Journal of the Korean Society of Water and Wastewater
Vol. 16, No. 6, pp. 710-716, 2002

상하수도학회지. 논문
16권 6호. pp. 710-716. 2002

새로운 파괴예측 모델을 이용한 상수도 관의 최적 교체

Optimal Pipe Replacement Analysis with a New Pipe Break Prediction Model

박수완[1,*] · G.V. Loganathan[2]

Park, Suwan[1,*] · G.V. Loganathan[2]

1 동서대학교 건설공학부
2 Dept. of Civil and Environmental Engineering, Virginia Tech, USA

(2002년 8월 13일 접수, 2002년 11월 25일 최종수정논문 채택)

Abstract

A General Pipe Break Prediction Model that incorporates linear and exponential models in its form is developed. The model is capable of fitting pipe break trends that have linear, exponential or in between of linear and exponential trend by using a weighting factor. The weighting factor is adjusted to obtain a best model that minimizes the sum of squared errors of the model. The model essentially plots a best curve (or a line) passing through "cumulative number of pipe breaks" versus "break times since installation of a pipe" data points. Therefore, it prevents over-predicting future number of pipe breaks compared to the conventional exponential model. The optimal replacement time equation is derived by using the Threshold Break Rate equation by Loganathan et al. (2002).

Key words: Pipe Break Prediction Model, Optimal Replacement, Threshold Break Rate
주제어: 상수도 관의 파괴 예측 모델, 최적 교체, 한계 파괴율

1. Introduction

In this paper a new mathematical model to predict pipe breaks — prediction of the number of breaks in the future time of a pipe — in water distribution systems is developed. The model uses past break history — break times versus number of breaks — of a pipe. It is coupled with the optimal threshold break rate estimator developed by Loganathan et al. (2002) to obtain the optimal replacement time of a pipe. It is composed of linear and exponential prediction models and determines the dominance of past break trends — linear, exponential or in between — by using a weighting factor. Thus, the model is coined as a "General Pipe Break Prediction Model" to reflect its capability of fitting any break trends. As a result, the tendency of a

*Corresponding author Tel.: +82-51-320-1830, Fax: +82-51-320-1818, E-mail: spark@dongseo.ac.kr (Park, S.W.)

상하수도학회지, 논문
16권 6호, pp. 710-716, 2002

pipe break trend can fall into one of the three categories: (1) linear, (2) exponential, and (3) somewhere between the linear and exponential break trend. In the past the pipe break trend had taken to be exponential [Shamir and Howard (1979), Walski and Pelliccia (1982), and Clark et al. (1982)]. The newly developed model overcomes the shortcomings of the exponential model, which is shown in the subsequent sections.

2. Literature Review

Shamir and Howard (1979) applied regression analysis to obtain a relationship for the breakage rate of a pipe as a function of time. This relationship, which is exponential, was used to find the optimal timing of pipe replacement to minimize the total cost of repair and replacement. It is clear that any error in the predictive model will alter the replacement time significantly. Walski and Pelliccia (1982) subscribed to the idea of the threshold break rate. They adopted Shamir and Howard's (1979) model for predicting break rates. They derived an optimal replacement time estimator by setting the total repair costs to be equal to the replacement cost. It is not clear why such a criterion for replacement is valid. Clark et al. (1982) suggested a model that combines two equations, one to predict the time elapsed until the first break occurred and the second to predict the number of subsequent breaks which were assumed to grow exponentially over time in an attempt to account for the relative impacts of various external agents. These equations had coefficients of determination (R^2) of 0.23 and 0.47, respectively. While Clark et al.'s (1982) procedure is a significant improvement in predicting pipe breaks, it does raise some concerns because of the low values of the coefficients of determination. Clark et al. (1982) have made the following observations: only a subset of pipes have recurrent repairs; the time to first repair is quite long, typically about fifteen years; the time between repairs becomes shorter as pipes get older; large diameter pipes tend to have fewer problems; and industrial development in general results in more repairs.

As can be found in the literature the mathematical model for the pipe failure prediction has been mostly based on the exponential model. However, it is conceivable that the break trend of a pipe can show a trend anywhere between the exponential and linear considering randomness of pipe failure. Furthermore, a case study for a large city area in the United States with 4,000 miles of pipes and 32,242 recorded break incidents confirmed the intermediate of linear and exponential break trend mentioned above. Therefore, a pipe break trend model that can span from the linear to exponential is formulated in this paper.

3. General Pipe Break Prediction Model

The accuracy of the fitted model plays a major role in the curve fitting approach for the prediction of future pipe breaks and subsequent timing of replacement of water mains. Poorly fitted models produce either unrealistically high or low number of breaks for the future years resulting in too early or late replacement. In this section a new model to better fit water main break data is developed, and the model is used with the Threshold Break Rate to obtain the optimal replacement time.

