Simulation of Fuzzy Reliability Indexes

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By means of the transformation from the problem of fuzzy reliability to the problem of general reliability, a model for analyzing fuzzy reliability is introduced in this paper. Because of the complexity of the problem of the fuzzy reliability, generally speaking, the analytical equations for calculating fuzzy reliability indexes of machine part cannot be obtained in most cases. Therefore, in this paper, an approach is given wherein progressions are employed to calculate them, or a simulation approach is used to estimate them by expressing general reliability indexes as progressions. By utilizing the approach put forwards in the paper, the calculating quantity for analyzing the fuzzy reliability will be reduced; even substantially reduced sometimes. Some examples are taken to explain the feasibility of the model and a simulation approach.

Key Words: Fuzzy Event Probability, General Reliability Theory, Reliability Index, Simulation

1. Introduction

There are some mature approaches to calculate reliability indexes of machine part in consideration of random information. But after the reliability problem of machine part was studied deep, it was found that there was a great deal of fuzzy information, which could not be evaded in design. The mechanical fuzzy reliability design, which takes random and fuzzy information into consideration at the same time, was studied only more than ten years ago. The aim of fuzzy reliability design is to analyze the real reliability of machine part by combining fuzzy mathematics with the general reliability theory. It is also an important developing direction of modern design approach of mechanical products, and will be widely used in mechanical engineering.

Strictly speaking, generalized stress and strength in machine design are both fuzzy to some degree, so they should be regarded as fuzzy variables in theory. But in practice, it is unnecessary to do so sometimes. For example, if stress or strength is regarded as an invariable or a random variable and calculated results can satisfy the accuracy requirement for application, it can be regarded as the invariable or the random variable.

The equation of calculating fuzzy event probability can be used to calculate fuzzy reliability indexes (Yang et al., 1992; Lin, 1994; Cheng, 1995). This approach is a basic approach. But from the view of methodology, any fuzzy mathematical problem can be solved by means of the transformation from the fuzzy mathematics problem to the general mathematical problem. Therefore, fuzzy reliability indexes can be calculated by means of the transformation from the problem of fuzzy reliability to the problem of general reliability in theory (Dong, 1999a; Dong, 1999b; Dong, 1999c; Dong et al., 2000; Dong, 2000).

In this paper, a model for calculating the fuzzy

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reliability indexes by means of the transformation from the problem of fuzzy reliability to the problem of general reliability is introduced when both fuzzy information and random information are taken into consideration. Because there are too many forms of probability density functions of random variables and membership functions of fuzzy variables, and because the analytical equations of calculating the fuzzy reliability indexes cannot be obtained in most cases, some other approaches should be taken to calculate them. One of the approaches is a simulation approach. When the simulation approach is used, to obtain adequately the accurate fuzzy reliability indexes, the number of simulation should be large enough. To make the calculating quantities for obtaining the fuzzy reliability indexes not so large, a good approach should be taken to calculate general reliability indexes. If a better approach is taken to calculate them, the calculating quantities will be reduced.

In this paper, to decrease the calculating quantities for obtaining the fuzzy reliability indexes, the general reliability indexes, sometimes the fuzzy reliability indexes may be expressed by some progressions.

2. Fuzzy Design Criterion

If both stress and strength of machine part are random variables, based on the general reliability theory, the reliability R of the part can be given by

$$R=P(s\leq r) \tag{1}$$

If the strength is an invariable and its numerical value is 120 MPa, according to Eq. (1), the part is in the state of safety when the random stress is 120 MPa, and the part is in the state of failure when the random stress is 120.01 MPa. But in fact, there is no much difference between 120 MPa and 120.01 MPa. Thus, it may be unsuitable to take 120 MPa as the limit of whether the part is in the state of safety or not. As an equation of calculating reliability, Eq. (1) should be revised. Because of the complexity of stress and strength, it is better to take a fuzzy design criterion to consider fuzzy information.

In consideration of varied fuzzy information, according to fuzzy reliability theory it is difficult to judge whether the part is safe or not when the numerical value of stress is only a bit bigger than that of strength (Wang, 1986). This is to say, the event of part safety should be considered as a fuzzy event, which can be expressed by "$s\tilde{\leq}r$". "$s\tilde{\leq}r$" means that it is fuzzy whether s is bigger than r or not. "$s\tilde{\leq}r$", which expresses that the part is safe to some degree, can be called a fuzzy design criterion to describe fuzzy information in design.

