DDM Rotordynamic Design Sensitivity Analysis of an APU Turbogenerator Having a Spline Shaft Connection

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An eigenvalue design sensitivity formulation of a general nonsymmetric-matrix rotor-bearing system is devised, using the DDM (direct differential method). Then, investigations on the design sensitivities of critical speeds are carried out for an APU turbogenerator with a spline shaft connection. Results show that the dependence of the rate of change of the critical speed on the stiffness changes of bearing models of spline shaft connection points is negligible, and thereby their modeling uncertainty does not present any problem. And the passing critical speeds up to the 4th critical speed are not sensitive to the design stiffness coefficients of four main bearings. Further, the dependence of the rate of change of the critical speed on the shaft-element length changes shows quantitatively that the spline shaft has some limited influence on the 4th critical speed but no influence on the 1st to 3rd critical speeds. With no adverse effect from the spline shaft, the APU system achieves a critical speed separation margin of more than 40% at a rated speed of 60,000 rpm.

Key Words: DDM Design Sensitivity, Spline Shaft Connection, FE Rotordynamics, Critical Speed Change Rate, Mode Shape

1. Introduction

A spline shaft connection may be utilized as the means of power transmission between two rotating systems in small and medium-sized gas turbines that are compact, light weight, and have high power. An ideally designed spline shaft should simply perform only a function of power transmission without incurring any adverse effects on the lateral dynamics of connected systems over the entre range of operating speeds. Before connecting two rotating systems with a spline shaft, it is quite important to analyze the dynamic characteristics of associated rotor-bearing systems before and after the connection, and to minimize any adverse effects of spline shaft or to have them within allowable limits through a detailed design process.

To obtain more accurate and favorable dynamic designs of mechanical systems, a design sensitivity analysis is getting more attention in these days. A dynamic design sensitivity analysis aims to investigate the rates of change of system dynamic responses with respect to the design variables and thereby to provide systematic information on the dependence of the dynamic characteristics of the system on the design variables.

For general structures, Wittrick (1962) and Fox and Kapoor (1968) performed eigenvalue design sensitivity analyses for systems having symmetric system matrices and Plaut and Huseyin (1973) dealt with systems having nonsymmetric system matrices. And Murthy and Haftka (1988) carried out an eigenvalue sensitivity analysis with a general complex system matrix, using the DDM

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(direct differential method). For rotor-bearing systems, Lund (1980) proposed an analysis scheme for determining critical speed sensitivities to design variable changes based on the TM (transfer matrix) formulation, and Rajan et al. (1986) performed eigenvalue design sensitivity analyses, using the AVM (adjoint variable method) based on the FE (finite element) formulation.

In this study, an eigenvalue design sensitivity formulation of a general nonsymmetric-matrix rotor-bearing system is devised, using the DDM along with FE based system equations of motion. A detailed critical speed design sensitivity analysis is carried out for an APU (auxiliary power unit) turbogenerator with a spline shaft connection. The effects of the spline shaft connection on the rotordynamic characteristics of the APU system are investigated.

2. DDM Desicn Sensitivity Formulation

Generalized FE homogeneous equations of motion of rotor-bearing systems (Lee's, 2001) are expressed in the state-space form by

$$[A]\{\dot{X}\}+[B]\{X\}=\{0\} \tag{1}$$

where $[A]=\begin{bmatrix}[0] & [M]\\ [M] & [C]\end{bmatrix}$, $[B]=\begin{bmatrix}-[M] & [0]\\ [0] & [K]\end{bmatrix}$

$$\{X\}=\begin{Bmatrix}\{q_2\}\\ \{q_1\}\end{Bmatrix} \text{ with } \begin{matrix}\{q_1\}=\{q\}\\ \{q_2\}=\{\dot{q}\}\end{matrix}$$

