Hybrid Vibration Control of Smart Laminated Composite Beams using Piezoelectric and Viscoelastic Material

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ABSTRACT

Active control of flexural vibrations of smart laminated composite beams has been carried out using piezoceramic sensor/actuator and viscoelastic material. The beams with passive constrained layer damping have been analyzed by formulating the equations of motion through the use of extended Hamilton's principle. The dynamic characteristics such as damping ratio and modal damping of the beam are calculated for various fiber orientations by means of iterative complex eigensolution method. This paper addresses a design strategy of laminated composite under flexural vibrations to design structure with maximum possible damping capacity.

Keywords : Hybrid vibration control, Dynamic characteristics, Piezoelectric sensor/actuator, Laminated composite beam, Passive constrained layer damping treatment (PCLD), Viscoelasticity

Nomenclatures

$\mathbf{D}$ = flexural stiffness matrix
D_z = electric displacement
$\mathbf{K}_D$ = damped stiffness matrix
$\mathbf{u}$ = displacement vector
w = transverse displacement
ε = strain
γ = shear strain
κ = curvature vector
ζ = damping ratio
$2\zeta\omega$ = modal damping
φ = specific damping capacity

1. Introduction

Over the past decade, there has been an increasing interest in the development of lightweight smart or intelligent structures for various applications, especially for vibration control of flexible structures. The composite materials are commonly used in these structures for higher strength-to-weight and stiffness-to-weight ratios. There are many papers on control of structural vibrations using passive constrained layer damping (PCLD) of viscoelastic(VE) materials[1,2]. Recently, the active constrained layer damping (ACLD) [3,4] has been very actively investigated by replacing the conventional constraining layer by the smart material such as PZT. The piezoceramic sensors/actuators[5,6] have been widely used in various research and applications such as structural vibration control since piezoceramic elements have demonstrated competitive characteristics, such as light weight, small size, and good dynamic performance. In the case of the piezoceramic, depending on the applied voltage to the piezoceramic, electromechanical coupling of the forcing transducer to the structure, and the location of the piezoelectric transducers, a degree of vibration control of flexible structures can be considered. There are many papers that have numerically or experimentally dealt with the active vibration control of plates or beams using piezoelectric materials. However, most of the research using the PZT and viscoelastic materials has been carried out separately, and the relation between active control by PZT and passive control by tailoring and VE has not been discussed in detail. In this paper, vibration control of laminated composite beam is carried out using PZT and viscoelastic patches under

PCLD(passive constrained layer damping) to address a design strategy of flexible structure with maximum possible damping capacity. Also, the interaction between the passive structural control such as tailoring and PCLD and the active control using PZT is investigated numerically and experimentally.

2. Method

Laminated composite beams with the piezoceramic sensor/actuator and viscoelastic materials are modeled as 2-D plates. Extended Hamilton's principle is used to derive the equation of motion for the beam. The piezoceramic sensor/actuator layers and the adhesive layers are treated as another layer with different material properties in deriving the kinetic and the potential energy. Extended Hamilton's principle states that

$$\delta\int_{t_1}^{t_2}[T-U+U_v+W]dt=0$$

where T is the kinetic energy, U is the total potential energy stored in the beam undergoing induced strain actuation, U_v is the shear deformation energy of viscoelastic layer, and W is the work done by the external forces. The beam with the piezoceramic sensor/actuator and viscoelastic patch is shown in Fig. 1.