If the fuzzy event "$s\tilde{\leq}r$" is written down as $\tilde{A}$, the reliability R of machine part is the probability of the fuzzy event $\tilde{A}$, i.e.,

$$R=P(\tilde{A})=P(s\tilde{\leq}r) \tag{2}$$

If stress is the random variable and strength is an invariable a_1, the membership function of the fuzzy event $\tilde{A}=\{s\tilde{\leq}a_1\}$ is as follows.

$$\mu(s)=\begin{cases} 1 & s\leq a_1 \\ \dfrac{a_2-s}{a_2-a_1} & a_1<s\leq a_2 \\ 0 & s>a_2 \end{cases} \tag{3}$$

The linear membership function in Eq. (3) is proven to be effective and reliable by a great deal of practice (Wang et al, 1988). a_1 is the numerical value of strength consulted from handbook and $a_2=(1.05\sim1.3)a_1$ can be determined by the expanding coefficient method (Wang et al, 1988).

Based on the fuzzy probability theory (Yang et al, 1992), the probability of the fuzzy event $\tilde{A}$, namely, reliability R of machine part, can be given by

$$\begin{aligned} R=P(\tilde{A})&=P(s\tilde{\leq}a_1)=\int_{-\infty}^{+\infty}f(s)\,\mu(s)\,ds \\ &=\int_{-\infty}^{a_1}f(s)\,ds+\int_{a_1}^{a_2}\frac{a_2-s}{a_2-a_1}f(s)\,ds \end{aligned} \tag{4}$$

where $f(s)$ is the probability density function of the random stress s.

Eq. (4) utilizes the fuzzy probability theory to calculate fuzzy reliability. According to the concept of the cut-set of fuzzy mathematics, the problem of fuzzy reliability can also be solved by means of the transformation from the fuzzy set

$\tilde{A}$ to the general set $\tilde{A}_\lambda$, namely, from the problem of fuzzy reliability to the problem of general reliability.

For any threshold level λ, the fuzzy event $\tilde{A}$ for safety of machine part can be transformed to $\tilde{A}_\lambda$. So, the following characteristic function $\theta(\mu(s)-\lambda)$ can be used to describe the state of safety or failure.

$$\theta(\mu(s)-\lambda)=\begin{cases} 1 & \mu(s)\geq\lambda \quad \text{Safety} \\ 0 & \mu(s)<\lambda \quad \text{Failure} \end{cases} \tag{5}$$

a_λ is the function of the threshold level λ and can be obtained based on $\mu(s)=\lambda$. It is obvious that substituting a_λ in $\mu(s)$ for s gives $\mu(a_\lambda)=\lambda$. Based on Eq. (5), it is known that the part is in the state of safety when the numerical value of the stress is not bigger than a_λ, and the part is in the state of failure when the numerical value of the stress is bigger than a_λ. Thus, to ensure safety of the part, a_λ is the allowable value of strength when the threshold level is λ. If the probability of $\tilde{A}_\lambda$ is written down as R_λ, R_λ can be expressed by

$$R_\lambda=P(\tilde{A}_\lambda)=P(s\leq a_\lambda) \tag{6}$$

Based on the equation of calculating general event probability, R_λ is given by

$$\begin{aligned} R_\lambda&=\int_{-\infty}^{+\infty} f(s)\,\theta(\mu(s)-\lambda)\,ds \\ &=\int_{-\infty}^{a_\lambda} f(s)\,\theta(\mu(s)-\lambda)\,ds+\int_{a_\lambda}^{+\infty} f(s)\,\theta(\mu(s)-\lambda)\,ds \\ &=\int_{-\infty}^{a_\lambda} f(s)\,ds \end{aligned} \tag{7}$$

Eq. (7) is the equation of calculating reliability when the value of the threshold level is set. Because λ can be any value on [0, 1], by using R_λ, R is given by (Dong, 1999a ; Dong, 1999b ; Dong, 1999c)

$$R=\int_0^1 R_\lambda d\lambda=\int_0^1\int_{-\infty}^{a_\lambda} f(s)\,ds\,d\lambda \tag{8}$$

If a_λ is obtained based on Eq. (3), using Eq. (8) yields the same result as using Eq. (4). Failure probability F can be calculated by using $F=1-R$.

