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Controllability for the fuzzy differential systems with nonlocal initial conditions

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Abstract

In this paper, we study the controllability of fuzzy differential systems with nonlocal initial conditions. Result of this paper has improved and expanded in [5].

Key words : fuzzy number, fuzzy differential systems, controllablity

1. Introduction

In [5], Z. Ding and A. Kandel studied the controllability of fuzzy dynamical system:

$$\begin{cases} x'(t) = A(t)x(t) + B(t)U(t), \\ x(0) = x_0 \end{cases}$$

where A, B are continuous matrices and U(t) is fuzzy set. Also, D. H. Jeong etc [6] is studied the controllability of fuzzy differential system with nonlocal initial condition:

$$\begin{cases} x'(t) = A(t)x(t) + B(t)U(t), \\ x(0) + g(x) = x_0 \end{cases}$$

where A(t), B(t) are continuous matrices, U(t) is fuzzy set and g is given function.

In this paper, we consider the existence of fuzzy solution and controllability for the following differential system with nonlocal initial condition:

$$(1.1)\ \begin{cases} x'(t) = A(t)x(t) + B(t)U(t) + F(t, x), \\ x(t) + g(x) = x_0 \end{cases}$$

where A(t), B(t) are continuous matrices, U(t) is fuzzy set, $g: E^n \to E^n$ is linear and satisfies a global Lipschitz condition, and $F: [0, T] \times E^n \to E^n$ is linear with respect to x, nonlinear with respect to t.

2. Preliminary

Let A and B be two nonempty bounded subsets of R^n. The distence between A and B is defined by Hausdorff metric. Denoted by $P_k(R^n) = \{A \subset R^n : A$ is nonempty closed compact convex $\}$.

Denoted by

$$E^n = \{u: R^n \to [0, 1] \mid u \text{ satisfies (1) - (4) below} \}.$$

where

(1) u is normal.
(2) u is fuzzy convex.
(3) u is upper semicontinuous.
(4) $[u]^0 = \overline{\{x \in R^n : u(x) > 0\}}$ is compact.

For $0 < \alpha \le 1$, denote $[u]^\alpha = \{x \in R^n : u(x) \ge \alpha\}$. Define $D: E^n \times E^n \to R \cup \{0\}$ by

$$D(u, v) = \sup_{0 < \alpha < 1} d_H([u]^\alpha, [v]^\alpha)$$

where d_H is the Hausdorff metric. We see that (E^n, D) is a complete metric space.

Theorem 2.1([8]) If $u \in E^n$, then
(1) $[u]^\alpha \in P_k(R^n)$ for all $0 \le \alpha \le 1$,
(2) $[u]^{\alpha_2} \subset [u]^{\alpha_1}$ for all $0 \le \alpha_1 \le \alpha_2 \le 1$,
(3) If $\{\alpha_k\}$ is a nondecreasing sequence converging to $\alpha > 0$, then $[u]^\alpha = \bigcap_{k \ge 1} [u]^{\alpha_k}$.

Conversely, if $\{A^\alpha : 0 \le \alpha \le 1\}$ is a family of subsets of R^n satisfying (1)-(3), then there exists a $u \in E^n$ such that $[u]^\alpha = A^\alpha$ for $0 \le \alpha \le 1$ and $[u]^0 = \bigcup_{0 \le \alpha \le 1} A^\alpha \subset A^0$.

Consider the fuzzy differential equation

$$(2.1) \quad x'(t) = F(t, x(t)), \quad x(0) = x_0$$

where $F: [0, T] \times E^n \to E^n$.

Definition 2.1([5]) A mapping $x: [0, T] \to E^n$ is a fuzzy

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weak solution to (2.1) if it is continuous and satisfies the integral equation

$$x(t)=x_0+\int_0^t F(s,x(s))ds, \text{ for all } t\in[0,T].$$

If F is continuous, then this weak solution also satisfies (2.1) and we call it fuzzy strong solution to (2.1).

It should be noted that $\Phi(t)=e^{A(t)}$ is the fundamental matrix of the equation

$$x'(t)=A(t)x(t), \quad t\geq 0.$$

3. Main Result

We assume the following hypotheses:

(H1) $M=\max_{t\in[0,T]}\|\Phi(t)\|$.

(H2) $N=\max_{t\in[0,T]}\|u(t)\|$,

where $u(t)\in[U(t)]^{\alpha}$.

(H3) $K=\max_{t\in[0,T]}\|B(t)\|$.