The exponential model used by Shamir and Howard (1979) tends to over-predict the number of future breaks of water mains when the history of pipe breaks does not show an exponential trend. In the General Pipe Break Prediction Model described in this section, a weighting factor is utilized to moderate the dominance of either exponential or linear model in prediction of breaks for a pipe. The General Pipe Break Prediction Model offers two advantages over the exponential model: (1) it provides a better fit for the pipe break history and better predicts the future number of breaks; (2) it is a break prediction model based on a cumulative number of breaks for a stretch of a pipeline defined. Shamir and Howard (1979) used "number of beaks in a year per 1,000 ft length of pipe." Considering the break patterns of a real pipeline underground, this approach may not generate satisfactory results since some pipelines simply don't fail frequently even though the length is great. Therefore, the break prediction model presented here incorporates cumulative number of breaks since installation.

The exponential part of the General Pipe Break Prediction Model has the following form:

$$N_c(t) = B\_exp \cdot e^{A\_exp(t-t_0)} \qquad (3.1)$$

where

$N_c(t)$ = cumulative number of breaks along the length of a defined pipe in year t

t = time in years

t_0 = base year for the analysis (pipe installation year, or the first year for which data are available)

A_exp = growth rate coefficient (1/year)

B_exp = coefficient of regression

The linear part of the General Pipe Break Prediction Model has following form:

$$N_c(t) = B\_lin + A\_lin(t - t_0) \qquad (3.2)$$

where

$N_c(t)$ = cumulative number of breaks along the length of a pipe in year t

t = time in years

t_0 = base year for the analysis (pipe installation year, or the first year for which data are available)

A_lin = growth rate coefficient (1/year)

B_lin = coefficient of regression

The General Pipe Break Prediction Model has the following form:

$$N_c(t) = (1 - WF)(B\_lin + A\_lin(t - t_0)) + WF \cdot B\_exp \cdot exp(A\_exp(t - t_0)) \qquad (3.3)$$

where

$N_c(t)$ = Cumulative number of breaks along the length of a pipe in year t

t = Time in years

t_0 = Base year for the analysis (pipe installation year, or the first year for which data are available)

WF = Weighting factor to determine the best model for the data given,

A_lin and B_lin = Coefficients of the linear model

A_exp and B_exp = Coefficients of the exponential model

The coefficients (A_lin, B_lin, A_exp, and B_exp) are determined from a curve fitting analysis. The weighting factor is determined between 0 and 1 that would result in the least sum of squared relative errors. The computational steps to determine the weighting factor, WF, is as follows:

First, the value of WF is set to 0, which represents the linear model, and the sum of squared errors is computed for the later 1/3 portion of the data to enhance the predictability of break events of the model. Then, WF is increased by some increment, ε, and the sum of squared errors is obtained. The optimal weighting factor is the one with least sum of squared errors. The procedure can be mathematically represented as follows:

$$\text{Minimize } SSE = \sum_{j=1}^{n}(O_j - C_j)^2 \qquad (3.4)$$

Subject to:

$C_j = (1 - \omega_i)L(t) + \omega_i E(t)$

$\omega_i = i \cdot \varepsilon$

where i = {0, 1, ..., $1/\varepsilon$}

$1/\varepsilon$ is an integer and $0 < \varepsilon < 1$

SSE = sum of squared errors for each i

O_j = each observed break time

C_j = computed value from the General Pipe Break Prediction Model for each j

ω_i = weighting factor for each i

$E(t)$ = the exponential model (Eq. (3.1))

상하수도학회지, 논문
16권 6호. pp. 710-716. 2002

$L(t)$ = the linear model (Eq. (3.2))
n = number of breaks

Fig. 1 illustrates the procedures of building the General Pipe Break Prediction Model from pipe break data.

4. Optimal Replacement Time by Using the General Pipe Break Prediction Model

The General Pipe Break Prediction Model (Eq. (3.3)) can be used to determine the optimal replacement time of a pipe by considering the equivalence relationship with the Threshold Break Rate developed by Loganathan et al. (2002). According to them,