3. Random Stress and Fuzzy Strength

If it is difficult to determine the undoubted numeric value of strength because there is obviously fuzzy uncertain information in design, it is better to use a fuzzy variable to express the strength of machine part (Chen, 1988). This section gives an approach to calculate reliability indexes of machine part when stress is a random variable and strength is a fuzzy variable. When stress is a fuzzy variable and strength is a random variable, a similar approach can be used.

Assuming that $\mu(r)$ is the membership function of fuzzy strength $\tilde{r}$, and $f(s)$ is the probability density function of the random stress s, based on the fuzzy probability theory, the reliability R is given by

$$R=P(s\leq\tilde{r})=\int_{-\infty}^{+\infty} f(x)\,\mu(x)\,dx \tag{9}$$

where $f(x)$ is the probability density function of the random stress $\tilde{r}$, $\mu(x)$ is the membership function of the fuzzy strength s.

Unfortunately, the calculated reliability by using Eq. (9) is false because it does not match practice and sometimes paradoxical result (For details, refer to example 2).

In fact, when the random stress s is exerted on the part and the strength of the part is the fuzzy variable $\tilde{r}$, whether the part is safe or not is a fuzzy event $A=\{s\leq\tilde{r}\}$. If R of the part is calculated based on Eq. (9) directly, the membership function of the fuzzy event $\tilde{A}$ must be obtained before Eq. (9) is used. Although the membership function of the fuzzy strength $\tilde{r}$ is known, it is not the membership function of the fuzzy event $\tilde{A}$. Because Eq. (9) takes $\mu(x)$ as the membership function of the fuzzy event $\tilde{A}$, the calculated reliability will not be the genuine reliability of the part. Therefore, Eq. (9) cannot be used to calculate the reliability R in this case.

In section 2, it is easy to obtain the membership function of the fuzzy event, so R can be calculated using the fuzzy event probability theory. Also, R can be obtained by means of the transformation from the problem of fuzzy reliability to the problem of general reliability. But in this section, the membership function, which describes whether the part is safe, cannot be obtained easily from the membership function of the fuzzy strength $\tilde{r}$. If the reliability R is calculated by means of the transformation from the problem of fuzzy reliability to the problem of general reliability, it is unnecessary to obtain the membership function of the fuzzy event $\tilde{A}$.

The concrete approach is as follows. For any threshold level λ, the internal number $\tilde{r}_\lambda=[a_\lambda, b_\lambda]$ of the fuzzy strength $\tilde{r}$ can be obtained based on the concept of the cut-set of fuzzy mathematics. So, whether machine part is safe or not is the general event $\tilde{A}_\lambda=\{s\le\tilde{r}_\lambda\}$. Assuming that strength is considered as a random variable of even distribution on $[a_\lambda, b_\lambda]$, the probability density function of the random variable $\tilde{r}_\lambda$ is given by

$$f_{\tilde{r}_\lambda}(r)=\frac{1}{b_\lambda-a_\lambda} \tag{10}$$

Based on the general reliability theory, when the threshold level is λ, the reliability R of machine part is given by (Dong, 1999c)

$$\begin{aligned} R_\lambda=P(s\le\tilde{r}_\lambda) &=\int_{a_\lambda}^{b_\lambda}\left[f_{\tilde{r}_\lambda}\int_{-\infty}^{r}f(s)\,ds\right]dr \\ &=\int_{-\infty}^{+\infty}f_s(s)\times\max\left[\min\left(\frac{b_\lambda-s}{b_\lambda-a_\lambda},\ 1\right),\ 0\right]ds \\ &=\int_{-\infty}^{a_\lambda}f(s)\,ds+\int_{a_\lambda}^{b_\lambda}\frac{b_\lambda-s}{b_\lambda-a_\lambda}f(s)\,ds \end{aligned} \tag{11}$$

where R_λ is the reliability of machine part when the threshold level is λ.