(H4) $\|g(x_1)-g(x_2)\|\leq L\|x_1-x_2\|$ for any $x_1,x_2\in E^n$.

(H5) $F(t,x)$ is linear with respect to x and nonlinear function with respect to t satisfied by $\|F(t,x(t))-F(t,y(t))\|\leq I\|x(t)-y(t)\|$.

Now, we will prove the following theorem.

Theorem 3.1 Assume that the hypotheses (H1)-(H5) are satisfied, $ML+MIT<1$ and $T>0$, then system (1.1) has a fuzzy solution $x(t)$.

Proof. Let T>0. Consider the differential inclusions

$$(3.1)\quad \begin{cases} x_\alpha'(t)\in A(t)x_\alpha(t)+B(t)U(t)+F(t,x_\alpha), & t\in[0,T],\\ x_\alpha(0)+g(x_\alpha)=x_0. \end{cases}$$

Let X^{α} be the solution set of inclusion (3.1).

Next we show that it is nonempty compact and convex in $C([0,T]:R^n)$. Nonempty is obvious since $[U(t)]^{\alpha}$ has measurable selections. Let $x_\alpha\in X^{\alpha}$, then there is a selection $u(t)\in[U(t)]^{\alpha}$ such that

$$x_\alpha(t)=\Phi(t)(x_0-g(x_\alpha))+\int_0^t\Phi(t-s)F(s,x_\alpha)ds+\int_0^t\Phi(t-s)B(s)u(s)ds$$

Then

$$\begin{aligned}&\|x_\alpha(t)\|\\ \leq\ &\|\Phi(t)(x_0-g(x_\alpha))\|+\int_0^t\|\Phi(t-s)F(s,x_\alpha)\|ds\\ &+\int_0^t\|\Phi(t-s)B(s)u(s)\|ds\\ \leq\ &M\|x_0\|+ML\|x_\alpha(t)\|+MKNT+\|x_\alpha(t)\|.\end{aligned}$$

So

$$\|x_\alpha(t)\|\leq\frac{M}{1-ML-MIT}(\|x_0\|+KNT)$$

Thus X^{α} is bounded.

Now we will prove that it is equi-continuous. For any $t_1,t_2\in[0,T]$ with $0<t_1\leq t_2<T$,

$$\begin{aligned}&x_\alpha(t_2)-x_\alpha(t_1)\\ =\ &\Phi(t_2)(x_0-g(x_\alpha))+\int_0^{t_2}F(s,x_\alpha(s))ds\\ &+\int_0^{t_2}\Phi(t_2-s)B(s)u(s)ds-\Phi(t_1)(x_0-g(x_\alpha))\\ &-\int_0^{t_1}\Phi(t_1-s)B(s)u(s)ds-\int_0^{t_1}F(s,x_\alpha(s))ds.\end{aligned}$$

It follows that

$$\begin{aligned}&\|x_\alpha(t_2)-x_\alpha(t_1)\|\\ \leq\ &\|\Phi(t_2)-\Phi(t_1)\|\,\|x_0-g(x_\alpha)\|\\ &+\int_0^{t_2}\|\Phi(t_2-s)-\Phi(t_1-s)\|\,\|B(s)u(s)\|ds\\ &+\int_{t_1}^{t_2}\|\Phi(t_1-s)B(s)u(s)\|ds\\ &+\int_0^{t_2}\|\Phi(t_2-s)-\Phi(t_1-s)\|\,\|F(s,x_\alpha(s))\|ds\\ &+\int_{t_1}^{t_2}\|\Phi(t_1-s)F(s,x_\alpha(s))\|ds\\ \leq\ &\|\Phi(t_2)-\Phi(t_1)\|(\|x_0\|+L\|x_\alpha\|)\\ &+KN\int_0^{t_2}\|\Phi(t_2-s)-\Phi(t_1-s)\|ds\\ &+MKN|t_2-t_1|+MI\int_{t_1}^{t_2}\|x_\alpha(s)\|ds\\ &+I\int_0^{t_2}\|\Phi(t_2-s)-\Phi(t_1-s)\|\,\|x_\alpha(s)\|ds.\end{aligned}$$

Since $\Phi(t)$ is uniformly continuous on [0,T],

$$\|x_\alpha(t_2)-x_\alpha(t_1)\|\to 0 \text{ as } t_2\to t_1.$$

Hence $x(t)$ is equicontinuous. Thus X^{α} is relatively compact. To prove X^{α} is also compact, it is suffice to show that it is close. Let $x_k\in X^{\alpha}$ and for each x_k, there is $u_k\in[U]^{\alpha}$ such that

$$(3.2)\quad \begin{aligned}x_k(t)=\ &\Phi(t)(x_0-g(x_k)+\int_0^t\Phi(t-s)B(s)u_k(s)ds\\ &+\int_0^t\Phi(t-s)F(s,x_k(s))ds.\end{aligned}$$

