\})$ is carried out in the physical space of design variables $\{x\}$ but the reliability index β is calculated in the normalized space of random variables $\{u\}$, which are the image of $\{y\}$ in the standard space.

4.2. Concurrent approach

In order to avoid the high computational time of the nested problems given in section 4.1, we propose a new formulation by combining deterministic and random

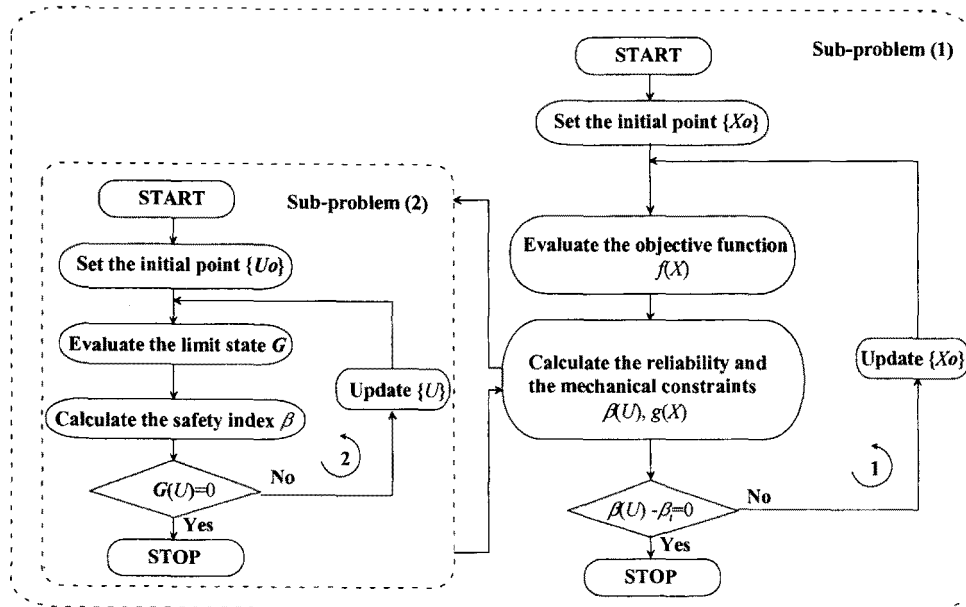


Fig. 6. Sequential RBDO algorithm (explicit problems).

spaces. The new optimization problem can be expressed by :

$$\begin{aligned}
 \min \quad & F(\{x\},\{y\}) = f(\{x\}) \cdot d_{\beta}(\{x\},\{y\}) \\
 \text{subject to} \quad & G(\{x\},\{y\}) \leq 0 \\
 & \beta(\{x\},\{y\}) \geq \beta_r \\
 & g_k(\{x\}) > 0
 \end{aligned} \tag{8}$$

where $F(\{x\},\{y\})$ is the new form of the objective function which integrates cost and reliability aspects and $d_{\beta}(\{x\},\{y\})$ is the image of $d(u)$ in the physical space. The optimization algorithm, which is illustrated in Fig. 7, supplies us with all information about the objective and constraint functions. This algorithm minimizes

the function $F(\{x\},\{y\})$, which is carried out in the hybrid space of deterministic variables $\{x\}$ and random variables $\{y\}$. In the numerical applications, we propose to solve the concurrent problem either by an extended penalty function or by the projected gradient method. At the optimal point, the limit state constraint $G(\{x\},\{y\})=0$ must be active for consistent reliability solution. The other constraints $g_k(\{x\}) \geq 0$ and $\beta(\{x\},\{y\}) \geq \beta_r$ are not necessarily active.

For analytical models, the efficiency of the proposed concurrent approach has been tested on several examples [9]. Furthermore, a hook structure is presented in this paper to show the advantage of the concurrent approach with respect to the sequential one.

5. RBDO with CAD & FE Models

In the CAROD system (Fig. 3), the coupling between RBDO, FEA and CAD models is represented by layer 2 as indicated in section 2. However, when FEA is involved, the computational time is a serious problem for practical applications. Furthermore, for complex geometries, the computational time will be a big problem and the coupling between several models (the geometrical modeling, the mechanical modeling, the reliability analysis and the optimization methods) will be very difficult. In this section, we show how the concurrent RBDO model becomes an efficient tool when the finite element model allows us to get the sensitivity information with respect to design and random variables. After the discussion of sensitivity equations in FEA, the concurrent RBDO is extended to nonlinear problems in order to demonstrate the efficiency of the concurrent methodology. Next, we present some difficulties in the complex geometry description when using local optimization variables.

