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A Conceptual Two-Layer Model of Thermohaline Circulation in a Pie-Shaped β-Plane Basin

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The three dimensional structure of thermohaline circulation in a β-plane is investigated using a conceptual two layer model and a scaling argument. In this simple model, the water mass formation region is excluded. The upper layer represents the oceans above the main thermocline. The lower layer represents the deep ocean below the thermocline and is much thicker than the upper layer. In each layer, geostrophy and the linear vorticity balance are assumed. The cross interfacial velocity that compensates for the deep water mass formation balances downward heat diffusion from the top. From the above relations, we can determine the thickness of the upper layer, which is the same as thermocline depth. The results we get is basically the same as that we get for an *f*-plane ocean or the classical thermocline theory. Mass budget using the velocity scales from the scaling argument shows that western boundary and interior transports are much larger than the net meridional transport. Therefore in the thermohaline circulation, horizontal circulation is much stronger than the vertical circulation occuring on a meridional plane.

Key words: Thermohaline circulation, Scaling law

INTRODUCTION

The local imbalance between the solar radiation and the earth's back radiation generates a meridional temperature gradient on the surface of the oceans. The temperature gradient drives a well-known convective oceanic circulation, the thermohaline circulation. Water in the polar oceans loses heat to the atmosphere, becomes dense, and then sinks to some depth, forming the deep water mass. The newly formed deep-water mass flows away from the polar regions toward the equator. While flowing to the equator, the deep flow slowly upwells to the surface to compensate for the sinking. This water gains heat from the solar radiation and becomes warm surface water, which then flows toward the polar oceans to satisfy continuity, thus completing the cycle.

The thermohaline circulation is believed to be important in maintaining the climate of the earth and oceans in two ways. First, the oceanic currents carry about half of the heat accumulated in the equatorial regions toward the pole (Vonder Haar and Oort, 1973), and compensate the imbalance of the solar radiation and the earth's back radiation. They help to reduce the temperature contrast between the equatorial region and the polar region. Second, the circulation supplies cold water to the upper ocean that balances downward heat diffusion from the surface and helps to maintain the thermocline where temperature changes rapidly. Therefore, the changes and variations of the climate of the earth are closely related to the thermohaline circulation.

Due to its role in the climate, the thermohaline circulation has caught much attention of many researchers. In many cases as sketched above the thermohaline circulation is considered as a pure two-dimensional phenomenon occurring on the meridional plane. If we are only interested in the net meridional transports of heat or mass flux, such a two-dimensional view, which comes from a zonal mean of the thermohaline circulation, would suffice. If we are interested in the variabilities or changes of the thermohaline circulation, we are to investigate the three dimensional structure of the circulation. The thermohaline circulation is a highly non-linear phenomenon and it is not easy to study its three-dimensional structure using analytical

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method. Therefore, studies based on simple scaling laws or one or two layer models are commonly used to investigate the structure of thermohaline circulation.

Stern (1975) derived a scaling law for convective circulation in a pie-shaped basin in β-plane. The thermal wind balance, and a balance between downward heat diffusion and upwelling of cold water were assumed. By assuming that the potential energy released by warm rising thermocline motion and cold sinking polar motion is balanced by dissipation in the Ekman layer, he derived the same scaling law for the thickness of a thermal boundary layer as that of a rotating annulus as in Hignett *et al.* (1981). He concluded that both the β-effect and meridional boundaries were not important in thermal boundary layer structures. Although meridional boundaries were included, the meridional geostrophic flow due to zonal pressure gradient was not included. The meridional mass transport was from the Ekman transport driven by zonal geostrophic flows as in a rotating annulus. In the oceans the main part of the meridional flow is geostrophic flows due to zonal pressure gradients that are maintained by the meridional boundaries and one can easily suppose that the theory by Stern (1975) can not be applicable to the oceans as shown by Park and Whitehead (1999).

Park and Whitehead (1999) and Park and Bryan (2000) showed that a simple scaling law using geostrophic relation and advective-diffusive heat balance is more plausible one for the thermohaline circulation of the oceans using a laboratory experiment and a numerical experiment. The scaling, in fact, is basically the same as the classical thermocline theory by Robinson and Stommel (1959) and explained in detail later in this paper, The scaling law, however, cannot explain the three dimensional structure of the thermohaline circulation.