It follows that

$$\begin{aligned}&\|x_k(t)-x(t)\|\\ \leq\ &\|\Phi(t)\|\,\|g(x_k)-g(x)\|\end{aligned}$$

$$+ \int_0^t \| \Phi(t-s)B(s)(u_k(s)-u(s)) \| ds$$

$$+ \int_0^t \| \Phi(t-s)(F(s,x_k(s))-F(s,x(s))) \| ds$$

$$\leq ML \| x_k(t)-x(t) \| + MI\int_0^t \| x_k(s)-x(s) \| ds$$

$$+ \int_0^t \| \Phi(t-s)B(s)(u_k(s)-u(s)) \| ds.$$

Then for $1-ML>0$,

$$\| x_k(t)-x(t) \| \leq \frac{MI}{1-ML} \int_0^t \| x_k(s)-x(s) \| ds$$

$$+ \frac{1}{1-ML} \int_0^t \| \Phi(t-s)B(s)(u_k(s)-u(s)) \| ds$$

From the Hahn Banach theorem, we know that we can find $x_k^* \in B_1^*$ (dual unit ball) such that

$$\| (\int_0^t \Phi(t-s)B(s)(u_k(s)-u(s))ds, x_k^*) \|$$

$$= \| \int_0^t \Phi(t-s)B(s)(u_k(s)-u(s))ds \|$$

Therefore

$$\| \int_0^t (u_k(s)-u(s))B^*(s)\Phi^*(t-s)x_k^* ds \|$$

$$= \| \int_0^t \Phi(t-s)B(s)(u_k(s)-u(s))ds \|$$

Since for $t>s$, $\Phi^*(t-s)$ and $B^*(s)$ are compact, then by Alaoglu's theorem, B_1^* is weak compact. So by passing to a subsequence, if necessary we may assume that x_k^* is weakly converges to x^* in B_1^*.

Hence $B^*(s)\Phi^*(t-s)x_k^*$ converges to some $z^*(t)$. We deduce that

$$\| \int_0^t \Phi(t-s)B(s)(u_k(s)-u(s))ds \| \to 0$$

as $k \to \infty$.

Setting

$$r_k(t) = \frac{1}{1-ML} \| \int_0^t \Phi(t-s)B(s)(u_k(s)-u(s))ds \|,$$

$$\| x_k(t)-x(t) \| \leq r_k(t) + \frac{MI}{1-ML} \int_0^t \| x_k(s)-x(s) \| ds.$$

By Gronwall's inequality, we have

$$\| x_k(t)-x(t) \| \leq r_k(t) e^{\int_0^t \frac{MI}{1-ML} ds} \leq r_k(t) e^{\frac{MIT}{1-ML}} \to 0$$

as $k \to \infty$.

Hence $x_k(t) \to x(t)$ as $k \to \infty$. Thus X^α is compact.

Next we will show that X^α is convex. if $x_1, x_2 \in X^\alpha$, then there are $u_1(t), u_2(t) \in [U(t)]^\alpha$ such that

$$x_1'(t) = A(t)x_1(t) + B(t)u_1(t) + F(t,x_1),$$

$$x_2'(t) = A(t)x_2(t) + B(t)u_2(t) + F(t,x_2)$$

and let $x = \lambda x_1 + (1-\lambda)x_2,\ 0 \leq \lambda \leq 1$.

$$\begin{aligned} x'(t) &= \lambda x_1'(t) + (1-\lambda)x_2'(t) \\ &= A(t)\{\lambda x_1(t) + (1-\lambda)x_2(t)\} \\ &\quad + B(t)\{\lambda u_1(t) + (1-\lambda)u_2(t)\} \\ &\quad + \lambda F(t,x_1) + (1-\lambda)F(t,x_2) \end{aligned}$$