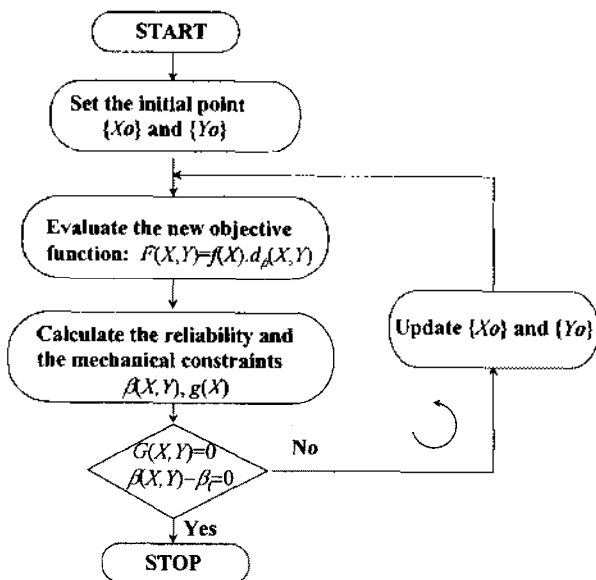


Fig. 7. Concurrent RBDO algorithm.

Finally, the advantage of the concurrent RBDO algorithm will be presented in order to compare it with the sequential one, especially for large-scale problems.

5.1. Sensitivity operators

Let us consider the case of RBDO using finite element model based on a geometrical and material linear elastic displacement method. For a given failure scenario, the limit state function is written as:

$$H(\{x\}, \{u\}, \{b(\{x\}, \{u\}), \{q\}\}) = 0 \quad (9)$$

where $\{q\}$ is the nodal displacement vector depending on the design variables $\{x\}$ and on the normalized ones $\{u\}$, and $\{b\}$ is a vector of response parameters associated with the limit state function, e.g. internal forces, stresses, strains or displacements. The nodal displacements are obtained by using the fact that a linear elastic finite element model is additive and the principle of superposition can be used. This is performed by applying the pseudo-loading technique in which a unit load or a load proportional to the load F_s is introduced for each load case $s=1, \dots, S$ in the model. The loads F_s are then modeled as stochastic variables $F_s(\{u\})$, depending on the stochastic variables u_i in the reliability problem.

In the optimization algorithms for the design point computation, the gradients of $G(\cdot)$ with respect to $\{u\}$ are needed. When the pseudo-load vector method is used to obtain the sensitivities of the response $\{b\}$, the finite element equations are written [14]:

$$[\mathbf{K}(\{x\}, \{u\})] \cdot \{q(\{x\}, \{u\})\} = \{f(\{x\}, \{u\})\} \quad (10)$$

where $\{f\}$ is the vector of external loads and $[K]$ is the structural stiffness matrix. For a given value of $\{x\}$, the material derivative dG/du_j is obtained by:

$$\frac{dG}{du_j} = \frac{\partial G}{\partial u_j} + \sum_{i=1}^{L_R} \frac{\partial G}{\partial b_i} \left[\frac{\partial b_i}{\partial u_j} + \sum_{p=1}^P \frac{\partial b_i}{\partial q_p} \frac{\partial q_p}{\partial u_j} \right] \quad (11)$$

where L_R is the dimension of the response vector $\{b\}$ and P is the number of nodal degrees of freedom, $\partial q_p / \partial u_j$ is selected from $\partial \{q\} / \partial u_j$ and obtained from (10) by:

$$\frac{\partial \{q\}}{\partial u_j} = [\mathbf{K}]^{-1} \left\{ \frac{\partial \{f\}}{\partial u_j} - \frac{\partial [\mathbf{K}]}{\partial u_j} \cdot \{q\} \right\} \quad (12)$$

In (11) and (12), the derivatives $\partial G / \partial u_j$, $\partial G / \partial b_i$, $\partial b_i / \partial u_j$, $\partial b_i / \partial q_p$, $\partial f / \partial u_j$ and $\partial [\mathbf{K}] / \partial u_j$ are obtained either by analytical or numerical approaches. In the RBDO problem with linear elastic analysis, it is seen that, at the sub-iteration level, the calculation of the limit state function and its gradients requires only one solution of the finite element equilibrium equations for

each sub-level (i.e. for each $\{x\}$), as long as the stiffness matrix is independent of $\{u\}$. Furthermore, the index sensitivities $\partial \beta / \partial x_i$ are necessary for the efficient use of first order optimization algorithms. It can be calculated by:

$$\frac{\partial \beta}{\partial x_i} = \frac{1}{\left| \frac{\partial G}{\partial u_j} \right|} \frac{\partial G}{\partial x_i} \quad (13)$$

The gradient $|\partial G / \partial u_j|$ is already known from the element reliability calculations. $\partial G / \partial u_j$ can be calculated analytically, semi-analytically or numerically by finite difference. The derivative $\partial b / \partial x_i$ is obtained after the determination of $\partial G / \partial x_i$ which for fixed values of the design point $\{u^*\}$ is written as in (13) where u_j is replaced by x_i , G is symmetrical in u_j and x_i , see (9). The derivatives $\partial G / \partial x_i$, $\partial G / \partial b_i$, $\partial b_i / \partial x_i$ and $\partial b_i / \partial q_p$ are similar to the case in (11). In general, they are easily obtained from the actual analytical expressions or by using the finite difference approach. $\partial q_p / \partial x_i$ is selected from $\partial \{q\} / \partial x_i$ determined from (10) as:

$$\frac{\partial \{q\}}{\partial x_i} = [\mathbf{K}]^{-1} \left\{ \frac{\partial \{f\}}{\partial x_i} - \frac{\partial [\mathbf{K}]}{\partial x_i} \cdot \{q\} \right\} \quad (14)$$