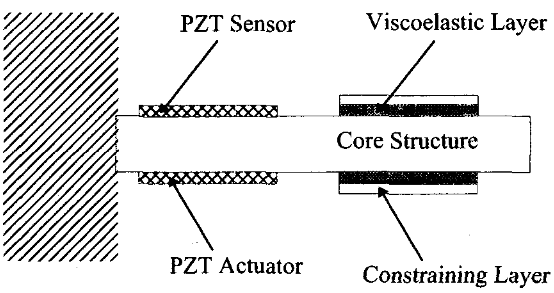

Fig. 1 Laminated composite beam with piezoceramic sensor/actuator and PCLD

When the beam is not very thin and not very long, the vibrational response is transmitted dominantly in flexural type motion and the vibration flow in wave type motion can be ignored. Thus the in-plane displacements can be ignored when only the transverse vibration is considered. Therefore, the displacement vector, **u**, and the strain vector, ε, can be expressed as follows :

$$\boldsymbol{u}=\begin{bmatrix}u_x\\u_y\\u_z\end{bmatrix}=\begin{bmatrix}-z\dfrac{\partial w}{\partial x}\\-z\dfrac{\partial w}{\partial y}\\w\end{bmatrix}=\begin{bmatrix}z\beta_x\\z\beta_y\\w\end{bmatrix}\tag{1}$$

$$\varepsilon=\begin{bmatrix}\varepsilon_x\\\varepsilon_y\\\gamma_{xy}\end{bmatrix}=\begin{bmatrix}z\dfrac{\partial\beta_x}{\partial x}\\z\dfrac{\partial\beta_y}{\partial y}\\z\dfrac{\partial\beta_y}{\partial x}+z\dfrac{\partial\beta_x}{\partial y}\end{bmatrix}=z\kappa\tag{2}$$

where w is the transverse displacement, β_x and β_y are the rotation about the x- and y-axis, respectively.

Shear deformation angle of viscoelastic layer under flexural vibration is expressed as follows:

$$\gamma_{xz}=\frac{\partial w}{\partial x}+\frac{\partial u_x}{\partial z}=\frac{\frac{t_D}{2}}{t_v}\left(\frac{\partial w}{\partial x}+\frac{u_{xc}-u_{xs}}{\frac{t_D}{2}}\right)\tag{3a}$$

$$\gamma_{yz}=\frac{\partial w}{\partial y}+\frac{\partial u_y}{\partial z}=\frac{\frac{t_D}{2}}{t_v}\left(\frac{\partial w}{\partial y}+\frac{u_{yc}-u_{ys}}{\frac{t_D}{2}}\right)\tag{3b}$$

where $t_D=t_s+2t_v+t_c$, and subscripts s, v, c represent the host structure, viscoelastic layer, and constraining layer, respectively.

From the relation between shear stress and shear deformation angle, the shear deformation energy of the viscoelastic layer is express as follows:

$$U_v=2\int_{V_v}\frac{1}{2}G_v\begin{Bmatrix}\gamma_{zx} & \gamma_{zy}\end{Bmatrix}\begin{bmatrix}\gamma_{zx}\\\gamma_{zy}\end{bmatrix}dV_v\tag{4}$$

where G_v is the complex shear modulus of the employed viscoelastic material.

The piezoelectric sensor equation for plate shape can be derived from the direct piezoelectric equations. Since no external electric field is applied to the sensor layer, the electric displacement developed on the sensor surface is directly proportional to the strain acting on the sensor at that surface of the structure. As for the plate shape sensor with poling direction 3, the electric displacement in directions 1 and 2 and the strains concerned with direction 3 are ignored. Therefore, the electric displacement in direction 3 is expressed as follows.

$$D_z = e_{31}\varepsilon_x + e_{32}\varepsilon_y + e_{36}\gamma_{xy} \tag{5}$$

The total charge developed on the piezoceramic sensor is as follows:

$$\Gamma(t) = \int_{A^s} (e_{31}\varepsilon_x + e_{32}\varepsilon_y + e_{36}\gamma_{xy})dA \tag{6}$$

where A^s is the area of the piezoceramic sensor layer.