Since $[U(t)]^\alpha$ is convex, $\lambda u_1(t) + (1-\lambda)u_2(t) \in [U(t)]^\alpha$. Also since $\lambda F(t,x_1) + (1-\lambda)F(t,x_2) = F(t, \lambda x_1 + (1-\lambda)x_2)$, there exists $u(t) \in [U(t)]^\alpha$ and $x \in X$ such that

$$x'(t) = A(t)x(t) + B(t)u(t) + F(t,x)$$

Also, since $x_1(0) + g(x_1) = x_0$, $x_2(0) + g(x_2) = x_0$,

$$\begin{aligned} x(0) &= \lambda x_1(0) + (1-\lambda)x_2(0) \\ &= \lambda x_0 - \lambda g(x_1) + (1-\lambda)x_0 - (1-\lambda)g(x_2) \\ &= x_0 - g(\lambda x_1 + (1-\lambda)x_2) \\ &= x_0 - g(x) \end{aligned}$$

So $x \in X^\alpha$. Thus X^α is convex. Therefore we have $[X(t)]^\alpha \in P_k(R^n)$ for every $t \in [0,T]$. Hence we proved the condition (1) of Theorem 2.1.

Now, in order to prove conditions (2), let $0 \leq \alpha_1 \leq \alpha_2 \leq 1$. Since $[U(t)]^{\alpha_2} \subset [U(t)]^{\alpha_1}$, we have $S^1_{[U(t)]^{\alpha_2}} \subset S^1_{[U(t)]^{\alpha_1}}$ and

$$\begin{aligned} x_{\alpha_2}'(t) &\in A(t)x_{\alpha_2}(t) + B(t)[U(t)]^{\alpha_2} + F(t,x_{\alpha_2}) \\ &\subset A(t)x_{\alpha_1}(t) + B(t)[U(t)]^{\alpha_1} + F(t,x_{\alpha_1}). \end{aligned}$$

Thus

$$\begin{aligned} &x_{\alpha_2}(t) \\ &\in \Phi(t)x_0 + \int_0^t \Phi(t-s)B(s)S^1_{[U(t)]^{\alpha_2}}(s)ds \\ &\quad + \int_0^t \Phi(t-s)F(s,x_{\alpha_2})ds \\ &\subset \Phi x_0 + \int_0^t \Phi(t-s)B(s)S^1_{[U(t)]^{\alpha_1}}(s) \\ &\quad + \int_0^t \Phi(t-s)F(s,x_{\alpha_1})ds. \end{aligned}$$

So $X^{\alpha_2} \subset X^{\alpha_1}$.

Finally we prove the condition (3) of theorem 2.1. Let (α_k) be a nondecreasing sequence converging to $\alpha > 0$. We need first show that $X^\alpha = \cap_{k \geq 1} X^{\alpha_k}$, then if this true we get $X^\alpha(t) = \cap_{k \geq 1} X^{\alpha_k}(t)$. Since $[U(t)]^\alpha = \cap_{k \geq 1}[U(t)]^{\alpha_k}$, we have $S^1_{[U(t)]^\alpha} = S^1_{\cap_{k\geq 1}[U(t)]^{\alpha_k}}$. Thus

$$\begin{aligned} & x'_\alpha(t) \\ \in\ & A(t)x_\alpha(t) + B(t)[U(t)]^\alpha + F(t, x_\alpha) \\ =\ & A(t)x_\alpha(t) + B(t)\cap_{k\geq 1}[U(t)]^{\alpha_k} + F(t, x_\alpha) \\ \subset\ & A(t)x_\alpha(t) + B(t)[U(t)]^{\alpha_k} + F(t, x_\alpha), k = 1, 2, \cdots. \end{aligned}$$

So we have $X^\alpha \subset X^{\alpha_k}$, $k = 1, 2, \cdots$ which yields $X^\alpha \subset \cap_{k\geq 1} X^{\alpha_k}$. Also, let x be the solution to the inclusions

$$x'_{\alpha_k}(t) \in A(t)x_{\alpha_k}(t) + B(t)[U(t)]^{\alpha_k} + F(t, x_{\alpha_k}),\ k\geq 1$$

Then

$$x(t) \in \Phi(t)(x_0 - g(x)) + \int_0^t \Phi(t-s)F(s, x)ds + \int_0^t \Phi(t-s)B(s)S^1_{[U(t)]^{\alpha_k}}(s)ds$$

and thus

$$\begin{aligned} x(t) \in\ & \Phi(t)(x_0 - g(x)) + \int_0^t \Phi(t-s)F(s, x)ds \\ & + \int_0^t \Phi(t-s)B(s)S^1_{\cap_{k\geq 1}[U(t)]^{\alpha_k}}(s)ds \\ =\ & \Phi(t)(x_0 - g(x)) + \int_0^t \Phi(t-s)F(s, x)ds \\ & + \int_0^t \Phi(t-s)B(s)S^1_{[U(t)]^\alpha}(s)ds \end{aligned}$$

This mean that $x \in X^\alpha$. Therefore $\cap_{k\geq 1} X^{\alpha_k} \subset X^\alpha$.