Stommel (1965) calculated circulation in a two-layer ocean using the linear vorticity balance and mass balance when the depth of the interface, which represents the thermocline, is given. The model is driven by uniform vertical motion, which represents the interior upwelling compensating deep water mass formation in high latitude. Later, Veronis (1976) solved a similar problem more rigorously and obtained circulation pattern and the shape of the interface, when the upper layer depth along the eastern boundary is given. In his model, however, the unknown locations and the rate of deep water mass formation in remote oceans that communicates with the basin of interest does not allow one to obtain exact solutions. If we confine our interest to a single hemispheric basin, and specify the location and the rate of water mass formation, we can obtain exact solution within his framework. However, there is no way of parameterizing the upper layer depth along the eastern boundary, which determines the scale depth of the interface.

In this study, the three dimensional structure of thermohaline circulation is investigated using a conceptual two-layer model and a simple scaling argument in a β-plane as described in Section 2. The vertical scale of convection that produces deep water is comparable to that of the horizontal scale so that water mass formation region is governed by non-hydrostatic dynamics. This area, therefore, cannot be explained using a simple scaling law based on geostrophy and excluded in this simple model. The effect of the interior upwelling compensating for the convection is investigated as in earlier studies such as Stommel (1965) and Veronis (1976). Although the analysis of circulation in a β-plane described in this paper is much simpler than those of Stommel (1965) and Veronis (1976), it was possible to obtain the three dimensional pattern of the thermohaline circulation and determine the thickness of upper layer by introducing the thermodynamics. In this simple study, surface wind stress is not included. Instead in Section 3, the effect of the wind stress on thermocline structure is discussed using a scaling argument.

CONVECTIVE CIRCULATION IN A β-PLANE

Assume a pi-shaped spherical sector in a β-plane as sketched in Fig. 1. Meridional temperature difference ΔT is applied over the surface. If we exclude the water mass formation region, which is governed by non-hydrostatic dynamics and confined to a narrow region near the northern end, the ocean can be considered as a two-layer system. The lower layer is equivalent to the homogeneous deep oceans below the thermocline investigated in Stommel and Arons (1960). The upper layer is equivalent to the surface layer above the thermocline. The interface, thus, is equivalent to the thermocline.

One might wonder what makes the western boundary current in the absence of wind stress as sketched in Fig. 1. As shown in the two-layer model of Kawase (1987) or the planetary geostrophic model by Colin de Verdiere (1988), in the interior of a β-plane basin, the vertical upwelling compensating water mass formation produces southward (upper layer) or northward

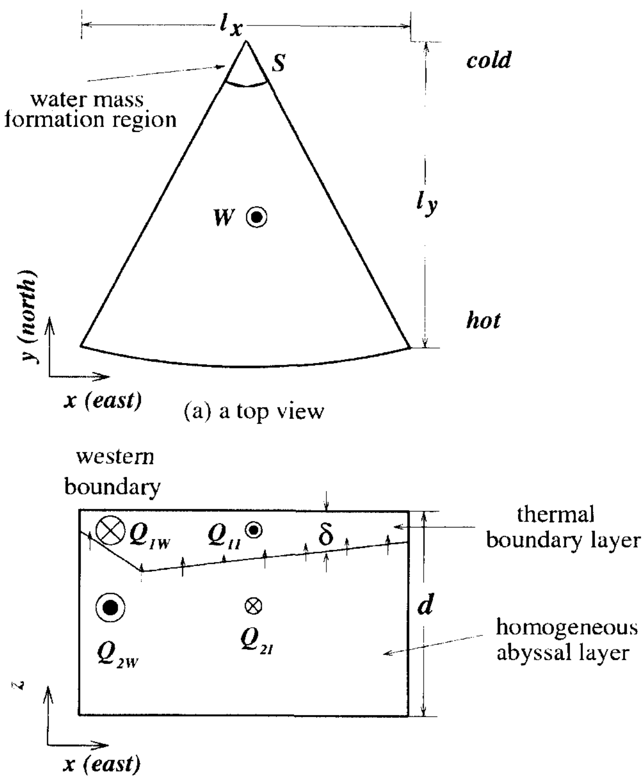

Fig. 1. A sketch of a pie-shaped spherical sector basin in a β-plane. (a) a top view and (b) a zonal section. Water mass formation occurs over the northen corner of the basin (shaded area) with strength S.