The in-plane strain in the piezoceramic actuator can be derived from the converse piezoelectric equation. Since no stress field is applied to the actuator layer, the strains induced by the electric field on the actuator layer are expressed as

$$\varepsilon^a = \mathbf{c}^{-1}\mathbf{e}^T\mathbf{E} = \mathbf{dE} \tag{7}$$

The plate load-deformation relations can be found by substituting strain into the stress-strain relation and integrating through the thickness t of the plate. The equivalent actuator moments M^a per unit length are

$$M^a = \int_{t_p} \mathbf{c}\varepsilon^a z \, dz \tag{8}$$

When the voltage u_c with the thickness of actuator t_p is applied across the thickness direction only, the electric field can be expressed as follows:

$$E = \begin{Bmatrix} 0 & 0 & \dfrac{u_c}{t_p} \end{Bmatrix}^T \tag{9}$$

The equivalent actuator moments M^a in terms of control voltage u_c can be written as follows:

$$M^a = Lu_c \tag{10}$$

where $L_i = c_{ij}d_{3j}\bar{z}^a$ (11)

, and $\bar{z}^a$ is the z coordinate of the mid-plane of the piezoceramic actuator. The work done by the active control force induced by the piezoceramic actuator is expressed as

$$W^c = \int_{A^a} \kappa^T M^a dA = \int_{A^a} \kappa^T L \, dA \, u_c \tag{12}$$

The displacement $\mathbf{u}$ and the curvature κ are expressed in terms of nodal displacements with shape functions using the 4-node, 12-degree-of-freedom quadrilateral plate bending element. Variational calculus yields the following equation of motion.

$$\mathbf{M\ddot{q}} + \mathbf{Kq} = \mathbf{F}_{\text{Ext}} + \mathbf{D_a u_c} \tag{13}$$

In Equation (13), $\mathbf{D_a}$ is the actuator influence matrix.

The damping property of composite materials exhibits anisotropic characteristics and can be controlled by changing the fiber orientations and the stacking sequences. In this paper, the damping analysis of laminated composite beams is carried out using the concept of specific damping capacity(SDC). The specific damping capacity is defined as follows.

$$\varphi = \Delta U/U \tag{14}$$

where ΔU is the energy dissipated during a stress cycle and U is the maximum strain energy during a stress cycle. When ΔU is discretized and expressed in nodal variables, the SDC, φ, can be rewritten as

$$\varphi = \frac{\mathbf{q}^T(\mathbf{K}_D + \mathbf{K}_{v_D})\,\mathbf{q}}{\mathbf{q}^T(\mathbf{K} + \mathbf{K}_v)\mathbf{q}} \tag{15}$$

where $\mathbf{K_D}$ is the damped stiffness matrix for elastic parts, $\mathbf{K}_v$ is the shear stiffness of the viscoelastic layers, and $\mathbf{K}_{v_D}$ is the damped shear stiffness of the viscoelastic layers.

When the beams are controlled by hybrid method, the dynamic characteristics such as damping ratio and modal damping of the beams with viscoelastic materials are calculated for various fiber orientations by means of iterative complex eigensolution method.

3. Results and Discussion

A simple test has been carried out to verify the finite element formulation for the passive constrained-layer damping, using a cantilevered aluminum beam of 200 x 20 x 2mm. The viscoelastic material ISD-112 of 50 x 20 x 0.127mm is located at 75mm to 125mm, with an aluminum constraining layer of 50 x 20 x 0.2mm. Table 1 shows the results for damping ratios and damped natural frequencies. From the results, it can be seen that the FE formulation has been verified experimentally.

Table 1 Comparison between finite element analysis and experiment

	FEM	Exp
1^{st} bending frequency(Hz)	40.904	39.50
1^{st} bending damping ratio	0.004685	0.005266
2^{nd} bending frequency(Hz)	255.03	251.0
2^{nd} bending damping ratio	0.02642	0.02743

Vibration control of the carbon/epoxy laminated composite beams using the piezoceramic sensor/actuator and viscoelastic material is investigated with the stacking sequence of $[\theta_4/0_2/90_2]_s$, where θ = 0, 15, 30, 45, 60, 75 and 90 deg. Specimen is made of the carbon prepreg(CU125NS). Thickness of the carbon prepreg is 0.125mm and size of the specimen is 230 x 20 x 2mm.

