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SELECTION PROCEDURES TO SELECT POPULATIONS BETTER THAN A CONTROL

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ABSTRACT

In this paper, we propose two selection procedures for selecting populations better than a control population. The bestness is defined in terms of location parameter. One of the procedures is based on two-sample linear rank statistics whereas the other one is based on a comparatively simple statistic, and is useful when testing time is expensive so that an early termination of an experiment is desirable. The proposed selection procedures are seen to be strongly monotone. Performance of the proposed procedures is assessed through simulation study.

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1. INTRODUCTION

Let $\pi_0, \pi_1, \ldots, \pi_k$ be $(k+1)$, $k \geq 2$ independent populations. The population π_0 is assumed to be control population and populations $\pi_1, \ldots, \pi_k$ are the treatment populations. In this paper, we define bestness in terms of location parameters and the problem of selecting all populations better than the control is considered in two cases. In the first case (Case 1), assume that the population π_i has the absolutely continuous distribution function $F_i(x) = F(x - \theta_i)$, where θ_i is the location parameter, $i = 0, 1, \ldots, k$ and $F(\cdot)$ is an (unknown) absolutely continuous distribution function. The treatment population π_0 is said to be better than the control population π_0 if $\theta_i \geq \theta_0$, $i = 0, 1, \ldots, k$. The goal is to select a subset of the k treatment populations, which contains all the populations better than the control. Any such selection is called a correct selection (CS). In the

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second case (Case 2) the underlying assumption is that the $(k+1)$ populations differ in their location parameters and have $F(0) = p$, so that θ_i, the location parameter of the i^{th} population, is its p^{th} quantile, $i = 0, 1, \ldots, k$.

Practical applications of the above formulation are encountered in agriculture, business concerns, *etc.* In agriculture, usually the aim of the experimenter is to select or recommend those varieties that has more average yield in comparison to a control variety. Similarly a business concern, using different advertising methods to enhance the sales, selects/prefers the advertising methods which produce the more average sales in comparison to the control case.

Rizvi *et al.* (1968) proposed non-parametric ranking procedures for comparison of treatment populations with a control in terms of α-quantiles. Deshpande and Mehta (1983) proposed procedures while comparing populations in terms of distribution functions. Gill and Mehta (1993) developed selection procedures for selecting population better than a control population while restricting to only scale parameters. Lehmann (1963), Puri and Puri (1968, 1969), Bartlett and Govindarajulu (1968) developed selection procedures based on joint ranking of sample observations from all the populations. However, Rizvi and Woodworth (1970) provided counter examples that the procedures based on joint ranking do not control the probability of correct selection over both the slippage parametric configuration (used under indifference zone) as well as the entire parametric space (used under subset selection approach). Hsu (1980, 1981) used pairwise ranking to propose subset selection procedures and has shown that these procedures control the probability of correct selection (PCS) over the entire parametric space. In a different approach, Lann (1991a, 1991b, 1992) considered subset selection procedures for an almost best population for location probability model. Here the goal was to select a subset containing at least one Δ-best population with confidence P^*. A treatment was called Δ-best if it is at a distance less than or equal to Δ ($\Delta \geq 0$) from the best population, whereas the best population was to be associated with the largest location parameter.

In this paper, we have used two-sample statistics for proposing subset selection procedures. These procedures control the PCS over the entire parametric space. Let $\Omega = \{\underline{\theta} : \underline{\theta} = (\theta_0, \theta_1, \ldots, \theta_k)^t, -\infty < \theta_i < \infty\}$ be the parametric space. The subset selection procedures for Case 1 and Case 2 are proposed in Section 2. These procedures are required to satisfy the P^*-condition $P_0(CS) \geq P^*$ for any $\underline{\theta} \in \Omega$, where $2^{-k} < P^* < 1$.

The proposed selection procedures are shown to be strongly monotone in Section 3. In Section 4 approximate implementation of the proposed procedures, with the help of existing tables, is discussed. Simulation study is made in Section 5 in order to see the relative performance of the proposed procedures.

