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A Study on Dynamic Crack-Tip Fields in a Strain Softening Material

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Abstract

The near-tip field of mode-I dynamic cracks steadily propagating in a strain softening material is investigated under plane strain conditions. The material is assumed to be incompressible and its deformation obeys the J_2 flow theory of plasticity. A power-law stress-strain relation with strain softening is adopted to account for the damage behavior of materials near the dynamic crack tip. By assuming that the stresses and strains have the same singularity at the crack tip, this paper obtains a fully continuous dynamic crack-tip field in the damage region. Results show that the stress and strain components possess the same logarithmic singularity of $(\ln(R/r))^{\delta}$, and the angular variations of field quantities are identical to those corresponding to the dynamic cracks in the elastic-perfectly plastic material.

1. Introduction

The stress and strain distribution near a crack tip is an important and complicated problem in fracture mechanics. It depends not only on the modes of a crack, but also on the motion states of the crack. A detailed review on dynamic fracture is recently given by Rosakis and Ravichandran[1]. The attention of this paper is primarily focused on the mode-I crack steadily propagating in a strain softening material under plain strain conditions.

To understand the mechanics behavior of near-tip fields for dynamic cracks in elastic-plastic materials, Nemat-Nasser and his coworker[2~6] have systematically investigated all three modes of dynamic cracks and the related issues in the

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elastic-perfectly plastic and power-law hardening materials, and obtained meaningful results. Thereafter, many other researchers further studied the mode-I elastic-plastic dynamic cracks using both theoretical and numerical methods[7~15]. Two differently featured dynamic crack-tip fields are thus proposed. One has discontinuous stresses and strains, but the other has continuous stresses and strains. Using the thermodynamics theory, Drugan[14] rigorously proved the correct-ness of the continuous dynamic crack-tip fields, and ruled out the possibility of discontinuous stresses and strains existed at the dynamic crack tip. Xu and Saigal[15] obtained the continuous crack-tip stress fields for both hardening and non-hardening materials using the element free Galerkin method, and thus directly support the conclusions of Drugan[14].

It is well known that the assumptions of perfect plasticity or strain hardening of materials are only valid for a certain stage of deformation. When the deformation develops largely enough, some softening and damage phenomena always appear in engineering materials. In order to consider such phenomena, it is essential to adopt a strain softening model to study the dynamic crack-tip fields. In fact, Gao[16] studied the mode-I dynamic crack using a strain damage model, but obtained an asymptotic solution containing strong discontinuities, which must be ruled out as discussed above. Therefore, a need remains to seek a continuous damage near-tip field for dynamic cracks.

By using a power-law strain softening model and the assumption of the same singularity of stresses and strains, the present paper constructs a fully continuous near-tip field of dynamic mode-I cracks in an incompressible strain softening material under plane strain conditions.

2. Basic Equations

Consider a semi-finite straight mode-I crack propagating steadily at speed V in an elastic-plastic strain softening material under plane strain conditions. The material is assumed to be incompressible, and to deform plastically according to the J_2 flow theory. Both rectangular Cartesian-coordinates (x_1, x_2) with x_1 aligned in the direction of propagation and polar coordinates (r, θ) with = 0 corresponding to the line ahead of the crack tip are introduced with their common origin at the tip of the crack and moving with the tip. Let $\sigma_{\alpha\beta}$ v_α represent the rectangular components of stress tensor and particle velocity, respectively. In the absence of body force, the motion equations for the crack propagation problem are

$$\sigma_{\alpha\beta,\beta} = \rho\dot{v}_\alpha \tag{1}$$

where ρ is the mass density, a comma implies the spatial derivatives with respect to the rectangular coordinate components, the dot superimposed on field quantities denotes the material time derivative. The summation convention to be followed throughout the paper for repeated indices is adopted. Greek indices α, β have the range of 1~2. Let $\varepsilon_{\alpha\beta}$ stand for the rectangular components of strain tensor. In terms of the particle velocity, v_α, the components of (small) strain rate tensor are

$$\dot{\varepsilon}_{\alpha\beta} = \frac{1}{2}\left(v_{\alpha,\beta} + v_{\beta,\alpha}\right) \tag{2}$$

It is noted that using the steady-state condition, $d(\ )/dt = -V\partial(\ )/\partial x$, the dimensions of left term and right term of equations (1) and (2) are consistent each other.