Similar to Eq. (8), while fuzzy strength and random stress, R of machine part can be expressed by

$$R=P(s\le\tilde{r})=\int_0^1 R_\lambda d\lambda=\int_0^1 P(s\le\tilde{r}_\lambda)\,d\lambda \tag{12}$$

For any threshold level λ, the failure probability F_λ of machine part is expressed by

$$\begin{aligned} F_\lambda=P(s>\tilde{r}_\lambda) &=\int_{a_\lambda}^{b_\lambda}f_{\tilde{r}_\lambda}(r)\left[\int_{r}^{+\infty}f_s(s)\,ds\right]dr \\ &=\int_{-\infty}^{+\infty}f_s(s)\times\min\left[\max\left(\frac{s-a_\lambda}{b_\lambda-a_\lambda},\ 0\right),\ 1\right]ds \\ &=\int_{a_\lambda}^{b_\lambda}\frac{s-a_\lambda}{b_\lambda-a_\lambda}f_s(s)\,ds+\int_{b_\lambda}^{+\infty}f_s(s)\,ds \end{aligned} \tag{13}$$

where F_λ is the failure probability of machine part when the threshold level is λ.

The failure probability F can be expressed by

$$F=P(s>\tilde{r})=\int_0^1 F_\lambda d\lambda=\int_0^1 P(s>\tilde{r}_\lambda)\,d\lambda \tag{14}$$

It is easily proven that $F+R=1$ based on Eqs. (12) and (14). Therefore, F can be calculated based on Eq. (14) or equation $F=1-R$.

Eq. (12) or Eq. (14) is the general equation of calculating R or F in theory. But in practical application, because of the complexity of the problem of fuzzy reliability, the analytical equations of calculating the fuzzy reliability indexes can be obtained from Eqs (12) and (14) only in some special cases. In most cases, the analytical equations cannot be obtained. In order to solve the problem, a simulation approach can be used to estimate the fuzzy reliability indexes, i.e. R or F.

4. Simulation of Reliability Indexes

When a simulation approach is used to estimate the fuzzy reliability indexes, it solves the problem of fuzzy reliability by means of the transformation from a big and complicated problem to two rela -ively small and simple problems. One of the two problems is to calculate general reliability, such as calculate the reliability for a given threshold level λ based on Eq. (7) or (11), or calculate the failure probability for the given threshold level λ according to Eq. (13). The other is to estimate fuzzy reliability indexes by using the above calculated general reliability indexes.

The steps of the simulation approach are as follows.

(1) Take a random number subjected to an even distribution on (0, 1) as a threshold level λ_i

(2) Calculate $R_{\lambda i}$ or $F_{\lambda i}$ using the corresponding equations when the threshold level is λ_i.

(3) Based on Eq. (8) or (12), it is easily known that Eq. (15) can be used to estimate the reliability R of machine part. According to Eq. (14), obviously, the failure probability F can be obtained from Eq. (16).

$$R\approx\frac{1}{n}\sum_{i=1}^{n}R_{\lambda i} \tag{15}$$

$$F \approx \frac{1}{n}\sum_{i=1}^{n} F_{\lambda i} \tag{16}$$

If n is big enough, adequately accurate reliability or failure probability can be estimated.

4.1 Simulation of Eq. (8)

In some special cases, the form of the integral function of standard normal distribution can be used to calculate the reliability of Eq. (4) or Eq. (8). For example, assuming that μ and σ are the mean and the standard deviation of the random stress of normal distribution individually, it is easy to obtain the analytical equation of calculating the reliability R from Eq. (4).

$$R = P(s \tilde{\leq} a_1) = \frac{1}{a_2 - a_1}\left\{\left[(a_2-\mu)\Phi\left(\frac{a_2-\mu}{\sigma}\right) - (a_1-\mu)\Phi\left(\frac{a_1-\mu}{\sigma}\right)\right] + \frac{\sigma}{\sqrt{2\pi}}\left[\exp\left(-\frac{(a_2-\mu)^2}{2\sigma^2}\right) - \exp\left(\frac{(a_1-\mu)^2}{2\sigma^2}\right)\right]\right\} \tag{17}$$

where $\Phi(t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{t}\exp\left(-\frac{x^2}{2}\right)dx$.