2. Proposed Selection Procedures

Case 1. Here the populations $\pi_0, \pi_1, \ldots, \pi_k$ are assumed to differ only in their location parameters. The selection procedures proposed in this case are based on two-sample linear rank statistics. Let $X_{i\alpha}, \alpha = 1, \ldots, n_i$ be a random sample of n_i observations from π_i, $i = 0, 1, \ldots, k$ and let $\underline{X} = (X_{01}, \ldots, X_{0n_0}, X_{11}, \ldots, X_{1n_1}, \ldots, X_{k1}, \ldots, X_{kn_k})$ be the vector of all the observations.

Let $R_{0\alpha}^{(i)}$ denote the rank of $X_{0\alpha}$ in the combined sample $X_{i1}, \ldots, X_{in_i}, X_{01}, \ldots, X_{0n_0}$. Define $S_0^{(i)} = (1/n_0 + 1/n_i)\big\{\sum_{\alpha=1}^{n_0}(a_m(R_{0\alpha}^{(i)}) - n_0\bar{a}_m)\big\}$, where $m = n_0 + n_i$ and $a_m(\beta)$ are some given scores satisfying the following two assumptions:

(A1) For any positive integer m, the scores $a_m(1), \ldots, a_m(m)$ are generated by a non-decreasing, non-constant, and square integrable function $J(u)$ $(0 < u < 1)$ in either of the following two ways: $a_m(\beta) = J(\beta/(m+1))$ or $a_m(\beta) = E\big\{J(U_m^{(\beta)})\big\}$, where $\beta = 1, \ldots, m$ and $U_m^{(1)} \le \cdots \le U_m^{(m)}$ denote the order statistic based on a sample of size m from the uniform distribution on the interval $(0,1)$. Define $\bar{a}_m = \{a_m(1) + \cdots + a_m(m)\}/m$.

(A2) $J(u)$ $(0 < u < 1)$, which is called as limiting score function, is such that $\sum_{\beta=1}^{m}(a_m(\beta) - \bar{a}_m)^2/(m-1) \to \sigma^2 = \int_0^1 (J(u) - \bar{J})^2 du < \infty$, where $\bar{J} = \int_0^1 J(u)du$.

The proposed selection procedure based on the statistic $S_0^{(i)}$ is as follows.

R_1 : For any i $(1 \le i \le k)$ include the population π_i in the subset if and only if $S_0^{(i)} \le C_0^{(i)}(\underline{n}, P^*)$, where $\underline{n} = (n_0, n_0, \ldots, n_0)^t$ and the constant $C_0^{(i)}(\underline{n}, P^*)$ are chosen so that for a pre-assigned probability $P^*(2^{-k} < P^* < 1)$,

$$P_0\left\{S_0^{(i)} \le C_0^{(i)}(\underline{n}, P^*),\ i = 1, \ldots, k\right\} \ge P^*.$$

Here P_0 indicates that the probability is computed under the parametric configuration $\theta_0 = \theta_1 = \cdots = \theta_k$.

Now we shall show that the procedure R_1 satisfies P^*-condition when the scores satisfy the (A1).

THEOREM 2.1. *Under (A1), procedure R_1 satisfies P^*-condition.*

PROOF. Assume without loss of generality that population π_i is better than the control population π_0. Since the scores satisfy (A1), $\max_{1\leq i\leq k} S_0^{(i)}$ is non-decreasing in $X_{01},\ldots,X_{0n_0}$ and non-increasing in other components of $\underline{X}$. Hence, by Lemma 4.1 of Mahamunulu (1967), we have for any $\underline{\theta}\in\Omega$,

$$\begin{aligned} P^* &\leq P_0\left\{S_0^{(i)}\leq C_0^{(i)}(\underline{n},P^*),\ i=1,\ldots,k\right\}\\ &\leq P_{\underline{\theta}}\left\{S_0^{(i)}\leq C_0^{(i)}(\underline{n},P^*),\ i=1,\ldots,k\right\}\\ &\leq P_{\underline{\theta}}(\mathrm{CS}\mid R_1). \end{aligned}$$