$\partial \{f\} / \partial x_i$ is again obtained analytically or numerically. It is seen that only one $[K]^{-1}$ is still needed for each configuration of the structural shape and dimensions. The main advantage of estimating the sensitivities of β using (11), (12), (13) and (14) instead of a simple numerical finite difference scheme is that a very large number of β calculations and stiffness assemblies and inversions can be avoided, thus reducing considerably the computational time consumption. Furthermore, the accuracy problem of taking finite difference in the iterative solutions is avoided. In fact, due to the multiple calculations of the design points, the calculation by finite difference of the derivative $\partial b / \partial x_i$ will not only be very expensive, but it will also be inaccurate because the estimates are obtained by the calculation of finite difference between iterative solutions. Therefore, semi-analytical sensitivities in RBDO become important, and, due to accuracy, it will in many cases be a fundamental requirement for the possibility of obtaining an optimal solution. It depends on the particular response calculation technique whether the derivatives of the limit state function can be calculated most efficiently by numerical finite difference, semi-analytical or analytical approaches. An alternative method to determine the derivatives of the response quantities such as stresses and displacements is the continuum method [6]. In the continuum method, the derivatives are obtained on the basis of variations of the continuum equilibrium equations and response functional. It does not require direct access to the finite element code to be used. The accuracy is the same as

the semi-analytical method described above for size optimization problems, but for shape optimization problems the continuum method is more stable. For the concurrent RBDO model, (9) to (14) can be formulated by replacing $\{u\}$ by the vector $\{y\}$ and β by d_β .

5.2. Efficiency in nonlinear analysis

The sequential model of RBDO including a linear finite element model is of course the simplest and least expensive finite element response model, which can be applied. In the cases where material or geometrical nonlinearities in the finite element model are involved, it is also possible to perform the RBDO but the computational time will increase significantly because the iterations must be performed at 3 levels:

- 1 - deterministic optimization in the design space $\{x\}$,
- 2 - reliability analysis in the normalized space $\{u\}$,
- 3 - nonlinear equilibrium iterations in the nodal displacement space $\{q\}$.

But the integrated form of the new concurrent method allows us to reduce significantly the computational time with respect to sequential approach. In order to prove the efficiency of this method, let us put together the random variables and the design variables in the same vector $\{z\} = \{x_1, \dots, x_n, y_1, \dots, y_m\}$, where n is the number of design variables and m is the number of random variables. The new form of the objective function can be expressed by:

$$F(\{z\}) = f(\{x\}) \times d_\beta(\{z\}) \quad (15)$$

and its derivative with respect to z_q can be written:

$$\frac{\partial F(\{z\})}{\partial z_q} = \frac{\partial f(\{z\})}{\partial z_q} \times d_\beta(\{z\}) + \frac{\partial d_\beta(\{z\})}{\partial z_q} \times f(\{z\}) \quad (16)$$

where $q=1, \dots, n+m$. Knowing that the objective function $f(\{z\})$ is independent of the random vector $\{y\}$, we get:

$$\frac{\partial f(\{z\})}{\partial z_q} = \frac{\partial f(\{x\})}{\partial x_i} \quad (17)$$

and since the derivative $\partial d_\beta(\{z\}) / \partial z_q$ can easily be determined, the concurrent method saves the computational time of the reliability analysis at each deterministic iteration during the optimization process. Therefore, the computational time of $\partial F(\{z\}) / \partial z_q$ almost equal to that of $\partial f / \partial x_i$. For nonlinear analysis, the concurrent RBDO is then very efficient because the number of derivatives is largely reduced and a lot of nonlinear iterations are avoided.

5.3. Complex geometry

When the RBDO is carried out for geometrical variables, the CAD model updating is necessary during

the design phase. Therefore, the parametrization step allows us to define the search directions of the optimization process. In the shape optimization case, these parameters or directions are chosen among the design variables that define the geometry of the boundary domain. In fact, the shape optimization process is piloted by the information corresponding to the geometrical boundary perturbation. The structural geometry that will be modified during the optimization process can be described by several methods such as element list (arcs of circles and straight segments), Bézier, B-spline or NURBS descriptions. The element list technique is very simple to implement, the design variables such as arc radius and center, angles or coordinates of straight segment ends can be chosen as optimization parameters. The boundary is described by the assembly of the elements in the list. The perturbation of the boundary design variables does not imply the change of all boundaries. But the discontinuity in the intersection of the different element constitutes a major problem for the optimization procedure. Because of these discontinuities, the geometric irregularities of the boundary influence much the evaluation of certain variable fields defined at the boundary. These irregularities represent a serious disadvantage for the functional minimization, as it creates artificial singularity in the model. Furthermore, the use of the element list such as straight segment, circular arc, parabolic curves represented by mathematical equations, does not ensure the free change of topology during the optimization process. Therefore, it is necessary to describe the structural geometry by using flexible curves or surfaces. When using Bézier curves, there exist two ways: the first one consists in using high-degree curves; in the second one, Bézier curves of modest order are pieced together using simple geometric rules to insure continuity at the different joints. For instance, to achieve zero-order continuity at a joint, it is sufficient to impose the end control points of the curve to coincide. First-order continuity can be obtained by stating that the edges of the two polygons adjacent to the common end point must lie on a straight line. But, the Bézier curves do not provide local control: moving any control point will change the shape of every part of the curve. However, the B-splines are on one hand that local control of the curve shape can be achieved and on the other hand, that additional control points can be introduced without increasing the degree of the curve. B-splines offer more parameters to the designer than Bézier curves: the degree can be selected, as well as the multiplicity of control points [15]. Therefore, the B-spline parametric curves representation is a very attractive tool for shape optimization by the design element technique.