For i-th eigenvalue and eigenvector of Eq. (1), the eigenvalue problem and eigenvector orthogonality relationship are referring to Murthy and Haftka (1988), given by

$$(\lambda_i[A]+[B])\{y\}_i=\{0\}_i \tag{2}$$

$$\{\bar{y}\}_i^T[A]\{y\}_i=R_i \tag{3}$$

where R_i is a system modal norm and $\{\bar{y}\}_i$ is a complex conjugate of eigenvector $\{y\}_i$. For an eigenvalue design sensitivity analysis, a general response function, which consists of eigenvalues, eigenvectors, and design variables, may be expressed by

$$\varphi=\varphi(\lambda_i, \{y\}_i, b) \tag{4}$$

where b is a design variable and λ_i and $\{y\}_i$ are implicit with respect to b. From Eq. (4), a total design derivative of the response function is obtained, using the variational principle (Arora and Cardoso, 1992)

$$\frac{d\varphi}{db}=\frac{\partial\varphi}{\partial b}+\left(\frac{\partial\varphi}{\partial\lambda_i}\right)\frac{d\lambda_i}{db}+\left(\frac{\partial\varphi}{\partial\{y\}_i}\right)^T\frac{d\{y\}_i}{db} \tag{5}$$

In Eq. (5), $d\lambda_i/db$ and $d\{y\}_i/db$ are the design derivatives (or synonymously, sensitivities) of the eigenvalues and eigenvectors to be sought. Taking direct derivatives of Eqs. (2) and (3) with respect to b, with the help of Eq. (5), gives

$$\begin{aligned}&(\lambda_i[A]+[B])\frac{d\{y\}_i}{db}+[A]\{y\}_i\frac{d\lambda_i}{db}\\ &=-\left(\lambda_i\frac{\partial[A]}{\partial b}+\frac{\partial[B]}{\partial b}\right)\{y\}_i\end{aligned} \tag{6}$$

$$\{\bar{y}\}_i^T[A]\frac{d\{y\}_i}{db}=-\frac{1}{2}\{\bar{y}\}_i^T\frac{\partial[A]}{\partial b}\{y\}_i \tag{7}$$

Equations (6) and (7) are written in the matrix form by

$$\begin{bmatrix}(\lambda_i[A]+[B]) & [A]\{y\}_i\\ \{\bar{y}\}_i^T[A] & 0\end{bmatrix}\begin{bmatrix}\dfrac{d\{y\}_i}{db}\\ \dfrac{d\lambda_i}{db}\end{bmatrix}=\begin{bmatrix}-\left(\lambda_i\dfrac{\partial[A]}{\partial b}+\dfrac{\partial[B]}{\partial b}\right)\{y\}_i\\ -\dfrac{1}{2}\{\bar{y}\}_i^T\dfrac{\partial[A]}{\partial b}\{y_i\}\end{bmatrix} \tag{8}$$

where $\dfrac{\partial[A]}{\partial b}=\begin{bmatrix}[0] & \dfrac{\partial[M]}{\partial b}\\ \dfrac{\partial[M]}{\partial b} & \dfrac{\partial[C]}{\partial b}\end{bmatrix}$,

$$\frac{\partial[B]}{\partial b}=\begin{bmatrix}-\dfrac{\partial[M]}{\partial b} & [0]\\ [0] & \dfrac{\partial[K]}{\partial b}\end{bmatrix}$$

Finally, the design derivatives of the eigenvalue and eigenvector are obtained by solving Eq. (8). Here, a design sensitivity of whirl natural frequency is predicted from an imaginary part of a design derivative of the eigenvalue. For an absolute comparison of contributions between various design variables with different magnitudes and dimensions, the following change rate of response function to a design variable is introduced

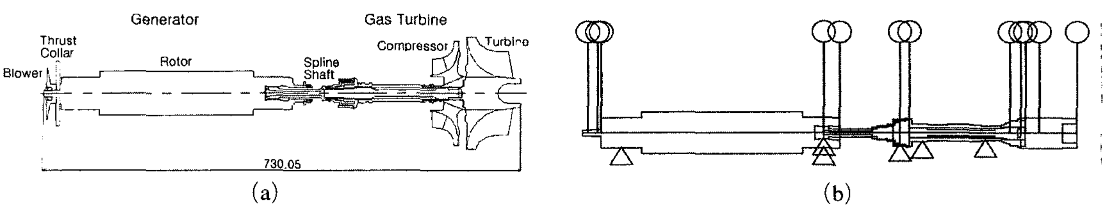
$$Change\ rate = \frac{\frac{d\varphi}{db} \times \Delta b}{\varphi} \times 100(\%) \qquad (9)$$