This proves the theorem. □

Case 2. Here once again the $(k+1)$ populations differ in their location parameters and it is further assumed that $F(0)=p$, so that θ_i, the location parameter of the i^{th} population, is its p^{th} quantile. For practical situations of this type, one may refer to interesting paper by Chakraborti and Desu (1989). In this case, the selection procedure is based on the following statistics. Let $U_0^{(i)}$ be the number of observations in the i^{th} $(1\leq i\leq k)$ sample not exceeding Q, where is s^{th} order statistic in the sample from control population. Here $s=[n_0p]+1$, where $[x]$ is the largest integer not exceeding x. Now

$$E\big(U_0^{(i)}\big)=n_iF_i(\theta_0),\ i=1,\ldots,k.$$

When $\theta_i=\theta_0$ for any i, $E(U_0^{(i)}/n_i)=F(0)=p,\ i=1,\ldots,k$.

Let $W_0^{(i)}=(U_0^{(i)}/n_i)-p,\ i=1,\ldots,k$. In this case the proposed selection procedure is based on statistic $W_0^{(i)}$ and is as follows.

R_2 : For any i $(1\leq i\leq k)$, include π_i in the subset if and only if $W_0^{(i)}\leq d_0^{(i)}(\underline{n},P^*)$. Here $\underline{n}=(n_0,n_1,\ldots,n_k)^t$, and the constant $d_0^{(i)}(\underline{n},P^*)$ are chosen such that for a pre-assigned probability $P^*(2^{-k}<P^*<1)$, we have

$$P^*\leq P_0\left\{W_0^{(i)}\leq d_0^{(i)}(\underline{n},P^*),\ i=1,\ldots,k\right\}$$

and again P_0 indicates that the probability is computed under $\theta_0=\theta_1=\cdots=\theta_k$.

On the lines of arguments used in Theorem 2.1, the procedure R_2 satisfies P^*-condition.

REMARK 2.1. Procedure R_2 is useful when testing time is expensive so that early termination of an experiment is desirable. The justification for this as under:

To apply procedure R_1, experimenter needs to observe the response variable on all the $n_1 + \cdots + n_k$ experimental units. This process of collecting the data often requires a considerable amount of time and resources. Generally, in life-testing experiments and drug-screening studies observations become available in an ordered fashion and it is desirable to terminate the experiment as soon as a pre-chosen quantile is observed in any of the k-samples. Procedure R_2 is for such experiments, which enables one to reach a decision as to include a population in the subset or not. Evidently, procedure R_2 could lead to substantial savings in time and resources if k and/or $n_1 + \cdots + n_k$ is large. For details refer to Chakraborti and Desu (1989).

In the following section we define the strong monotonicity of a selection procedure and then establish this property for procedures R_1 and R_2.

3. Strong Monotonicity of Procedures R_1 and R_2

Gupta and Nagel (1971), and Santner (1975) have defined unbiasedness, monotonicity, and strong monotonicity properties of a selection procedure while proposing selection procedures for parametric families of probability distributions. Now below we define the strong monotonicity properties of a selection procedure R when the parameters of interest are location parameters. For any $\underline{\theta} \in \Omega$, let $P_{\underline{\theta}}(i) = P_{\underline{\theta}}\{\pi_i \text{ is included in the subset}|R\}$, $i = 1, \ldots, k$.

Definition 3.1. *The selection procedure R is strongly monotone in π_i iff $P_{\underline{\theta}}(i)$ is increasing in θ_i when all other components of $\underline{\theta}$ are fixed, and $P_{\underline{\theta}}(i)$ is decreasing in θ_j, $(j \neq i)$ when all other components of $\underline{\theta}$ are fixed.*

The following theorem shows strong monotonicity of selection procedure R_1.