Equation (17) can be obtained from Eq. (8). but it is much more difficult to obtain Eq. (17) from Eq. (8).

The analytical equation of calculating the reliability R cannot be obtained sometimes if the membership function is not the linear membership function in Eq. (3). When the random stress and the membership function in Eq. (8) are any distributions, it is better to use the previous simulation approach to estimate the reliability or the failure probability. The simulation approach is a general approach. When the simulation approach is used, in order to obtain the adequately accurate reliability or failure probability, the number of simulation should be big enough. Therefore, to decrease calculating quantities, a relatively simple approach should be taken to calculate R_λ or F_λ. The following is a simple example.

It is familiar that stress is a random variable of normal distribution. If so, Eq. (7) can be expressed by

$$R_\lambda = \frac{1}{2} + \frac{1}{\sqrt{2\pi}}\sum_{i=0}^{\infty}(-1)^i \frac{1}{(2i+1)\cdot 2^i \cdot i\,!}\left(\frac{a_\lambda - \mu}{\sigma}\right)^{2i+1} \tag{18}$$

In Eq. (18), a_λ can be obtained from the membership function $\mu(s)$. If $\mu(s)$ is linear, $a_\lambda = a_2 - (a_2 - a_1)\lambda$ can be obtained based on Eq. (3). Substituting the above a_λ into Eq. (18) yields

$$R_\lambda = \frac{1}{2} + \frac{1}{\sqrt{2\pi}}\sum_{i=0}^{\infty}(-1)^i \frac{1}{(2i+1)\cdot 2^i \cdot i\,!} \times \left[\frac{(a_2-\mu) - (a_2 - a_1)\lambda}{\sigma}\right]^{2i+1} \tag{19}$$

Substituting Eq. (19) into Eq. (8) yields

$$R = \frac{1}{2} + \frac{1}{\sqrt{2\pi}}\sum_{i=0}^{\infty}(-1)^i \frac{1}{(2i+1)\cdot 2^{i+1}\cdot (i+1)\,!} \times \frac{\sigma}{a_2 - a_1}\left[\left(\frac{a_2-\mu}{\sigma}\right)^{2(i+1)} - \left(\frac{a_1-\mu}{\sigma}\right)^{2(i+1)}\right] \tag{20}$$

Equation (20) is the progression expression of Eq. (8) when stress is the normal distribution, and a_λ in Eq. (8) is determined based on Eq. (3). Obviously, Eq. (20) can also be obtained from Eq. (17) or Eq. (4), but the obtaining process is more complicated.

In the above case, because the progression expression of calculating R can be given by Eq. (20), it is easy to calculate the reliability R. Unfortunately, the progression expressions of calculating R cannot be obtained in most cases. For this reason, a better approach to obtain R is to calculate R_λ based on Eq. (7) and estimate R by using of Eq. (15).

If the general reliability R_λ can be expressed by a progression and the previous simulation approach is used to estimate R, calculating quantity will be reduced to a certain extent sometimes. In some special cases, if R can be expressed by a progression, such as Eq. (20), the complicated problem of fuzzy reliability is transformed to a relatively very simple problem, and the calculating quantity will be reduced greatly.

Section 4.2 shows that Eq. (20) is very useful in some cases.

4.2 Simulation of Eq. (12)

When strength of machine part is a fuzzy variable $\tilde{r}$ and stress is a random variable s, it is known that Eq. (12) can be used to calculate the reliability R of the part. Unluckily, the analytical equation of calculating R can hardly be obtained from Eq. (12) in any case. Therefore, the previous simulation approach is an alternative to estimate R. Here is an example.