3. APU and Modeling

Figure 1(a) shows a schematic of 100 kW APU turbogenerator designed for a rated speed of 60, 000 rpm. The APU is composed of two main subsystems, a generator in the left and a gas turbine in the right. Power from the gas turbine is transferred to the generator through a spline shaft. The generator is supported by two air foil bearings, while a blower supplies cooling air and a thrust collar transfers axial load to a thrust air foil bearing. The gas turbine rotor consists largely of a shaft and a radial turbine and compressor and is supported by a ball bearing in the left and a roller bearing in the right. The stiffness and damping of its two rolling bearings are determined by their flexible supports, i.e., a squirrel cage for the ball bearing and an Allison ring for the roller bearing. Figure 1(b) shows the APU's FE model with the circles representing lumped inertias such as various disks/wheels and the triangles representing the four main radial bearings and two connection points between the spline shaft and each rotor shaft. The generator, spline, and gas turbine shafts are modeled as 16, 9, and 23 shaft elements, respectively. Table 1 summarizes lumped disk inertias, shaft material properties, and bearing coefficients for the APU model.

Table 1 Lumped disk inertias, shaft material properties, and bearing coefficients for the APU rotor-bearing system FE model

	Lumped disk inertia			Shaft Material property		
	m (kg)	I_p (kg·m³)	I_t (kg·m³)			
D1	0.168	8.090×10^{-5}	4.296×10^{-5}	$E=2.0\times10^{11}$ N/m² $\rho=8{,}100$ kg/m³		
D2	0.378	4.973×10^{-4}	2.498×10^{-4}			
D3	0.010	1.285×10^{-6}	6.557×10^{-7}			
D4	0.017	8.938×10^{-7}	9.750×10^{-7}	Bearing or support coefficients $K=K_{XX}=K_{YY}$ (N/m)		
D5	0.162	4.450×10^{-5}	5.926×10^{-5}			
D6	0.038	1.214×10^{-5}	9.501×10^{-6}		LHS	RHS
D7	0.128	9.171×10^{-5}	9.501×10^{-5}	Generator rotor	3.0×10^{6}	3.0×10^{6}
D8	0.372	2.700×10^{-3}	1.484×10^{-3}			
D9	0.019	5.435×10^{-6}	2.757×10^{-6}	Spline shaft	1.0×10^{8}	1.0×10^{8}
D10	0.033	8.874×10^{-6}	4.578×10^{-6}			
D11	2.658	5.313×10^{-3}	3.037×10^{-3}	Gas turbine	5.0×10^{6}	5.0×10^{7}
D12	0.076	2.830×10^{-5}	1.668×10^{-5}			



(a) (b)

Fig. 1 (a) Layout of the APU rotor system (b) FE model of the APU rotor

4. Results and Discussions

For a systematic and quantitative analysis of spline shaft effects on the critical speeds of the APU system, a critical speed design sensitivity analysis has been carried out, taking as design variables all the bearing stiffnesses (including the bearing model stiffness coefficient of 1×10^8 N/m of the spline connection points) and all the shaft element lengths. For an absolute comparative evaluation of each design variable's contribution, a 10% increase of each design variable has been considered.