Theorem 3.1. *The selection procedure R_1 is strongly monotone.*

Proof. Define the indicator function $I(\cdot)$ as

$$I(S_0^{(i)}) = \begin{cases} 1, & \text{if } S_0^{(i)} \leq C_0^{(i)}(\underline{n}, P^*), \\ 0, & \text{otherwise.} \end{cases}$$

Then by using above Theorem 2.1 and Lemma 4.1 of Mahamunula (1967), we have

$$P_{\underline{\theta}}(i) = E_{\underline{\theta}}\{I(S_0^{(i)})\} \leq E_{\theta}^*\{I(S_0^{(i)})\} = P_{\underline{\theta}^*}(i)$$

where $\underline{\theta} = (\theta_0, \theta_1, \ldots, \theta_i, \ldots, \theta_k)$ and $\underline{\theta}^* = (\theta_0, \theta_1, \ldots, \theta_i^*, \ldots, \theta_k)$ with $\theta_i \leq \theta_i^*$. This proves the theorem. □

As strong monotonicity implies monotonicity which in turn implies unbiasedness (see Santner, 1975), we have the following corollary.

COROLLARY 3.1. *The selection procedure R_1 is monotone and unbiased.*

On the lines of Theorem 3.1 we can easily show that the selection procedure R_2 is strongly monotone and hence monotone and unbiased. In the next section, we see that with the help of existing tables selection procedures R_1 and R_2 can be approximately implemented.

4. APPROXIMATE IMPLEMENTATION OF PROCEDURES R_1 AND R_2

Here, we first establish the asymptotic normality of vectors $\underline{S} = (S_0^{(1)}, \ldots, S_0^{(k)})^t$ and $\underline{W} = (W_0^{(1)}, \ldots, W_0^{(k)})^t$. The asymptotic normality of vector $\underline{S}$ under the configuration $\theta_0 = \theta_1 = \cdots = \theta_k$ follows immediately from a result of Koziol and Reid (1977) and is stated below in Lemma 4.1.

LEMMA 4.1. *Under $\theta_0 = \theta_1 = \cdots = \theta_k$ and as $\min(n_0, n_1, \ldots, n_k) \to \infty$ such that $n_i/N \to \lambda_i$, $0 < \lambda_i < 1$, for $i = 0, 1, \ldots, k$, the random vector $(N/\sigma_N^2)^{1/2}\underline{S}$ is asymptotically normally distributed with mean vector $\underline{0}$ and dispersion matrix as*

$$\frac{N}{\sigma_N^2} E(S_0^{(i)} S_0^{(j)}) = \begin{cases} 1/\lambda_0 + 1/\lambda_i, & \text{for } i = j, \\ 1/\lambda_0, & \text{for } i \neq j, \end{cases}$$

where $\sigma_N^2 = \sum_{\beta=1}^{N} (a_N(\beta) - \overline{a}_N)^2/(N-1) \to \int_0^1 (J(u) - \overline{J})^2 du$ and $N = n_0 + n_1 + \cdots + n_k$.

Now let $n_1 = n_2 = \cdots = n_k = n$ (say) and as $n(n_0) \to \infty$, $n/N(n_0/N) \to \lambda_1(\lambda_0)$, where $N = n_0 + nk$. Let $Z_0(i)$ be $(1/\lambda_0 + 1/\lambda_1)^{-1/2}(N/\sigma_N^2)^{1/2} S_0^{(i)}$. It follows from Lemma 4.1, that the limiting distribution of the random vector $\underline{Z} = (Z_0^{(1)}, \ldots, Z_0^{(k)})^t$ under $\theta_0 = \theta_1 = \cdots = \theta_k$ is asymptotically multivariate normal of equally correlated normal variables. The limiting value of this correlation coefficient is $r = (1/\lambda_0)/(1/\lambda_0 + 1/\lambda_1)$. The constant $C_0^{(i)}(\underline{n}, P^*)$ of the selection procedure R_1, when $n_1 = n_2 = \cdots = n_k$ is determined such that

$$\begin{aligned} P^* &= P_0\Big\{Z_0^{(i)} \leq z,\ i = 1, \ldots, k\Big\} \\ &= P_0\Big\{\max_{1<i<k} Z_0^{(i)} \leq z\Big\}, \end{aligned}$$

where $z = (1/\lambda_0 + 1/\lambda_1)^{-1/2}(N/\sigma_N^2)^{1/2}C_0^{(i)}(\underline{n}, P^*)$, $i = 1, \ldots, k$.