Provided that μ and σ are the mean and the standard deviation of the random stress of normal distribution respectively, $\tilde{r}_\lambda=[a_\lambda, b_\lambda]$ is the interval number of the fuzzy strength $\tilde{r}$, for any threshold level λ, from Eq. (11), R_λ can be obtained as

$$R_\lambda=\frac{1}{b_\lambda-a_\lambda}\left\{\left[(b_\lambda-\mu)\Phi\left(\frac{b_\lambda-\mu}{\sigma}\right)-(a_\lambda-\mu)\Phi\left(\frac{a_\lambda-\mu}{\sigma}\right)\right]+\frac{\sigma}{\sqrt{2\pi}}\left[\exp\left(-\frac{(b_\lambda-\mu)^2}{2\sigma^2}\right)-\exp\left(-\frac{(a_\lambda-\mu)^2}{2\sigma^2}\right)\right]\right\} \tag{21}$$

By comparing Eq. (21) with Eq. (17), if a_2 and a_1 in Eq. (20) are replaced by the above b_λ and a_λ individually, the progression expression of Eq. (21) can be obtained. This is to say, the progression of the reliability R_λ can be given by

$$R_\lambda=\frac{1}{2}+\frac{1}{\sqrt{2\pi}}\sum_{i=0}^{\infty}(-1)^i\frac{1}{(2i+1)\cdot 2^{i+1}\cdot(i+1)\,!}\times\frac{\sigma}{b_\lambda-a_\lambda}\left[\left(\frac{b_\lambda-\mu}{\sigma}\right)^{2(i+1)}-\left(\frac{a_\lambda-\mu}{\sigma}\right)^{2(i+1)}\right] \tag{22}$$

Therefore, if a_λ and b_λ have already been obtained based on the concept of the cut-set of fuzzy mathematics, R_λ can be calculated based on Eq. (22) and R can be estimated according to Eq. (15).

5. Examples

To explain the previous approaches, the probability density functions and the membership functions in the first two examples are in the simplest form.

Example 1 Provided that the probability density function $f(s)$ of random stress and the membership function $\mu(s)$, which expresses a fuzzy design criterion, are as follows.

$$f(s)=\begin{cases}\frac{s}{9} & 0<s\leq 3\\ \frac{6-s}{9} & 3<s\leq 6\\ 0 & \text{Else}\end{cases}$$

$$\mu(s)=\begin{cases}1 & s\leq 4\\ 5-s & 4<s\leq 5\\ 0 & s>5\end{cases}$$

We want to calculate the reliability R in consideration of this fuzzy information.

Based on the fuzzy probability theory, the use of the known conditions yields

$$R=\int_{-\infty}^{+\infty}f(s)\mu(s)\,ds=\int_0^3 f(s)\mu(s)\,ds+\int_3^4 f(s)\mu(s)\,ds+\int_4^5 f(s)\mu(s)\,ds$$

Substituting the related data into the above equation gives $R=47/54=0.870\,4$. The above method utilizes the fuzzy probability theory to calculate R directly. Eqs. (7) and (8) can be used to calculate R too.

Based on $\mu(s)=\lambda$, $a_\lambda=5-\lambda$ can be obtained. Because $a_\lambda=5-\lambda\geq 4>3$, according to Eq. (7), the following equation can be obtained.

$$R_\lambda=\int_{-\infty}^{a_\lambda}f(s)\,ds=\int_0^{a_\lambda}f(s)\,ds=\int_0^3\frac{s}{9}ds+\int_3^{a_\lambda}\frac{6-s}{9}\,ds=\frac{1}{2}+\frac{1}{18}(8-2\lambda-\lambda^2) \tag{23}$$

Substituting the above equation into Eq. (8) yields

$$R=\int_0^1\left[\frac{1}{2}+\frac{1}{18}(8-2\lambda-\lambda^2)\right]d\lambda=47/54=0.870\,4$$

Because the membership function, which expresses the fuzzy design criterion, is the same as the membership function of fuzzy event, the results by using of the two approaches are the same. Calculating R_λ based on Eq. (23), and then simulating R by using Eq. (15) yields $R\approx 0.870\,2$. The number of simulation is 10000.