The first four passing critical speeds below the

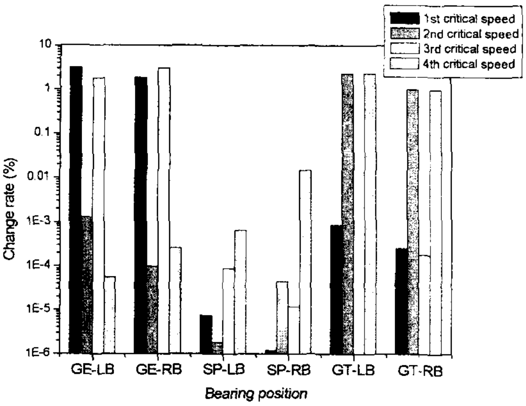
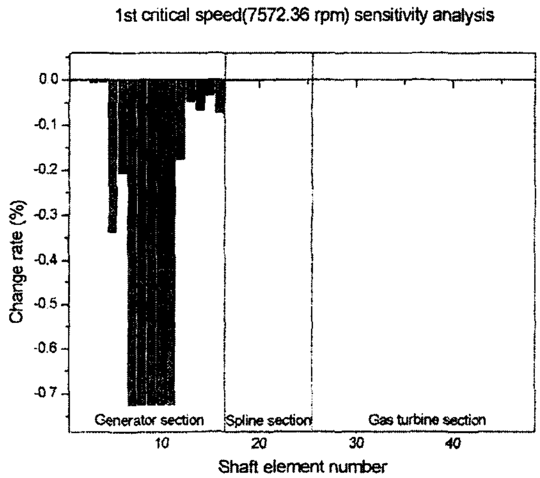
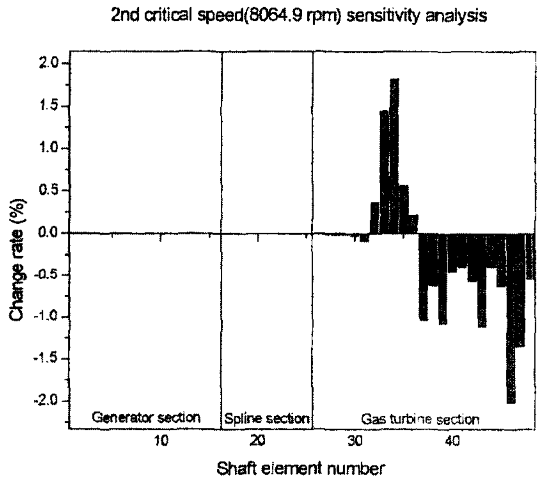


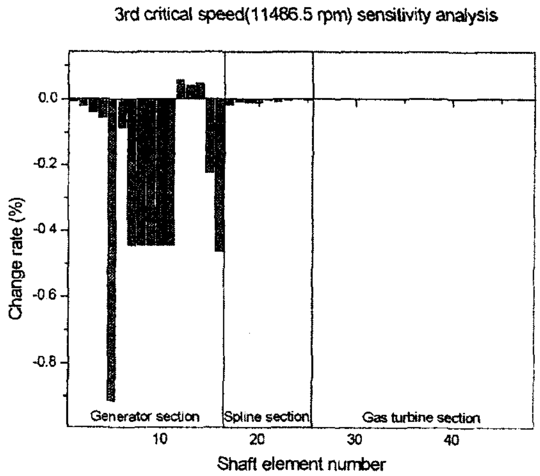
Fig. 2 Critical speed change rates for 10% bearing stiffness increases