Now we can make use of Table-I of Gupta *et al.* (1973) (reading N as k, α as $1 - P^*$ and ρ as $(1/\lambda_0)/(1/\lambda_0 + 1/\lambda_1)$ in that table) to read the constant z and thereby get the value of constant $C_0^{(i)}(\underline{n}, P^*)$, $i = 1, \ldots, k$. Although extensive knowledge of the asymptotic properties of the sample quantiles exist, the asymptotic distribution of $\underline{W} = (W_0^{(1)}, \ldots, W_0^{(k)})^t$ follows from a well known result (*e.g.* David, 1981, p. 255) and is stated below in Lemma 4.2.

LEMMA 4.2. *The asymptotic distribution of* $N^{1/2}(\underline{W} - E(\underline{W}))$ *as* $\min(n_0, n_1, \ldots, n_k) \to \infty$ *such that* $n_i/N \to \lambda_i$, $0 < \lambda_i < 1$, *for* $i = 0, 1, \ldots, k$, *is normal with mean vector* $\underline{0}$ *and dispersion matrix* $\sum = ((\sigma_{ij}))$, *where* $N = n_0 + n_1 + \cdots + n_k$ *and*

$$\sigma_{ij} = \begin{cases} Q_i^2 p_0/\lambda_0 + p_i/\lambda_i, & \text{for } i = j, \\ Q_i Q_j p_0/\lambda_0, & \text{for } i \neq j, \end{cases}$$

where $p_0 = p(1-p)$, $p_i = F_i(\theta_0)(1 - F_i(\theta_0))$, $Q_i = f_i(\theta_i)/f_0(\theta_0)$, $i = 1, \ldots, k$, *and we assume that* $F_i'(\theta_0) = f_i(\theta_0)$ *exists and is positive for* $i = 0, 1, \ldots, k$.

Under $\theta_0 = \theta_1 = \cdots = \theta_k$, it is easy to see that $E(\underline{W}) = \underline{0}$ and

$$\sigma_{ij} = \begin{cases} p_0/\lambda_0 + p_0/\lambda_i, & \text{for } i = j, \\ p_0/\lambda_0, & \text{for } i \neq j. \end{cases}$$

Consequently, we have the following theorem.

THEOREM 4.1. *Under* $\theta_0 = \theta_1 = \cdots = \theta_k$ *and as* $\min(n_0, n_1, \ldots, n_k) \to \infty$ *such that* $n_i/N \to \lambda_i$, $0 < \lambda_i < 1$, *for* $i = 0, 1, \ldots, k$, *the random vector* $(N/p_0)^{1/2}\underline{W}$ *is asymptotically normally distributed with mean vector and dispersion matrix as*

$$\frac{N}{p_0}E(W_0^{(i)}W_0^{(j)}) = \begin{cases} 1/\lambda_0 + 1/\lambda_i, & \text{for } i = j, \\ 1/\lambda_0, & \text{for } i \neq j. \end{cases}$$

Using Theorem 4.1, the selection procedure R_2 can also be implemented with the help of Gupta *et al.* (1973) tables as explained above for procedure R_1. In the following section, we carry out simulation study in order to see the relative performance of procedures R_1 and R_2.

5. Simulation Study

In this section, we present the results of our simulation study carried out to assess the relative performance of procedures R_1 and R_2. Simulation is carried out in the following steps.

(i) Three sets of parametric families namely Normal, Double Exponential, and Cauchy are considered.