If the fuzzy information is not taken into account, the membership function is simply given by

$$\mu(s)=\begin{cases}1 & s\leq 4\\ 0 & s>4\end{cases}$$

Therefore, the reliability R is

$$R=\int_{-\infty}^{+\infty}f(s)\mu(s)\,ds=\int_0^3 f(s)\mu(s)\,ds+\int_3^4 f(s)\mu(s)\,ds=\int_0^3 f(s)\,ds+\int_3^4 f(s)\,ds=\frac{7}{9}=0.78$$

Example 2 Assuming that stress is a random variable and its probability density function is

$$f(s)=\begin{cases} s-5 & 5<s\le 6 \\ 7-s & 6<s\le 7 \\ 0 & \text{Else} \end{cases}$$

Provided strength is a fuzzy variable $\tilde{r}$ and its membership function is

$$\mu(r)=\begin{cases} \frac{r-6}{4} & 6<r\le 10 \\ \frac{14-r}{4} & 10<r\le 14 \\ 0 & \text{Else} \end{cases}$$

Let us calcalate R.

If R is calculated based on Eq. (9), we get

$$R=P(s\le\tilde{r})=\int_{-\infty}^{+\infty} f(x)\mu(x)\,dx$$
$$=\int_{6}^{7}(7-x)\frac{x-6}{4}\,dx=0.041\ 667$$

In the following, R is calculated based on Eqs. (11) and (12). For any threshold level $\lambda\in[0,1]$, according to the concept of the cut-set of the fuzzy mathematics, the interval number of the fuzzy strength is

$$\tilde{r}_\lambda=[a_\lambda, b_\lambda]=[6+4\lambda, 14-4\lambda]$$

From the given membership function of the fuzzy strength $\tilde{r}$, it is known that $a_\lambda>6$ and $b_\lambda\ge 7$. Based on Eq. (11), R_λ is given by

$$R_\lambda=P(s\le\tilde{r}_\lambda)$$
$$=\int_{-\infty}^{a_\lambda} f(s)\,ds+\int_{a_\lambda}^{b_\lambda}\frac{b_\lambda-s}{b_\lambda-a_\lambda}f(s)\,ds$$
$$=\int_{5}^{6} f(s)\,ds+\int_{6}^{a_\lambda} f(s)\,ds+\int_{a_\lambda}^{7}\frac{b_\lambda-s}{b_\lambda-a_\lambda}f(s)\,ds$$

Based on Eq. (12), R is as follows.

$$R=P(s\le\tilde{r})=\int_{0}^{1} R_\lambda d\lambda$$

Substituting a_λ, b_λ and the given $f(s)$ into the above equation yields $R=0.998\ 627$. When the number of simulation is 10000, using the simulation approach gives $R=0.998\ 632$.

Obviously, if the value of strength is an invariable 10, R is equal to 1. When the strength is a fuzzy variable, which means that the strength is about 10, it is impossible for the reliability R be so low as 0.041 667. The reason is that the membership function of the fuzzy strength is not that of the fuzzy event of part safety. Therefore, the above conclusion illustrates that it is unsuitable to calculate R directly by using of Eq. (9) in this case.

Example 3 Stress is a random variable of normal distribution and its mean and standard deviation are 175.76 MPa and 33.33 MPa differently. Strength is a constant 260 MPa. Provided that fuzzy information is taken into consideration, the membership function is shown as Eq. (3) and $a_2=274$ MPa is taken. We want to calculate R.

To compare calculated results, the accurate value of R should be obtained. Therefore, a program was written based on Eq. (17) and $R=0.996\ 589\ 6$ was obtained. Of course, the calculating quantity is too large. The calculated reliabilities based on Eq. (20) are shown in Table 1. The reliabilities are calculated when only the first n items in Eq. (20) are taken. From table 1, only taking the first 19 items is enough to ensure that R has seven significant numbers. Of course, when R can be expressed by a progression, the calculating quantity of obtaining R will be relatively very small.

If different number of simulation and the different items are taken when R is estimated based on Eqs. (19) and (15), the estimated reliabilities are shown in Table 2. In Table 2, when the number of simulation is 10000, to ensure that R converges to 0.996 585 0, the first 19 items should be taken. If only the first 17 items are taken, R is 0.996 585 8, which does not converge to 0.996 585 0.

When R is estimated by the previous simulation approach, the calculating quantity will be not so large if R_λ can be expressed by a progression. The errors of the calculated reliabilities are caused by the simulation approach itself.