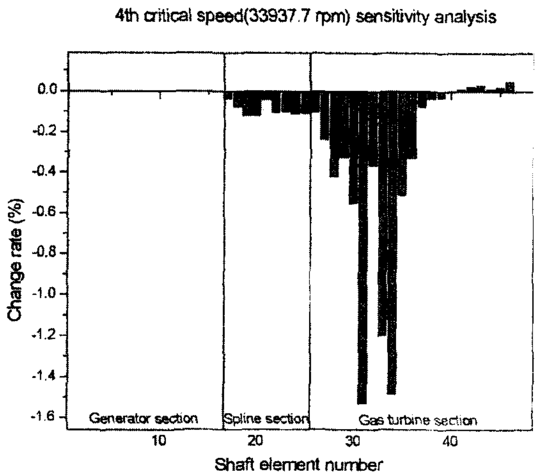
(a) 1st critical speed change rate



(b) 2nd critical speed change rate



(c) 3rd critical speed change rate

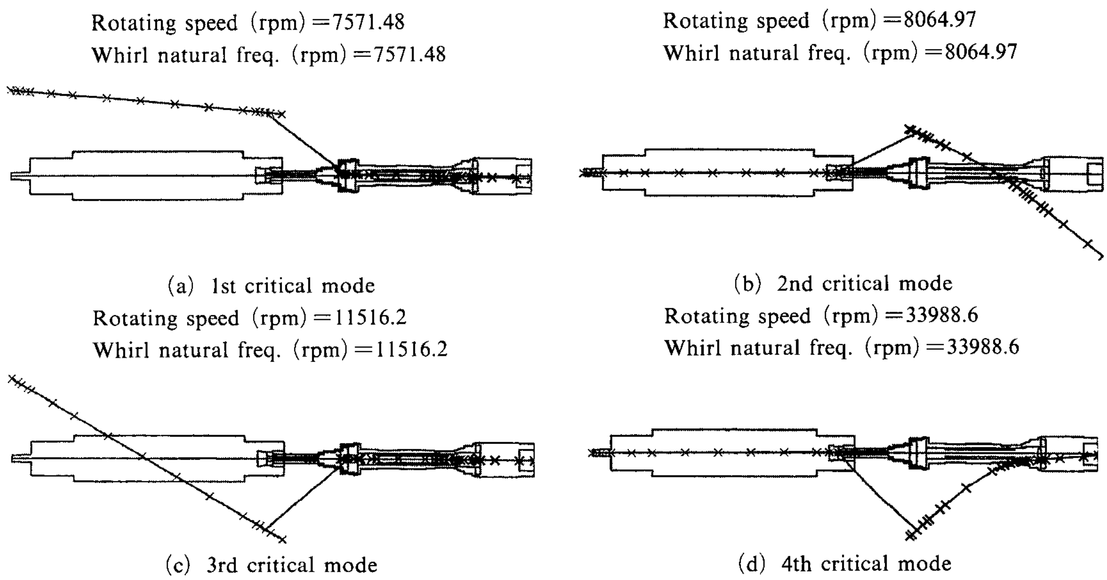


(d) 4th critical speed change rate

Fig. 3 Critical speed change rates for 10% shaft element length increases rated speed are 7,572, 8,065, 11,487, and 33,938 rpm. Figure 2 shows the rate of change of the critical speed on each bearing stiffness variation where GE represents the generator, SP the spline shaft, GT the gas turbine, LB the LHS bearing, and RB the RHS bearing. The rate of change of the critical speed on variations in the generator and gas turbine bearing stiffness are all less than 4%. Hence, the stability of the critical speeds is assured even if the stiffness of the four main support bearings deviates from their design values. It is also noticed that the 1st and 3rd critical speeds are attributed to the generator and that the 2nd and 4th critical speeds are attributed to the gas turbine. Particularly, the rate of change of the critical speed to varying bearing model stiffness of the spline shaft connection points is confirmed to be quite negligible, and hence their modeling uncertainty is not a critical issue.

Figure 3 shows the rate of change of the critical speed to variations in shaft element lengths. It is clearly observed again that the 1st and 3rd critical speeds are attributed to the generator and that the 2nd and 4th critical speeds are attributed to the gas turbine. Particularly, it is quantitatively confirmed that the spline shaft does have some influence on the 4th critical speed but little influence on the other 1st to 3rd critical speeds.

On the other hand, by modeling the spline shaft connection points as pivots, the first four critical speeds are predicted to be 7,571, 8,065, 11,516, and 33,989 rpm. They are almost identical to those obtained by modeling the spline shaft connection points as bearings, as can be expected from the previons sensitivity analysis results. The corresponding mode shapes are shown in Fig. 4. It is observed that the 1st and 3rd modes are rigid body modes of the generator, the 2nd mode is a rigid body mode of the gas turbine, and the 4th mode is a flexible mode of the gas turbine. It is also seen that the spline shaft are almost straight for the 1st, 2nd, and 3rd modes, and hence does not influence the 1st to 3rd critical speeds. Whereas the spline shaft does bend a little at the 4th mode, and thereby it eventually constrains the gas turbine to some extent to influence the 4th critical speed. All the above findings are in good agreement with the sensitivity analysis results. Table 2 summarizes the critical speeds obtained for separate models of the generator and the gas turbine and for the model of the entire APU assembly with the spline connection points modeled as pivots or bearings. For separate independent models, each share of the spline shaft