(ii) Four populations say π_0, π_1, π_2 and π_3 with particular parametric configurations from each family are taken. The following two sets of configurations are considered:

Set-I : 1. Normal distribution
$\pi_0 \sim N(2,1)$, $\pi_1 \sim N(2.8,1)$, $\pi_2 \sim N(0.5,1)$, $\pi_3 \sim N(2.1,1)$;

2. Double Exponential distribution
$\pi_0 \sim DE(2,1)$, $\pi_1 \sim DE(2.8,1)$, $\pi_2 \sim DE(0.5,1)$, $\pi_3 \sim DE(2.1,1)$;

3. Cauchy distribution
$\pi_0 \sim C(2,1)$, $\pi_1 \sim C(2.8,1)$, $\pi_2 \sim C(0.5,1)$, $\pi_3 \sim C(2.1,1)$.

Set-II : 1. Normal distribution
$\pi_0 \sim N(1.8,1)$, $\pi_1 \sim N(0.5,1)$, $\pi_2 \sim N(1.5,1)$, $\pi_3 \sim N(2.1,1)$;

2. Double Exponential distribution
$\pi_0 \sim C(1.8,1)$, $\pi_1 \sim C(0.5,1)$, $\pi_2 \sim C(1.5,1)$, $\pi_3 \sim C(2.1,1)$;

3. Cauchy distribution
$\pi_0 \sim C(2,1)$, $\pi_1 \sim C(2.8,1)$, $\pi_2 \sim C(0.5,1)$, $\pi_3 \sim C(2.1,1)$.

In the Set-I and Set-II, notation $DE(\mu,\lambda)$ denotes the Double Exponential distribution with location parameter μ and scale parameter λ and notation $C(\mu,\lambda)$ denotes the Cauchy distribution with location parameter μ and scale parameter λ.

(iii) In procedure R_1 we restrict to the situation when the underlying scores are Wilcoxon scores, *i.e.* $J(u) = u$, $0 < u < 1$. For procedure R_2 we assume that, the location parameter of the i^{th} population $(i = 0,1,2,3)$ is the median, *i.e.* $p = 1/2$.

(iv) Random samples of size n_0 and common sample size n $(n = 6, 10, 15, 20, 30$ $40)$ are generated through computer from control population and the three populations, respectively in a set and size of the selected subset along with the probability of correct selection is noted by taking $P^* = 0.95, 0.90$ and 0.75 for both procedures R_1 and R_2. This process is repeated 10,000 times. Note that n_0 and n may be chosen to take different values.

(v) The estimated expected subset size ($E(S)$) and estimated probabilities of correct selection for the above mentioned values of n_0 and n are obtained for both procedures by taking the average of the subset sizes and probabilities of correct selection in 10,000 repetitions, respectively.

(vi) As a measure of "goodness" of a subset selection procedure, we use the ratio of the estimated expected subset size $E(S)$ to the estimated probability of correct selection, *i.e.* $E(S)/\widehat{P(\text{CS})}$. A rule R is said to be "better" than a rule R^* if the ratio for R is less than the ratio of R^*. The relative efficiency of the procedure R_1 relative to the procedure R_2 is an inverse ratio of the measures of goodness, *i.e.* $e(R_1, R_2) = \{E(S|R_2)/E(S|R_1)\} \times \{E(\text{CS}|R_1)/E(\text{CS}|R_2)\}$. The computed values of $e(R_1, R_2)$ for different parametric configurations of families of distributions, as in Set-I and Set-II, and for above mentioned choices of n_0, n and P^* are represented in Figures 1 and 2.