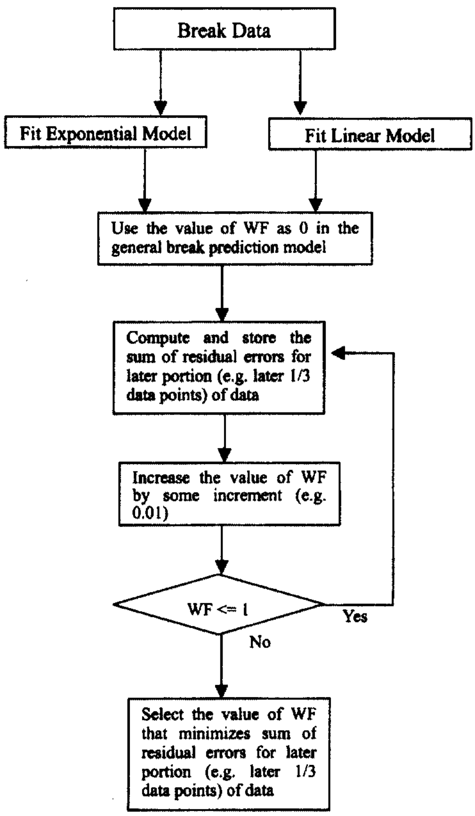

Fig. 1. General pipe break prediction model building process.

$$\frac{dN_c(t)}{dt} = Brk_{tb} \tag{4.1}$$

where the left-hand side of Eq. (4.1) is the pipe breakage rate at time t and the right-hand side of Eq. (4.1) represents the Threshold Break Rate of a pipe, which is

$$Brk_{tb} = \frac{\ln(1+R)}{\ln(1+\frac{C}{F_l \cdot L})} \tag{4.2}$$

where F_l = replacement cost per unit length of a pipe ($/ft)
L = the length of a pipe (ft)
C = repair cost of a pipe
and R = discount ratio.

Therefore, the optimal replacement time of a pipe can be determined by taking a derivative of Eq. (3.3) with respect to time t and substituting it to Eq. (4.1). The optimal replacement time is obtained as

$$t^* = \frac{1}{A\_exp} \ln\left(\frac{Brk_{tb} - (1-WF) \cdot A\_lin}{WF \cdot A\_exp \cdot B\_exp \cdot e^{-A\_exp \cdot t_0}}\right) \tag{4.3}$$

Example

By using the recorded historic break times of a pipe, obtain the General Pipe Break Prediction Model and the optimal replacement time. Use the interest rate as 7% per year.

Solution

The properties of a pipe is obtained from a break database as follows:

PIPE ID	Installation Year (year)	Length (ft)	ICPF ($/ft)	RCPB ($)	Brkth (breaks/year)
14449-1952-CI-6	1952	1363.5	92.77	2814	3.08

This pipe is a cast-iron 6 inch pipe with a length of 1363.5 ft. ICPF is the Installation Cost Per Foot, and RCPB means the Repair Cost Per Break of a pipe. The Threshold Break Rate (Brkth) is obtained by using Eq. (4.2). The recorded historic break times of the pipe, 14449-1952-CI-6, obtained from the break database are as follows;

MTB(1)	MTB(2)	MTB(3)	MTB(4)
277	361	373	426
MTB(5)	MTB(6)	MTB(7)	MTB(8)
437	469	480	546

The value in the parentheses after MTB (months to break from installation time) provides the cumulative number of breaks at each MTB and the order of breaks.

First, the entire data (break times and corresponding cumulative number of breaks) are fitted to the linear model, that is

$$y_i = A\_lin + B\_lin \cdot x_i + \varepsilon_i, \qquad i = 1, 2, \ldots, n$$

where
y_i = the cumulative number of breaks at ith break
x = the time of ith break (year)
ε = error at ith break
and n = total number of breaks of a pipe

The method of least squares is used to estimate A_lin and B_lin. Estimates of A_lin and B_lin are obtained by solving equation

$$A\_lin = \bar{y} - B\_lin \cdot \bar{x}. \qquad (4.4)$$

where

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \text{ and } \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i.$$

An estimate of B_lin is obtained by solving equation

$$B\_lin = \frac{S_{xy}}{S_{xx}}$$

where

$$S_{xx} = \sum_{i=1}^{n}(x_i - \bar{x})^2 \text{ and } S_{xy} = \sum_{i=1}^{n} y_i(x_i - \bar{x})^2.$$

The ith residual error is

$$\varepsilon_i = y_i - (A\_lin + B\_lin\, x_i), \qquad i = 1, 2, \ldots, n$$

The error sum of squares is expressed as

$$SS_E = \sum_{i=1}^{n} \varepsilon_i^2 = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2.$$

The estimated values of A_lin and B_lin for the pipe, 14449-1952-CI-6, are

$$A\_lin = 0.378 \text{ and } B\_lin = -8.65.$$

Second, the entire data is fitted to the exponential model, that is

$$y_i = B\_exp \cdot x_i^{A\_exp} + \varepsilon_i, \qquad i = 1, 2, \ldots, n$$

By taking natural logarithms to both sides we obtain a linear model, that is

$$\ln(y_i) = \ln(B\_exp) + A\_exp \ln(x_i).$$

Let $\ln(B\_exp) = a$ and $A\_exp = b$, then the estimates of B_exp and A_exp are obtained by solving

$$a = \ln y - b \cdot \ln x. \qquad (4.5)$$

Therefore, by substituting x for $\ln x$ and y for $\ln y$ in Eq. (4.5) and from Eq. (4.4) we obtain the estimates of A_exp and B_exp as

$A\_exp = 0.117$ and $B\_exp = 0.0676$.