(lower layer) motion according to the linear vorticity balance $\beta v = f w_z$. The northward western boundary current in the upper layer and the southward western boundary current in the lower layer are required to balance the interior flows that are in the opposite directions as in the wind driven circulations.

Assume uniform vertical motion W, which compensates the water mass formation, from the lower layer into the upper layer across the interface. Numerical experiment by Park and Bryan (2001), in fact, shows that diapycnal velocity at the base of thermocline is relatively uniform. The water mass formation in the northern region drives the circulation in the lower layer as in Stommel and Arons (1960). The interior vertical motion stretches the lower layer so the linear vorticity balance in the lower layer yields

$$\beta V_2(d-\delta) = f_0 W$$

since vertical motion vanishes at the bottom. Here V_2 is the meridional velocity scale in the lower layer, and δ is a depth scale of thermocline and we derive this later in this section. In the oceans the thermocline is much thinner than the depth of the basin, d i.e., $d \gg \delta$,

$$V_2 \sim \frac{f_0 W}{\beta d} > 0$$

so the interior flow is northward.

To the north of a latitude line θ, the mass balance in the lower layer among the interior mass transport $Q_{2I} \approx d l_x V_2$, the western boundary transport Q_{2W}, the vertical mass flux $WA(\theta)$, and the water mass formation rate $S = WA(\theta_S)$, requires

$$Q_{2W} = -Q_{2I} - S + WA(\theta) < 0$$

Here, $l_x = R\Delta\Phi \cos\theta$, the width of the basin at a latitude line θ, where R is the radius of the earth and $\Delta\Phi$ is the width of the basin in radian, is much larger than that of the western boundary. The surface area of the basin to the north of latitude circle θ $A(\theta) = R^2\Delta\Phi(\sin\theta_N - \sin\theta)$, where θ_N is the latitude of the source region. The western boundary current is southward, and its transport is larger than that in the interior since the interior has to supply the upwelling. The net meridional transport in the lower layer at latitude θ, $M_2(\theta) = Q_{2W} + Q_{2I} = -S + WA(\theta) < 0$. The net flow is southward and smaller than $|Q_{2W}|$ and $|Q_{2I}|$ as in Stommel and Arons (1960).

The vertical motion shrinks the upper layer so the vorticity equation yields the meridional velocity scale of the upper layer V_1.

$$V_1 \sim -\frac{f_0 W}{\beta \delta} < 0$$

and the interior flow is southward. If we consider the mass balance in the upper layer to the north of θ,

$$Q_{1W} = -Q_{1I} + S - WA(\theta) > 0$$

The western boundary transport is northward and its transport is larger than that in the interior. The upwelled water should return to the north to satisfy continuity. The interior flow is southward and it cannot carry the upwelled water. Instead, the northward western boundary current should take the water to the north, and the boundary transport is larger than the interior transport. The net meridional transport $M_1(\theta) = Q_{1W} + Q_{1I} = -S - WA(\theta) = -M_2(\theta) < 0$, and is northward. As in the lower layer, the transport is smaller than $|Q_{1W}|$ and $|Q_{1I}|$.

The interior flows in the lower layer and the upper layer are opposite direction so that the thermal wind relation requires

$$\frac{f_0(V_1-V_2)}{\delta} \sim \frac{g\alpha\Delta T}{l_x} \sim \frac{g\alpha\Delta T}{l_y}$$

Here, we use $T_y \sim T_x$ as proved by Park and Bryan (2000). Since $\delta \ll d$, and therefore $|V_1| \gg |V_2|$, we get

$$\delta \sim \left(\frac{l_y W f_0^2}{g\alpha\beta\Delta T}\right)^{1/2}$$

The upward advection of cold water across the interface should be balanced by the downward diffusion of heat ($wT_z \approx \kappa T_{zz}$) as in Munk (1965), so

$$W \sim \frac{\kappa}{\delta} \quad (1)$$

where κ is the thermal diffusivity of sea water. If we eliminate W from the above two relations, the thermal boundary layer thickness δ becomes

$$\delta \sim \left(\frac{\kappa l_y f_0^2}{g\alpha\beta\Delta T}\right)^{1/3} \quad (2)$$

This is the same as that in the classical thermocline theory by Robinson and Stommel (1959).