(a) 1st critical mode (b) 2nd critical mode (c) 3rd critical mode (d) 4th critical mode

Fig. 4 Critical mode shape of the entire APU rotor-bearing system with a pivot spline connection model

Table 2 Critical speeds for different system modelings

Critical speed for separate independent rotor-bearing systems (rpm)		Critical speed for entire connected APU rotor-bearing systems (rpm)	
Generator	Gas Turbin	Spline Connection → Bearing model	Spline Connection → Pivot model
	6,522(1B)	6,574(1B)	6,574(1B)
7,550(1B)		7,568(2B)	7,567(2B)
7,533(1F)		7,572(1F)	7,571(1F)
	7,968(1F)	8,065(2F)	8,065(2F)
10,814(2B)		10,874(3B)	10,902(3B)
11,407(2F)		11,487(3F)	11,516(3F)
	28,732(2B)	33,006(4B)	33,048(4B)
	29,002(2F)	**33,938(4F)**	**33,989(4F)**
	43,928(3B)	44,837(5B)	44,864(5B)
48,941(3B)		48,946(6B)	48,954(6B)
85,727(4B)		85,727(7B)	85,729(7B)

(note) B : Backward critical speed, F : Forward critical speed

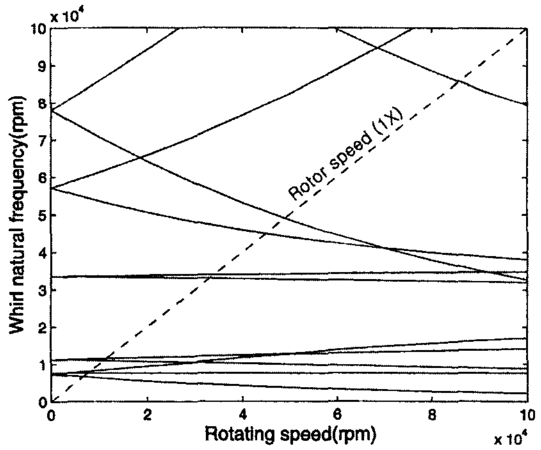


Fig. 5 Campbell diagram of the entire APU rotor-bearing system with a pivot spline connection model

mass has been lumped and added to the corresponding generator and gas turbine shaft positions. The critical speeds of the entire APU system are practically identical to the corresponding ones of the separate independent systems except for the 4th critical speed. The 4th critical speed of the entire APU is about 14% higher than the corresponding one of the gas turbine (33,938 rpm to 29,002 rpm). Figure 5 shows the Campbell diagram of the entire APU rotor system with the pivot spline connection model. It is noticed that the APU system achieves a critical speed separation margin of more than 40% from the rated speed.

5. Conclusions

In this study, an eigenvalue design sensitivity formulation of a general nonsymmetric-matrix rotor-bearing system has been devised by using the DDM. The formulation has been successfully implemented for the critical speed design sensitivity analysis of an APU turbogenerator with a spline shaft connection.

Results have shown that the rate of change of the critical speed to variations in the bearing model stiffness of the spline shaft connection points is extremely negligible, and hence their modeling uncertainty is not a critical issue. It also has been shown that the critical speeds up to the 4th critical speed are not sensitive to the stiffness coefficients of the four main bearings or supports. Further, the rate of change of the critical speed to variations in the shaft element lengths has shown quantitatively that the spline shaft has limited influence on the 4th critical speed but not on the other 1st to 3rd critical speeds. It is concluded that the APU system considered here achieves a separation margin of more than 40% from the rated speed without any adverse effect from the spline shaft.

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