When using the sequential RBDO procedure with FEA-CAD model, we need a high computational time to solve an implicit model because of several repeated loops, as illustrated in Fig. 8. However, our concurrent methodology (second layer of CAROD system) will

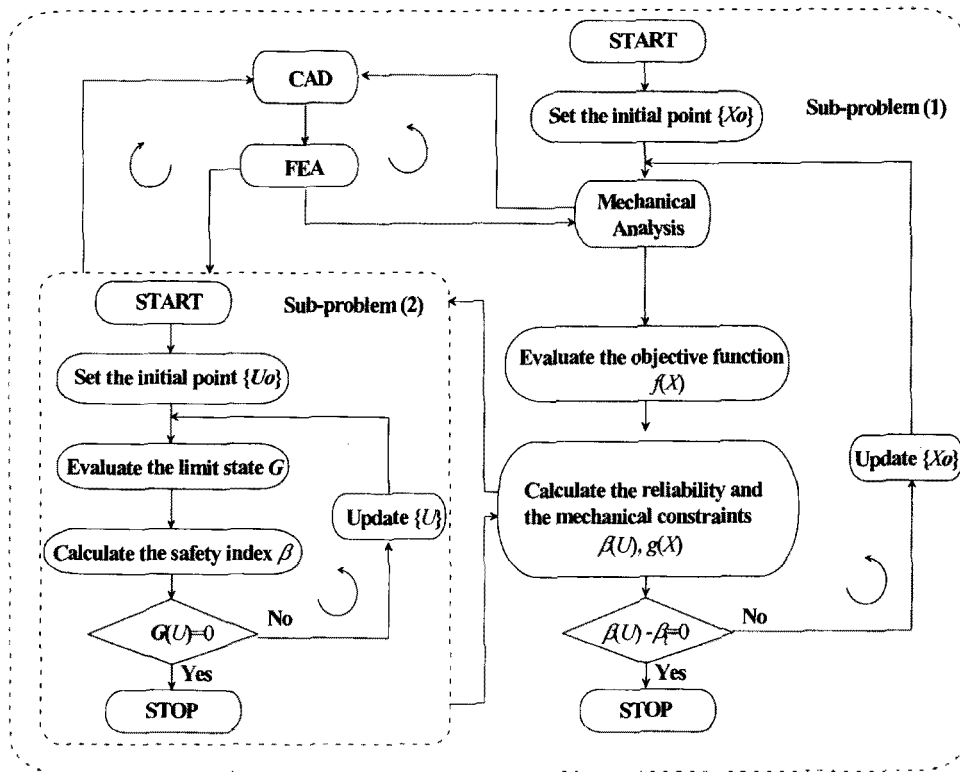


Fig. 8. Sequential algorithm for RBDO-FEA-CAD model(implicit problems).

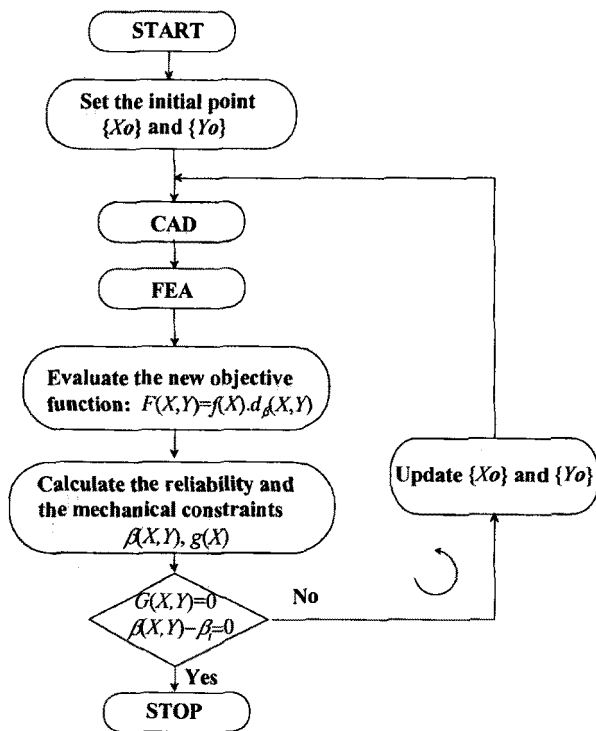


Fig. 9. Concurrent algorithm for RBDO-FEA-CAD model.

efficiently reduce the computational time because of the decrease of the loops number in the optimization

algorithm (Fig. 9). The computational time will be largely reduced when solving large-scale problems. In section 5.2, we analytically demonstrated that the computational time of the gradient calculation of $F(\{x\},\{y\})$ almost equal to that of $f\{x\}$. Thus, the concurrent methodology saves the computational time of the reliability analysis at each deterministic iteration during the optimization process. Furthermore, a hook application shows the efficiency of the concurrent methodology with respect to the sequential one.