Third, the error sum of squares in later 1/3 of break data points (e.g. calculate the residual error starting from 6[th] break if a pipe has total 9 breaks) are calculated by varying the weighting factor (e.g. from 0 to 1 by an increment of 0.01) in the General Pipe Break Prediction Model. The General Pipe Break Prediction Model for the pipe by using the estimated coefficients are expressed as

$$y_i = (1 - wf)(A\_lin + B\_lin \cdot x_i) + wf \cdot B\_exp \cdot e^{A\_exp \cdot x_i}$$
$$= (1 - wf)(0.378 - 8.65 \cdot x_i) + wf \cdot 0.0676 \cdot e^{0.117 \cdot x_i}$$

where i = 6, 7, 8. Since n = 8 for the pipe, $8/3 \cong 3$. Therefore, take the later 3 break data and calculate the sum of squared errors for $x_6 = 469$, $x_7 = 480$, and $x_8 = 546$. The lowest value of the sum of the squared errors is calculated as 0.2957 when the weighting factor is 0.47. Therefore, the General Pipe Break Prediction Model for the pipe is

$$y = 0.53(0.378 - 8.65 \cdot x) + 0.32 \cdot e^{0.117 \cdot x}$$

The optimal replacement time of the pipe is obtained by using Eq. (4.3), that is

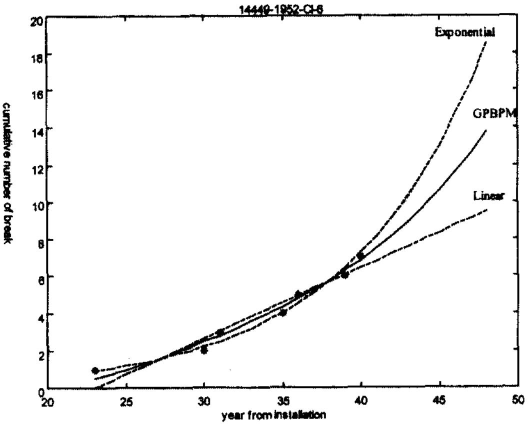

Fig. 2. Break versus time plots of the GPBPM, exponential and linear models.

$$t_1^* = \frac{1}{A\_exp} \ln\left(\frac{Brk_{th} - (1 - WF)A\_lin}{WF * A\_exp * B\_exp * e^{-A\_exp * t_0}}\right)$$
$$= \frac{1}{0.117} \ln\left(\frac{3.08 - (1 - 0.47)0.378}{0.47 \times 0.117 \times 0.0676 \times e^{-0.117 * 0}}\right)$$
$$\cong 57$$

Since installation year is used as time '0' in this calculation, actual replacement year is expressed as $\underline{t}^*$ = [installation year] + t_1^* = 1952 + 57 = 2009.

Fig. 2 shows the plot of the resulting General Pipe Break Prediction Model (GPBPM) of the pipe. It also shows the plots of the exponential and the linear model fitted to the break data.

5. Summary

A new pipe break prediction model, which is coined as a "General Pipe Break Prediction Model" that can accommodate linear, exponential or in-between of linear and exponential break trend is developed in this paper. The model overcomes the shortcomings of the exponential model found in the literature which can over-predict future pipe break incidents. The model is used with the Threshold Break (Loganathan et al., 2002) to obtain the optimal replacement time equation (Eq. (4.3)) of a pipe. Detailed model building processes of the General Pipe Break Prediction Model is provided with an example of optimal replacement time analysis using the Threshold Break Rate. The newly developed model and replacement analysis method is expected to contribute to the reduction of pipe maintenance costs in water distribution systems.

References

Clark, R.M., C.L. Stafford, and J.A. Goodrich (1982) Water distribution systems: A spatial and cost evaluation, *Journal of Water Resources Planning and Management, ASCE*, **108**(3), pp. 243-256.

Loganathan, G..V., S. Park, and H.D. Sherali (2002) A Threshold Break Rate for Pipeline Replacement in Water Distribution Systems, *Journal of Water Resources*

Planning and Management, ASCE, **128**(4), pp. 271-279.

Shamir, U., and C.D.D. Howard (1979) An analytic approach to scheduling pipe replacement, *Journal of American Water Works Association,* **71**(5), pp. 248-258.

Walski, T.M., and A. Pelliccia (1982) Economic Analysis of Water Main Breaks, *Journal of American Water Works Association,* **74**(3), pp. 140-147.