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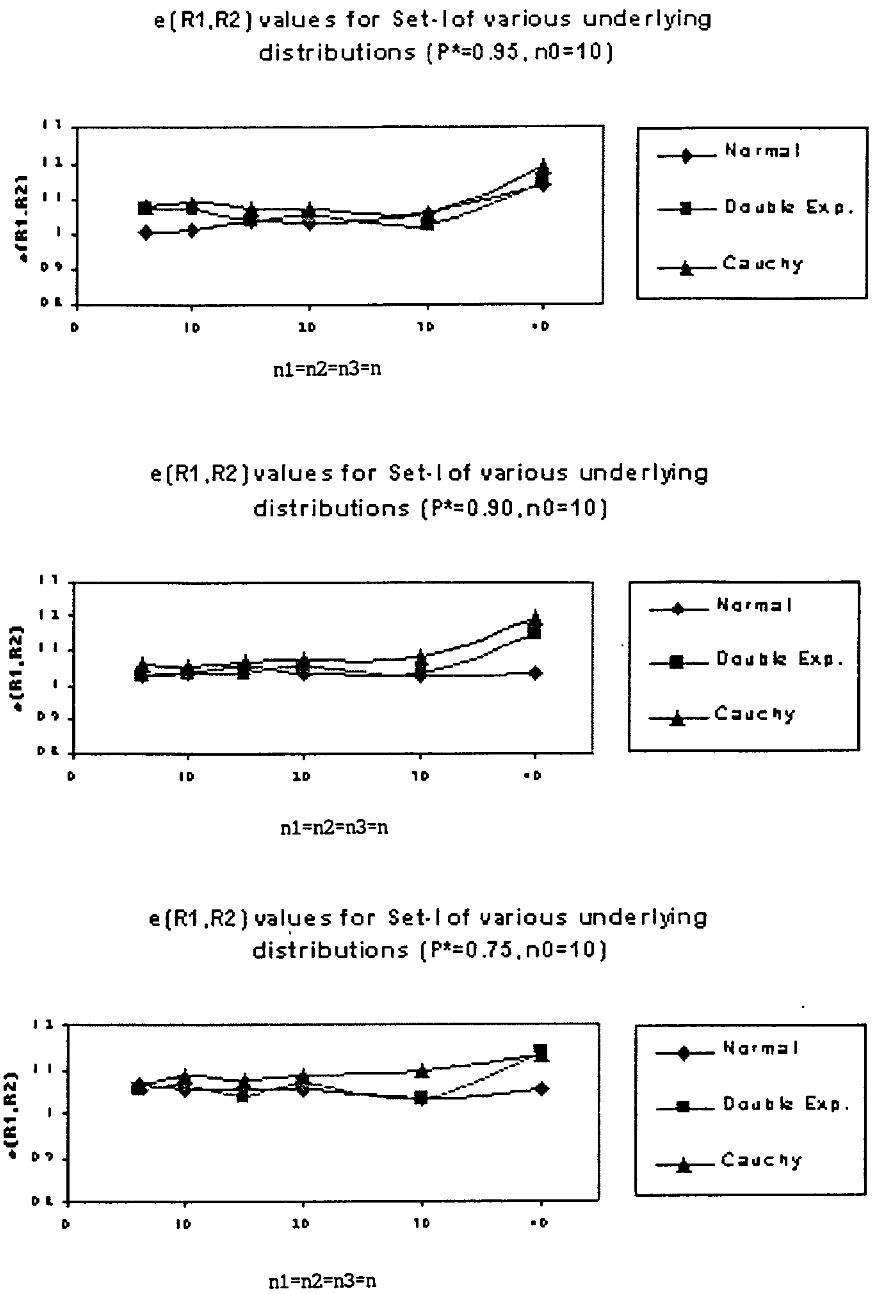
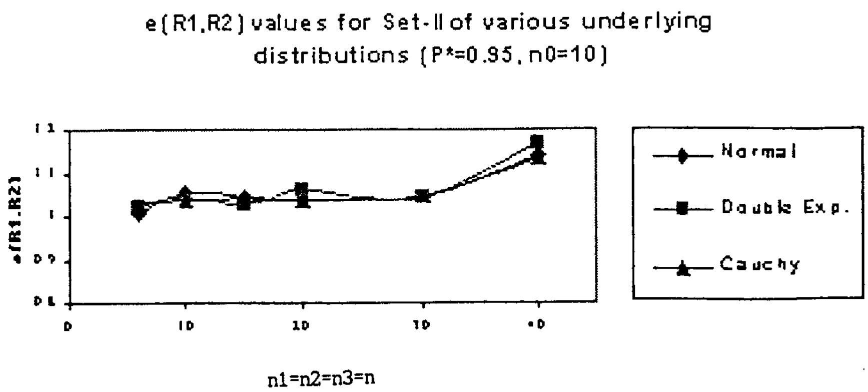
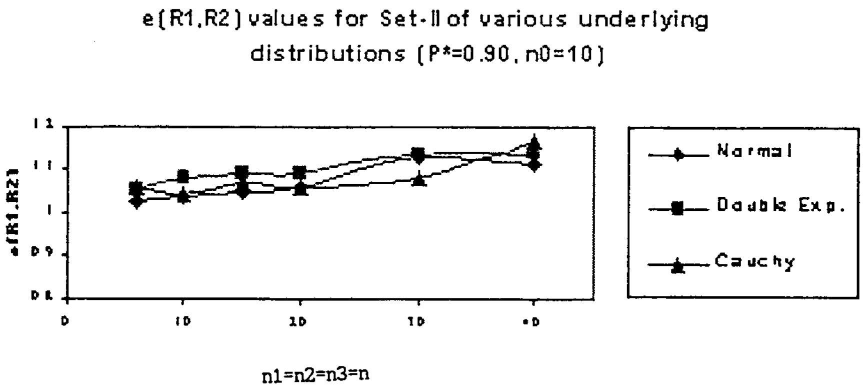