7. RBTO Model

In the classical topology optimization, we search to minimize the compliance for a given volume of material, the material density is used as a continuous design variable. The material density and the associated effective properties are controlled by the mean of shape variables of microstructure cells. The problem is thus formulated as:

$$\begin{aligned}
 \min & & : & L(w) \\
 \text{subject to} & & : & a_d(w, v) = L(v) \text{ for all } v \in H \\
 \text{and} & & : & \text{volume} \leq V
 \end{aligned} \tag{18}$$

In problem 18, we use the energy bilinear from the internal work and the load linear for the external work. We minimize the mean compliance in order to achieve the stiffest structure. It is given by:

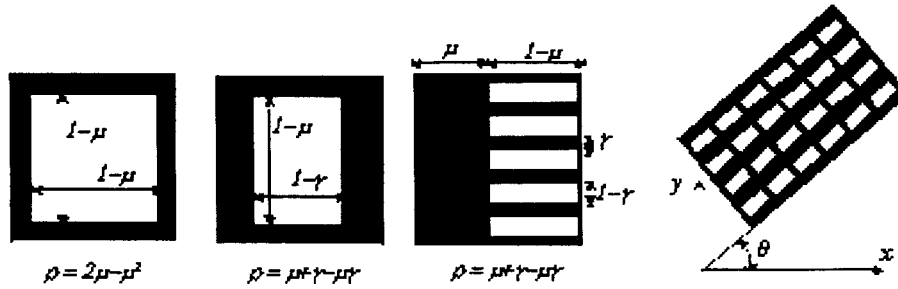


Fig. 10. Shape variables in microstructure cells.

$$L(v) = \int_{\Omega} f v d\Omega + \int_{\Gamma} t v d\Gamma \tag{19}$$

and the external work is given by:

$$a_d(w, v) = \int_{\Omega} C_{ijkl}(d) \epsilon_{ij}(w) \epsilon_{kl}(v) d\Omega \tag{20}$$

where f and t are respectively the body load and surface traction, ϵ_{ij} are the strain tensor components, C_{ijkl} is the effective stiffness of the microstructure cells and H is the set of kinematically allowable displacement field. The problem is defined on a fixed reference domain Ω and the stiffness C_{ijkl} depends on the used design variables.

For a so-called second rank layering, such as the third cell in Fig. 10, we have the relationship:

$$C_{ijkl} \equiv C_{ijkl}(\mu, \gamma, \theta) \tag{21}$$

where μ and γ denote the densities of the layering and θ is the rotation angle of the layering. The relation (21) can be computed analytically and the volume is evaluated by:

$$\text{Volume} = \int_{\Omega} (\mu + \gamma - \mu\gamma) d\Omega \tag{22}$$

Alternative microstructures such as square or rectangular holes in square cells can also be used (such as the first two cells in Fig. 10, the important feature being the possibility of having density values covering the full interval $[0, 1]$). The optimization problem can now be solved either by optimality criteria methods or by duality methods, where the advantage is to take into account the fact that the problem has just one constraint. The angle θ of layer rotation is controlled via the results on optimal rotation of orthotropic materials as presented in [5]. It turns out that this method allows for the prediction of the shape of the body and it is possible to predict placement and shape of holes in the structure. Our proposal is to introduce the reliability analysis in the classical topology optimization problem in order to obtain the optimal topology by introducing

the reliability constraints. This model is called Reliability-Based Topology Optimization (RBTO). When using this model, the resulting optimal topologies are more reliable than the resulting classical topologies for the same weight of the structures [9].

8. Numerical Applications

Two applications show the interest of CAROD system with respect to the traditional DDO procedure and the sequential optimization process including the RBDO model.

8.1. Rotating disk

The first example consists of designing an axisymmetric turbine disk. The meridian cross-section of the disk is made of four parts, as illustrated in Fig. 11(a): the hub of uniform thickness, the disk itself, the rim and the blades. The optimization problem is to find the optimal shape of the disk and the thickness of the hub that yield to minimum weight while satisfying upper limits on radial and circumferential stresses under thermal and centrifugal loads.