If we use the above scales

$$Q_{1I}(\theta) = V_1 l_x \delta = -\frac{f_0 W}{\beta} l_x \text{ and}$$

$$M_1(\theta) = S - WA(\theta) = W\{A(\theta_S) - A(\theta)\} = WRl_o(\sin\theta - \theta_S)$$

Therefore, $r_{1I}(\theta)$ the ratio between Q_{1I}, the interior transport, and M_1, the net meridional transport,

$$r_{1I}(\theta) = \frac{Q_{1I}}{M_1} = \frac{f_0\cos\theta}{\beta R(\sin\theta - \sin\theta_S)}$$

Since $Q_{1W}(\theta) = M_1 - Q_{1I}$, the ratio between the western boundary transport and the net meridional transport

$$r_{1W}(\theta) = 1 - R_{1I}$$

If we do the same calculation for the second layer, we get

$$r_{2I}(\theta) = r_{1I}, \text{ and}$$

$$r_{2W}(\theta) = 1 - r_{2I}$$

since the motions driven by the buoyancy forcing are purely baroclinic.

At mid-latitude of a hemispheric basin, we can set l_x=6000 km, $f_0 = 1\times10^{-4}\,s^{-1}$, $\beta = 2\times10^{-11}\,s^{-1}m^{-1}$, and y=3000 km. Then we get $r_{1I} = r_{2I} \approx -1.5$ and $r_{1W} = r_{2W} \approx 2.5$. As explained earlier, the interior flows in each layer are opposite to the corresponding net meridional flows and r_{iI}'s, where i=1 and 2, are negative. The western boundary transports and the net meridional transport are in the same direction in each layer so r_{iW}'s are positive. In any case, the values of r's are significantly larger than 1, in other words the net meridional flow is significantly weaker than the western boundary currents or the interior flows. Therefore, we can see that the horizontal component of the thermohaline circulation is much stronger than the vertical component occurring on a meridional plane.

Table 1. The interior and western boundary transports driven by deep water mass formation in the North Atlantic.

Interior	upper layer	−15 Sv to -22.5 Sv
	lower layer	15 Sv to 22.5 Sv
western boundary	upper layer	25 Sv to 37.5 Sv
	lower layer	−25 Sv to -37.5 Sv

The strength of the North Atlantic meridional overturing circulation is known to be between 10 Sv and 15 Sv. If we apply the above results to mid-latitude North Atlantic, we get the strength of the interior flow and the western boundary flow in each layer as in Table 1.

The interior Sverdrup transport in the North Atlantic is about 40Sv (Munk, 1950), but the strength of the Gulf Stream is more than 80Sv off Cape Hatteras (Pickard and Emery, 1982) and much larger and many researchers have tried to explain the difference mostly with a recirculation gyre. As can be seen in Table 1, the southward interior flow due to buoyancy driven circulation is comparable to that of the Sverdrup transport and make significant contribution to the northward western boundary current. If we include the horizontal circulation driven by the thermohaline circulation, we can reduce the difference between the interior Sverdrup transport and the western boundary current significantly. With a four-mooring, transport-resolving current meter array deployed East of Abaco, Bahamas at 26.5°N, Lee et al. (1990) reported that the southward transport of the Deep Western Boundary Current of the North Atlantic is about 30Sv. The estimation of the deep western boundary current shown in Table 1, although it is a very crude estimate, is comparable to the observed value.

The total meridional heat flux H is

$$H \sim \rho_o C_p \Delta T\, S$$

and since $S \sim W\, l_x l_y$

$$H \sim \rho_o C_p = \left(\frac{g\alpha l_y^2 l_x^3 \kappa^2 \beta \Delta T^4}{f_0^2} \right)^{1/3}$$

If $\beta = f_0/l_y$ is introduced, and H in Eqs. (1) and (2) are the same as those applied to an f-plane as in Park and Whitehead (1999), who also show that prediction from the scaling law and the observations are comparable, or to a spherical sector as in Park and Bryan (2000). The main difference between an f-plane and a β-plane in terms of the above scaling argument is vertical velocity scale. In an f-plane the continuity is used, while in a β-plane the linear vorticity balance is adopted. The vertical velocity scales from the two equations happen to be the same and yield the same scaling law. This does not necessary mean that thermohaline circulations from an f-plane and a β-plane are the same, because the scaling law only predicts zonally averaged properties. Since we are in a β-plane, we can assume a narrow western boundary current as depicted in Fig. 1.