Example 4 Stress is a random variable of normal distribution and its mean and standard deviation are 175.76 MPa and 33.33 MPa differently. Strength is a fuzzy variable $\tilde{r}$ and its membership function is

$$\mu(r)=\begin{cases} \exp\left[-\frac{(r-265.5)^2}{10^2}\right] & r\le 265.5 \\ \exp\left[-\frac{(r-265.5)^2}{5^2}\right] & r>265.5 \end{cases}$$

Let us calculate R when the strength is the fuzzy

Table 1 Calculated reliabilities by taking the first n items

n items	10	15	17	19	20	25
Reliability	0.989 322 0	0.996 607 3	0.996 590 4	0.996 589 6	0.996 589 6	0.996 589 6

Table 2 Estimated reliabilities by the simulation approach of Eq. (19)

Number of simulation	n items					
	10	15	17	19	20	25
	Reliability					
1000	0.989 370 2	0.996 590 4	0.996 576 8	0.996 576 0	0.996 576 0	0.996 576 0
10000	0.989 342 3	0.996 599 6	0.996 585 8	0.996 585 0	0.996 585 0	0.996 585 0

Table 3 Estimated reliabilities by the simulation approach of Eq. (22)

Number of simulation	n items					
	10	15	17	19	20	25
	Reliability					
1000	0.991 505 1	0.995 365 4	0.995 359 7	0.995 359 4	0.995 359 4	0.995 359 4
10000	0.991 498 3	0.995 360 4	0.995 354 7	0.995 354 4	0.995 354 4	0.995 354 4

Table 4 Estimated reliabilities with different data

Fuzzy Strength	(265.5, 10, 5)	(265.5, 20, 5)	(265.5, 10, 10)	(250,10, 5)	(270,10, 5)
Reliability	0.995 354 4	0.992 090 0	0.996 010 1	0.983 859 1	0.996 878 5

variable.

In this example, the progression expression of R cannot be obtained, and the equation of calculating R cannot be obtained too. From the previous discussion, it is known that Eq. (22) can be used to calculate R_λ, and Eq. (15) can be used to estimate R. According to the membership function of the fuzzy strength $\tilde{r}$, for any threshold level λ, the interval number of the fuzzy strength $\tilde{r}$ is given by

$$\begin{aligned}\tilde{r}_\lambda &= [a_\lambda, b_\lambda] \\ &= [265.5-10\sqrt{-\ln\lambda},\ 265.5+5\sqrt{-\ln\lambda}\,]\end{aligned}$$

For the different number of simulation and the different items of calculating R_λ based on Eq. (22), simulation results are shown in Table 3. Assuming that (m, α, β) is used to express a fuzzy variable, the above fuzzy strength $\tilde{r}$ can be expressed by (265.5, 10, 5).

To ensure that R converges to a certain value, how many items should be taken relates to stress and strength. Generally speaking, the more reliability R is close to 1, the more items should be taken. For example, in example 4, if fuzzy strength is (290, 10, 5) MPa, the first 25 items should be taken to ensure that R converges to 0.999 563 2.

If m, α or β of the fuzzy strength $\tilde{r}$ is changed, the calculated reliabilities are shown in Table 4. The number of simulation is 10000.

Table 4 shows that R will rise if m or β of the fuzzy strength is increased and R will be reduced if m of the fuzzy strength is decreased or α of the fuzzy strength is increased.

6. Conclusion

From the view of methodology, the model of analyzing fuzzy reliability introduced in this paper is of universal significance. In the paper, two basic cases are given, but the model can be used to analyze the fuzzy reliability of machine part when there are both random information and fuzzy information at the same time in machine design. It is known that the problem of the fuzzy reliability will surely become so complicated that the fuzzy reliability indexes cannot be calculated directly sometimes. The analytical equations of calculating the fuzzy reliability indexes cannot be obtained in most cases, and therefore the simulation approach can be used to estimate them.

By means of the transformation from the problem of fuzzy reliability to the problem of general reliability, the complicated fuzzy reliability problem may be transformed to two simple ones. If the progression expressions of the fuzzy reliability indexes can be obtained, the calculating quantities of obtaining the fuzzy reliability indexes will decrease greatly. If only the progression expressions of the general reliability indexes can be obtained, the calculating quantities of obtaining the fuzzy reliability indexes will decrease to a certain extent. The approach discussed in the paper is very effective in some cases.

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