The thickness of the rim and attached blades are considered to be fixed with predetermined values. Due to axi-symmetry, only one half-cut of the structure is needed to be modeled. In this case, we apply the Deterministic Design Optimization (DDO) process and the CAROD system in order to show the advantage of the integration of the reliability criteria into the optimization problem. The geometric description of the

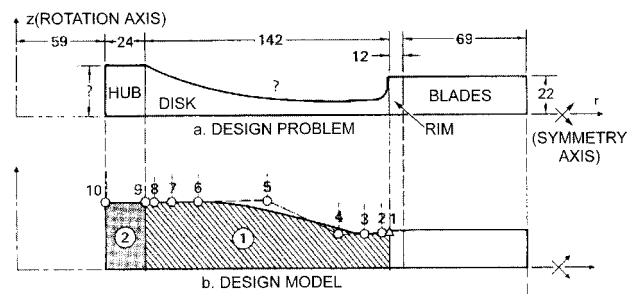


Fig. 11. Rotating disk configuration.

design model begins with defining all its boundaries and selecting appropriate parametric curves. In the present case many parts of the boundary are required to be straight segments. Each segment can be considered, for example, as a particular case of a Bézier curve. The only curved part of the boundary will be represented by a B-Spline with 6th order continuity (i.e. degree five). To control it, nine master nodes are employed, as indicated in Fig. 11(b). The second step is to subdivide the structure into design elements, which is quite obvious for this problem. As shown in Fig. 11(b), two design elements are sufficient to fully describe the moving boundaries, corresponding to the disk and its hub. The two other sub-regions, representing the rim and the blades, have a fixed geometry. Next the design variables must be selected in order to monitor the acceptable changes in the shape of the two design elements. It is to remind that the design variables provide the positions of the master nodes describing the boundary curves. In this study, the hub thickness and the disk shape have to be determined while keeping the rim and blades thickness constant. Therefore, only numbered 2 through 10 in Fig. 11(b) will be allowed to move. Finally, the fourth step consists of expressing constraints restricting the control node displacements. For example the structure may not move into the negative side of the Z-axis. Also, all the design elements must keep reasonable aspects. To facilitate the introduction of these requirements, the design variables are defined as the distances separating each moving node from its corresponding reference node. In addition the move direction of each control node is kept constant. In the present case the control nodes are required to move in the Z direction. With this definition of the design variables, the geometric requirements can be easily stated and treated by the optimization algorithm:

- 1 - the hub must have a uniform thickness: it is sufficient to impose that the displacements of nodes 9 and 10 be the same; this is a simple equality constraint between two variables, which can be linked to only one parameter before entering the optimizer;
- 2 - in order to prevent the moving nodes to penetrate the negative Z-space, the design variables are imposed to remain positive.

Having constructed a proper design model, involving only 8 independent variables, an finite element analysis model can be created. The Deterministic Design Optimization (DDO) problem is to minimize the structure volume subject to the mechanical stress constraint $\sigma_{max} - \sigma_w \leq 0$, however, the RBDO model aims to minimize the structural volume subject to the reliability constraint $\beta \geq \beta_t$, and the allowable stress one. For this problem, the target reliability level is also $\beta_t=3.8$.

Table 1 provides the difference between the Deter-

Table 1. Axisymmetric disk results

Model	DDO	CAROD
$V(\text{mm}^2)$	4.38×10^8	4.78×10^8
β	1.73	3.82
P_f	4.2×10^{-2}	6.7×10^{-5}

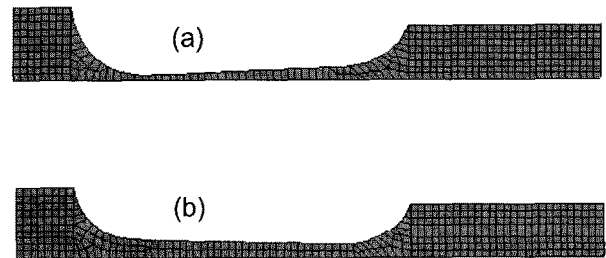


Fig. 12. (a) Optimal configuration by DDO process, (b) Optimal configuration by CAROD system.

ministic Design Optimization(DDO) and the CAROD system for the rotating disk; Fig. 12 compares the two optimal designs. Using CAROD system, we increase the structure volume (weight) by 8.4% with respect to the volume produced by the DDO model. However, this small increase will largely improve the reliability level of the structure: the failure probability P_f is reduced for 4.2% to 0.007%. So, a better ratio of reliability per unit cost is achieved.

8.2. Hook structure

To illustrate the efficiency of the proposed approach, the steel hook structure, illustrated in Fig. 13(a), is analyzed. The hook is supported at its top by a shaft in the hanging hole of radius $R2$ and the load is hanged on the lower circular arc of radius $R1$. The hook thickness

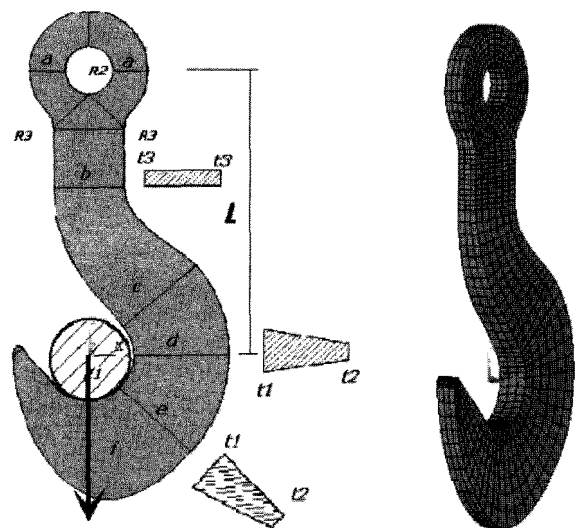


Fig. 13. Hook models.