e(R1,R2) values for Set-I of various underlying distributions (P*=0.95, n0=10)
e(R1,R2)
Normal
Double Exp.
Cauchy
n1=n2=n3=n
e(R1,R2) values for Set-I of various underlying distributions (P*=0.90, n0=10)
e(R1,R2)
Normal
Double Exp.
Cauchy
n1=n2=n3=n
e(R1,R2) values for Set-I of various underlying distributions (P*=0.75, n0=10)
e(R1,R2)
Normal
Double Exp.
Cauchy
n1=n2=n3=n

FIGURE 1


e(R1,R2) values for Set-II of various underlying distributions (P*=0.95, n0=10)
e(R1,R2)
Normal
Double Exp.
Cauchy
n1=n2=n3=n


e(R1,R2) values for Set-II of various underlying distributions (P*=0.90, n0=10)
e(R1,R2)
Normal
Double Exp.
Cauchy
n1=n2=n3=n

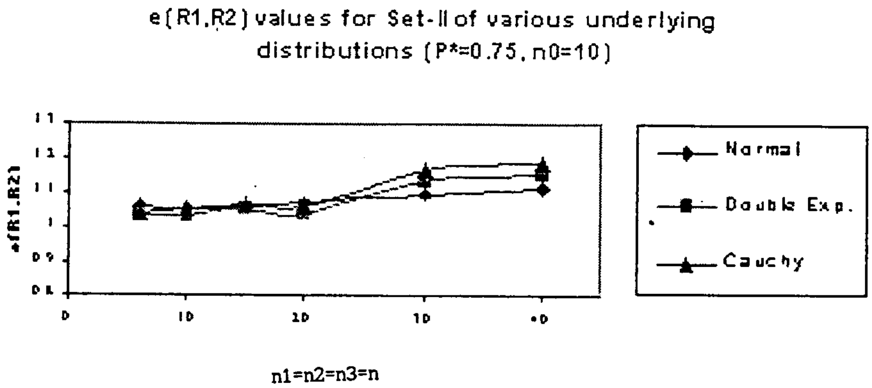

e(R1,R2) values for Set-II of various underlying distributions (P*=0.75, n0=10)
e(R1,R2)
Normal
Double Exp.
Cauchy
n1=n2=n3=n

FIGURE 2

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