Table 2 Mechanical Properties of Carbon/Epoxy laminates (CU125NS)

Property	Symbol	Value
Young's modulus(0 deg)	E_1	114.7 x 10^9 Pa
Young's modulus(90 deg)	E_2	7.589 x 10^9 Pa
Shear Modulus	G_{12}	4.77 x 10^9 Pa
Poisson's ratio	ν_{12}	0.28
Volume density	ρ	1510 kg/m^3
Damping capacity(0deg)	φ_{S1}	0.013966
Damping capacity(90deg)	φ_{S2}	0.049120
Damping capacity(Shear)	φ_{S12}	0.074344

The material properties of composites are listed in Table 2.

The finite element model of the beam consists of 46 elements with 72 nodes. The size of piezoceramic in Fig. 1 is 50 x 20 x 0.5mm, and the center of piezoceramic is 35mm from the clamping point. The material properties of PZT are listed in reference 6. Collocated piezoceramic sensor/actuator functions to control the first and second modal vibrations simultaneously and the control technique is implemented on a digital system. The viscoelastic material used is ISD-112 from 3-M, and its size is 50 x 20 x 0.127mm. The center of VE is located at 115mm from the clamping point, to suppress the second bending mode effectively with the passive constrained-layer damping technique. And the size of constraining layer made of aluminum is 50 x 20 x 0.254mm.

The damping and the stiffness of the beams are controlled passively by changing the fiber orientation in the outer layer. Fig. 2 shows the passive damping ratio(ζ) of the first bending and the second bending modes of the beams with PCLD only. The flexible specimen has larger damping ratio than the specimen with higher stiffness for the two modes. As Figure 2 shows, the second mode has been suppressed effectively with PCLD due to the location of viscoelastic and constraining layer. Also one can notice that the second mode is very sensitive to the fiber orientation

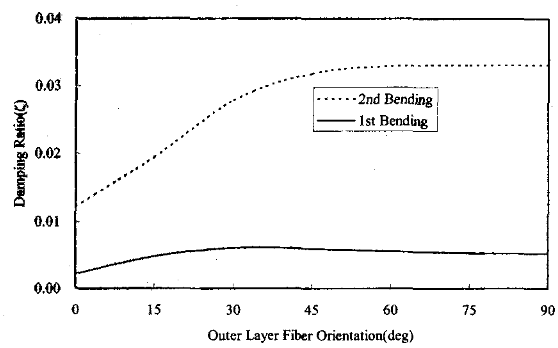

Fig. 2 Damping ratio of laminated composite beams with PCLD only

The modal damping ($2\zeta\omega$) which takes into account the damping ratio and the natural frequency at the same time, is also dependent upon the location and size of the PCLD. Fig. 3 and Fig. 4 show the modal damping of the first and second bending modes of the beam for various fiber orientations. Although the flexible specimen of 90 deg has better damping ratio than the stiff specimen of 0 deg, the 15 deg and 30 deg specimens have the best modal damping for the first and second mode, respectively.

Piezoceramic is used to provide active damping. The center of the PZT is located at 35mm from the clamping point, and the concept of structural damping index (SDI)[6] is used for the placement of PZT. Fig. 5 shows the active modal damping ($2\zeta\omega$) of the first bending mode, with negative velocity feedback control using the piezoceramic only. The beam with larger bending stiffness is very effective in vibration control by only using the PZT.