varies linearly between inner and outer faces: trapezoidal cross-section is chosen for the lower hanging part and rectangular cross-sections are taken for the rest of the hook. For functioning considerations, the fixed dimensions are the hanging circular arc radius $R1=190$ mm, the hole radius $R2=100$ mm, the fillet radius $R3=100$ mm and the hook height $L=1200$ mm. The used material is the construction steel with Young's modulus $E=200$ GPa and allowable stress $\sigma_w=235$ MPa. The applied load is $F=400$ kN, which is distributed on the 30 contact elements at the circular. The hook is modeled by 1602 solid finite elements with 20-nodes quadratic shape functions that leads to 6200 nodes with 18600 degrees of freedom (Fig. 13(b)). In this study, the objective is to minimize the hook volume under the design constraints, whose the reliability constraint. To optimize the structure, the mean values of the dimensions $m_a, m_b, m_c, m_d, m_e, m_f$ and the thickness m_{t1}, m_{t2} and m_{t3} are the control design parameters. The external applied load F and the physical dimensions $a, b, c, d, e, f, t_1, t_2$ and t_3 are the random variables, which are supposed to be normally distributed. Table 1 gives the RBDO variables, as well as the corresponding standard-deviations and initial values. In this problem, we have 19 optimization variables: 10 random variables $\{y\}$ and 9 design variables $\{x\}$. For this design, the target reliability level is $\beta_t=3.35$ with convergence tolerance equal to 1. The equivalent maximum failure probability is $P_f=4 \times 10^{-4}$.

Sequential approach: The problem can be written in two sub-problems:

1 - optimization of the objective function:

$$\begin{aligned} \min & : f(\{x\}) \\ \text{subject to} & : \sigma_k(\{x\}) - \sigma_w \leq 0 \\ \text{and} & : \beta(\{x\}, \{u\}) > \beta_t \end{aligned} \quad (23)$$

2 - calculation of the reliability index $\beta(\{x\}, \{u\})$:

$$\begin{aligned} \min & : d(\{u\}) \\ \text{subject to} & : H(\{x\}, \{u\}) \leq 0 \end{aligned} \quad (24)$$

Concurrent approach: Using the hybrid reliability-based design model, we can simplify the two last sub-problems into one problem:

$$\begin{aligned} \min & : F(\{x\}, \{y\}) = f(\{x\}) \cdot d_\beta(\{x\}, \{y\}) \\ \text{subject to} & : \sigma_{\max}(\{x\}, \{y\}) - \sigma_w = 0 \\ & : \beta(\{x\}, \{y\}) \geq \beta_t \\ & : \sigma_k(\{x\}) - \sigma_w \leq 0 \end{aligned} \quad (25)$$

Table 2 gives the optimal solutions of the two approaches. By comparing their results, we find that the optimal solutions are very close and the reliability constraint is satisfied for the hybrid and classical models. In considering the same initial volume $V_0=0.6688 \times 10^8$ mm³ for both approaches, the classical RBDO approach requires 439 Finite Element Analyses (FEA) to reach the minimal volume $V^*=0.2373 \times 10^8$ mm³ and to satisfy the target reliability level $\beta=3.38 > \beta_t$ (i.e. 0.9% higher than the target). However, the hybrid method needs only 84 evaluations to reach the minimal volume $V^*=0.2345 \times 10^8$ mm³ and to satisfy the target reliability level $\beta=3.37 > \beta_t$ (i.e. 0.6% higher than the target).

At each deterministic iteration, the classical method needs a complete reliability analysis in order to calculate the reliability index. Furthermore, for each reliability iteration we need 10 FEA (equal to the random variables number $m=10$) that leads to a very high FEA (for this example: 7 reliability iterations for the first deterministic iteration and 3 ones for the following optimization iterations). By comparing their results, the hybrid method gives a computational time clearly reduced with respect to the classical approach (almost 80%). In addition, for each deterministic iteration, we need a gradient calculation ($n+1=10$ FEA, n is the design variables number) and one FEA for evaluating the stresses. In the hybrid RBDO procedure, as demonstrated in section 4, a gradient calculation for the design variables ($n+1=10$ FEA) and two FEA (one for the design variables and the other for the random ones) are necessary for each iteration. Table 4 gives the reduction of the FEA between the two methods,

Table 2. Initial points

Variables $\{y\}$	a	b	c	d	e	f	t_1	t_2	t_3	F
Means $\{x\}$	m_a	m_b	m_c	m_d	m_e	m_f	m_{t1}	m_{t2}	m_{t3}	400
Variances	3	2	4	4	4	4	1	1	1	20
Initial points	150	100	200	200	200	200	200	200	200	200