Next we consider the controllability conditions of fuzzy systems (1.1). The concept of controllability is concerned with the following problem: given system (1.1), for the initial state $x_0 - g(x)$, the state at time T is a fuzzy set x^1, find the input $u(t)$, $t \in [0, T]$ that transfers $x_0 - g(x)$ (at 0) x^1 (at T). We need the following definition.

Definition 3.1([6]) The state $x_0 - g(x)$ of system (1.1) is said to be controllable on the interval [0,T] where T is a finite time if some control U over [0,T] exists which transfers $x_0 - g(x)$ to the fuzzy state at T. Otherwise the state $x_0 - g(x)$ is said to be uncontrollable on [0,T].

Lemma 3.1([5]) Let $f(t) \neq 0$ be a continuous function and U, V are two fuzzy sets. If

$$\int_0^T f(t)\, U dt = \int_0^T f(t)\, V dt$$

then U = V.

Theorem 3.2([5]) System (1.1) ($g(x) = 0$) is controllable over the interval [0,T], if $\Phi(T-t)B(t)$ is nonsingular or equivalently, if the matrix

$$M(0, T) = \int_0^T \Phi(T-t)B(t)B^*(t)\Phi^*(T-t)dt$$

is nonsingular.

Furthermore, the control $U(t)$ which transfer the state of the system from $x(0) = x_0$ to a fuzzy state $x(T) = x^1$ can be chosen as

$$\begin{aligned} U(t) = & \frac{1}{T} B^{-1}(t)\Phi^{-1}(T-t)x^1 \\ & - B^*(t)\Phi^*(T-t)M^{-1}(0, T)\Phi(T)x_0 \end{aligned}$$

Theorem 3.3 System (1.1) is controllable over the interval [0,T], if $\Phi(t)B(t)$ is nonsingular. Furthermore, the control $U(t)$ which transfer the state of the system from $x(0) = x_0 - g(x)$ to a fuzzy state $x(T) = x^1$ can be chosen as

$$\begin{aligned} U(t) = & B^{-1}(t)\Phi^{-1}(T-t) \\ & \{\frac{1}{T}(x^1 - \Phi(T)(x_0 - g(x))) \\ & - \Phi(T-t)F(t, x)\} \end{aligned}$$

Proof.

Since $\Phi(T-t)B(t)$ is nonsingular, there exists $\{\Phi(T-t)B(t)\}^{-1} = B^{-1}(t)\Phi^{-1}(T-t)$.

If $U(t)$ exists such that $U(t)$ ransfer $x_0 - g(x)$ to x^1 over $[0, T]$, then we get

$$x(T) = x^1 = \Phi(T)(x_0 - g(x)) + \int_0^T \Phi(T-s)F(s, x)ds + \int_0^T \Phi(T-s)B(s)U(s)ds.$$

Thus

$$\begin{aligned} & \int_0^T \Phi(T-s)B(s)U(s)ds \\ =\ & \int_0^T \{\frac{1}{T}(x^1 - \Phi(T)(x_0 - g(x))) \\ & - \Phi(T-s)F(s, x)\}ds \\ =\ & \int_0^T \Phi(T-s)B(s)B^{-1}(s)\Phi^{-1}(T-s) \\ & \{\frac{1}{T}(x^1 - \Phi(T)(x_0 - g(x))) \\ & - \Phi(T-s)F(s, x)\}ds. \end{aligned}$$

Hence from Lemma 3.1,

$$\begin{aligned} & U(t) = B^{-1}(t)\Phi^{-1}(T-t) \\ & \{\frac{1}{T}(x^1 - \Phi(T)(x_0 - g(x))) - \Phi(T-t)F(t, x)\}. \end{aligned}$$

Example 3.1 Let us consider the system

$$x'(t) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} x(t) + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} U(t) + \begin{pmatrix} t^2 x \\ t^2 x \end{pmatrix}$$

$$x(0)=\begin{pmatrix}1\\1\end{pmatrix}-\begin{pmatrix}g_1(x)\\g_2(x)\end{pmatrix}.$$

We assume that α-level sets of fuzzy sets x^1 are

$$[x^1]^\alpha=\begin{pmatrix}[-0.1(1-\alpha),0.1(1-\alpha)]\\ [-0.1(1-\alpha),0.1(1-\alpha)]\end{pmatrix}.$$

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