DISCUSSIONS

Using a conceptual two layer model in a β-plane, three dimensional structures of thermohaline circulation have been investigated. In the model, the thickness of the upper layer, which was given without physical reasoning in previous studies, is determined using a scaling argument based on geostrophy, advective-diffusive heat balance and the linear vorticity balance. We have quantified the ratio between the interior transport and the boundary transport and shown that thermohaline circulation could make a significant contribution to western boundary transport.

In this study, however, the effect of the surface wind stress is neglected. In Bryan and Cox (1967), the wind stress at the surface had no apparent effect on the meridional heat transport, although the wind stress changed horizontal circulation from a single gyre structure to a faster double gyre one. The wind driven gyres were faster than the thermally driven one, but a large part of the former is simply recirculation and did not contribute to the meridional overturning circulation much. The surface wind stress, however, is another important factor in shaping the thermocline as shown in Luyten *et al.* (1983). Here, we discuss the effect of the surface wind stress on the thermocline using a scaling argument.

The Effect of Wind Stress

The upwelling velocity scale (Eq. 1) used in the scaling argument in Section 2 is internally generated by the thermal forcing and independent of the Ekman pumping. If the Ekman pumping is specified at the surface, two quantities (V_1 and δ_T) have to be determined using three independent equations (thermal wind relation, the linear vorticity equation and the heat equation); the problem becomes over-specified. Thus, the thermocline scale, Eq. 2 is not compatible with the Ekman pumping. Equation 2, which is basically the same as the classical thermocline theory by Robinson and Stommel (1959), is believed to be valid in the deep part of the oceanic thermocline below the wind driven thermocline (Welander, 1971)

Stommel and Webster (1962) show how Ekman pumping modifies the advective-diffusive thermocline, Eq. 2, in the interior of an ocean with a similarity solution. The zonal advection of heat was neglected, and a balance between the advection and diffusion was assumed. The ocean could be divided into two vertical regions. The upper region is above the thermocline with $W<0$, and the lower region is below the thermocline with $W>0$. The interface between the two layer represents the depth of the thermocline. The diffusion acts only at the interface so that the density balance is advective away from the interface. The location and the thickness of the interface are determined by matching temperature, vertical velocity and their vertical gradients through the interface.

In a subtropical ocean, the thermocline becomes an internal boundary layer. The depth of the internal boundary layer is the same as the thermocline thickness from the ideal fluid (wind driven) thermocline theory (Welander, 1971).

$$\delta_a \sim \left(\frac{W_E f_0 l_y^2}{g\alpha\Delta T} \right)^{1/2} \quad (3)$$

The thickness of the internal boundary layer (Stommel and Webster, 1962) is

$$\delta_I \sim \left(\frac{d f_0^2 \kappa}{g\alpha\Delta T \delta_a \beta} \right)^{1/2} = \left(\frac{d l_y f_0 \kappa}{g\alpha\Delta T \delta_a} \right)^{1/2}$$

if we use $\beta = f_0/l_y$. If we estimate the above scales with canonical values from the oceans, which are ΔT=20°C, κ=0.1~1×10^{-4}m^{2}s^{-1}, f_o=10^{-4}s^{-1}, l_y=6×10^6 m, g=10 ms^{-2} and α=2×10^{-4}°C^{-1}, we get $\delta_I \approx 100$ m and smaller than δ_a, which is about when Ekman pumping velocity $W_E = 10^{-6}$ ms^{-1}. In subpolar ocean, $w<0$ from the top to the bottom so that the interface becomes the same as the sea surface. Ekman suction at the surface lifts the thermocline and makes it shallow.

Salmon (1990) showed similar results with a similarity solution of a simpler form.

The wind stress may not change the meridional heat transport as suggested by Bryan and Cox (1967), but the wind stress does change the thermocline depth; where $W_E<0$, it makes the thermocline deep, and where $W_E>0$, it makes the thermocline shallow. At this stage, however, it is not possible to give a quantitative answer to how the surface wind stress modifies the thermocline thickness and the meridional heat flux of the diffusive thermocline theory.

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