Table 3. Sequential and concurrent RBDO results

$\{x\}$	m_a	m_b	m_c	m_d	m_e	m_f	m_{t1}	m_{t2}	m_{t3}	400
Seq. RBDO	111.0	80.56	196.5	200.6	196.1	154.7	31.6	10.42	10	--
Con. RBDO	110.7	80.00	198.2	198.2	198.1	151.6	27.8	13.06	10	--
$\{y\}$	a^*	b^*	c^*	d^*	e^*	F^*	t_1^*	t_2^*	t_3^*	F^*
Seq. RBDO	111.0	80.67	195.83	196.66	195.13	154.8	30.7	9.35	10	451
Con. RBDO	110.1	79.50	198.05	198.04	197.97	152.5	27.6	10	10	427

Table 4. Efficiency comparison

Model	Seq. RBDO	Con. RBDO
$V(\text{mm}^3)$	23.73×10^6	23.45×10^6
β	2.38	2.37
n_{det}	9	7
n_{fiab}	$3 \times 9 + 7$	0
n_{calls}	439	84

where n_{det} and n_{rel} are the number of deterministic and reliability iterations, respectively, and n_{calls} is the number of finite element analyses.

The results show that the concurrent method clearly reduces the computational time particularly for large-scale problems. The steel hook problem shows the efficiency of the concurrent RBDO including FE-CAD models with respect to the sequential one. In the proposed formulation, the integration of the reliability does not represent a significant increase of computational time but makes it very reasonable.

9. Conclusion

CAROD system is an efficient computer-aided tool, integrating reliability analysis and concurrent engineering concepts in the classical design phase. The first application (rotating disk) shows that when using the sequential DDO procedure, the reliability level of the deterministic point is very low with respect to the target reliability level. However, using CAROD, the solution respects the required reliability level but with a small increase of volume. The second application (steel hook structure) demonstrates the efficiency of the concurrent RBDO including FE-CAD models with respect to the sequential one. Furthermore, the efficiency is confirmed in section 5.2. This new system also allows us to use the concurrent engineering concept as a practical tool to give all information during the design process. The coupling of different models is ensured by adapted protocol allowing the exchange of variable states and sensitivities. CAROD system is appropriate to solve complex engineering problems by supplying the designers with all information in the design process.

References

[1] Der Kiureghian, A. and Polak, E. (1998), Reliability-

- based optimal structural design: A Decoupled approach. In: Andrzej S. Nowak (ed.), Reliability and Optimization of Structural Systems, 197-205.
- [2] Ditlevsen, O. and Madsen, H. (1996), *Structural Reliability Methods*. In: John Wiley & Sons.
- [3] Gheng, G., Li, G. and Cai, Y. (1998), Reliability-based structural optimization under hazard loads. *Structural Optimization* 16(2-3), (1998), 128-135.
- [4] Hasofer, A.M. and Lind, N.C. (1974), An Exact and Invariant First Order Reliability Format. *J. Eng. Mech., ASCE, EMI*, 100, 111-121.
- [5] Hassani, B. and Hinton, E. (1999), *Homogenization and Structural Topology Optimization*, Springer-Verlag, New York.
- [6] Haug, E.J., Choi, K.K. and Komkov, V. (1986), *Design Sensitivity Analysis of Structural Systems*, Academic Press, New York.
- [7] Jagannathan, V., Cleetus, K.J., Kannan, R., Matsumoto, A.S. and Lewis, J.W. (1991), Computer Support for Concurrent Engineering: Four Strategic Initiatives. *Concurrent Engineering*, September/October, 14-30.
- [8] Kharmanda, G., Mohamed, A. and Lemaire, M. (June 2001), New hybrid formulation for reliability-based optimization of structures. The Fourth World Congress of Structural and Multidisciplinary Optimization (WCSMO-4), Dalian, China.
- [9] Kharmanda, G. and Olhoff, N. (2001), Reliability-Based Topology Optimization, Report N. 110, Institute of Mechanical Engineering, Aalborg University, Denmark.
- [10] Lee, K. (1999), *Principles of CAD/CAM/CAE Systems*, Addison-Wesley.
- [11] Lemaire, M. and Mohamed, A. (September 2000), Finite Element and Reliability: A Happy Marriage? In: Nowak, A. and Szerszen, M. (eds), Reliability and Optimization of Structural Systems, (185), 3-14.
- [12] Madsen, H.O. and Friis Hansen, P. (1991), Comparison of some algorithms for reliability-based structural optimization and sensitivity analysis. In: Brebbia, C.A. and Orszag, S.A. (eds), Reliability and Optimization of Structural Systems, 443-451.
- [13] Marsh, D. (1999), *Applied Geometry for Computer Graphics and CAD*, Springer, London.
- [14] Mohamed, A. and Lemaire, M. (1998), The use of sensitivity operators in the reliability analysis of structures. In: Spanos, P.D. (ed.): Computational Stochastic Mechanics, Greece, 211-220.
- [15] Piegl, L. and Tiller, W. (1995), *The NURBS Book*, Springer-Verlag, USA.
- [16] Tu, J., Choi K.K. and Park, Y.H. (1999), A new study on reliability-based design optimization. *Journal of Mechanical Design*, 121, 557-564.
- [17] Wu, K.C. (1995), Multi-Criterion Preliminary Design of a Tetrahedral Truss Platform, presented at the 36th AIAA/ASME/ASCE/AHE/ASC Structures.