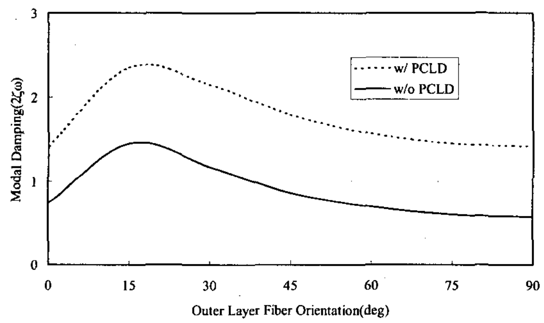

Fig. 3 1st bending modal damping of laminated composite beam with PCLD only

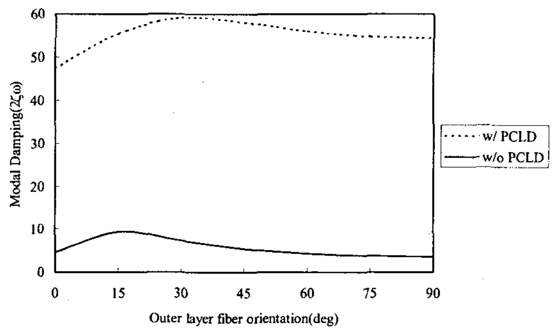

Fig. 4 2nd bending modal damping of laminated composite beam with PCLD only

Fig. 6 and Fig. 7 show the active bending modal damping of the laminated composite beams with hybrid control using PZT and PCLD. As Fig. 6 shows, the beam with higher bending stiffness has good performance for the first bending mode as the feedback gain increases. However for the second mode, the beam with 15 deg fiber orientation has the best dynamic characteristics as the gain increases.

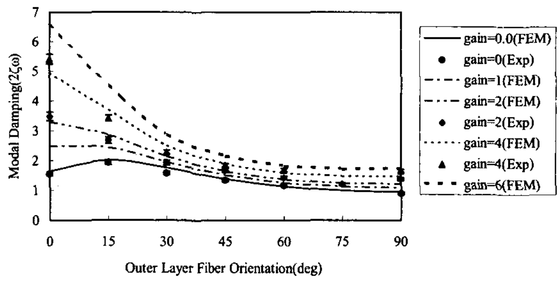

Fig. 5 1st bending modal damping of laminated composite beam with PZT Sensor/Actuator only

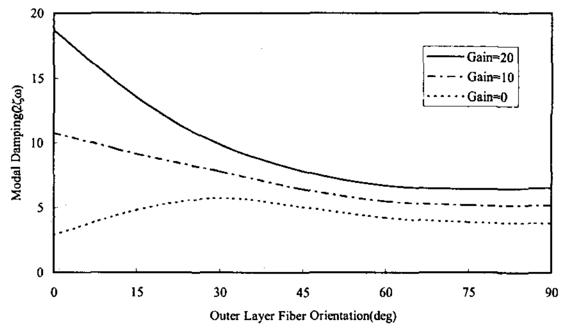

Fig. 6 1st bending modal damping of laminated composite beams with hybrid control

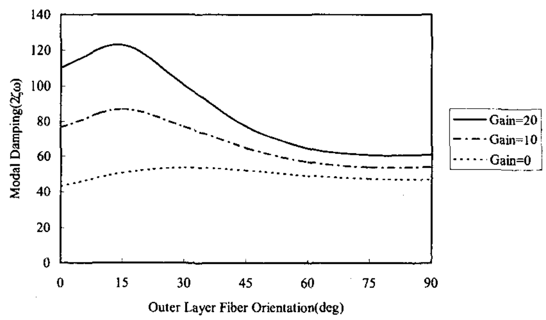

Fig. 7 2nd bending modal damping of laminated composite beams with hybrid control

From the results of active control and PCLD, it can be seen that the integration of passive structural control and active control using smart material can maximize the performance of a structure.

4. Conclusions

Control/structure integrated vibration control of laminated composite beams with the collocated piezoelectric sensor/actuator and viscoelastic material has been investigated numerically and verified experimentally. The finite element method is used to predict the structural characteristics of the laminated composite beams. The following conclusions have been drawn.

Tailoring and the attachment of the viscoelastic materials passively increase the damping ratio and the modal damping of the flexible beams significantly. Also vibration control using PZT only is very effective for the stiffer beams. For the first mode, damping ratio and modal damping of the beam with larger bending stiffness increase greatly through the combination of active control and PCLD. However for the second mode, the 15 deg specimen has the best performance in the hybrid control. From the results of active control and PCLD, it can be concluded that the integration of passive structural control and active control using smart material can maximize the performance of a structure.

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