The material incompressibility condition implies that the particle velocity is divergence free, which can be expressed under the plane strain conditions by

$$v_{1,1} + v_{2,2} = 0 \tag{3}$$

From the J_2 flow theory of plasticity, the two dimensional constitutive equations for an incompressible elastic-plastic material can be written as

$$\begin{aligned} \dot{\varepsilon}_{11} = -\dot{\varepsilon}_{22} &= \frac{1}{4\mu}\left(\dot{\sigma}_{11} - \dot{\sigma}_{22}\right) + \frac{1}{2}\lambda\left(\sigma_{11} - \sigma_{22}\right) \\ \dot{\varepsilon}_{12} &= \frac{1}{2\mu}\dot{\sigma}_{12} + \lambda\sigma_{12} \end{aligned} \tag{4}$$

where μ is the shear modulus, λ is the plastic flow factor which can be determined by the magnitude of stress components in the plastic regions.

In order to model the softening behavior of materials in plastic damage regions, we assume that the uniaxial tensile elastic-plastic stress-strain relation follows the power-law curve

$$\sigma = \begin{cases} E\varepsilon & , \quad \varepsilon < \varepsilon_0 \\ E\varepsilon\left[\dfrac{\varepsilon/\varepsilon_0 + n\eta}{1 + n\eta}\right]^{-(1+n)} & , \quad \varepsilon \ge \varepsilon_0 \end{cases} \tag{5}$$

where E is Young's modulus, n is the material softening exponent, $\eta > 1$ is a material constant, ε_0 is the initial yield strain, and $\sigma_0 = E\varepsilon_0$ is the initial yield stress. In particular, $\eta\varepsilon_0$ is the strain value corresponding to the peak of stress σ. Fig. 1 illustrates the power-law softening stress-strain curves for n = 3, 5, 10 and $\eta = 1.5$ based on (5). It is observed that stresses decrease gradually with increasing strains in the post-peak region of the strain softening curves. Therefore, the power-law strain softening curves can account for the effects of material damage on the crack-tip field.

In term of the uniaxial stress σ and total strain, ε, the uniaxial plastic strain is represented by $\varepsilon_p = \varepsilon - \sigma/E$. From this equation and (5), a plastic potential function, F, is defined as

$$F(\sigma,\varepsilon_p) = \sigma - (\sigma + E\varepsilon_p)(1+n\eta)^{(1+n)}\left[\frac{\varepsilon_p}{\varepsilon_0} + \frac{\sigma}{\sigma_0} + n\eta\right]^{-(1+n)} \tag{6}$$

For the plane strain case ($\varepsilon_{33} = 0$), equation (6) is also valid if the uniaxial tensile stress, σ, and the uniaxial plastic strain, ε_p, are replaced by the effective stress, σ_e, and the effective or accumulative plastic strain, $\bar{\varepsilon}_p$, respectively,

$$\sigma_e = \sqrt{3}\left[\frac{1}{4}\left(\sigma_{11} - \sigma_{22}\right)^2 + \sigma_{12}^2\right]^{\frac{1}{2}} \tag{7}$$

$$\bar{\varepsilon}_p = \frac{2}{\sqrt{3}}\int_0^t\left[\frac{1}{4}\left(\dot{\varepsilon}_{11}^p - \dot{\varepsilon}_{22}^p\right)^2 + \left(\dot{\varepsilon}_{12}^p\right)^2\right]^{\frac{1}{2}} dt \tag{8}$$

where t is a time variable, and $\dot{\varepsilon}_{\alpha\beta}^p$ are the components of plastic strain rate tensor.

By use of the normality rule and the consistency condition, $\dot{F}=0$, in the theory of plasticity, the plastic flow factor λ can be determined by

$$\lambda=-\frac{3\dot{\bar{\varepsilon}}_e}{2\sigma_e}\left(\frac{\partial F}{\partial \bar{\varepsilon}_p}\right)^{-1} \tag{9}$$

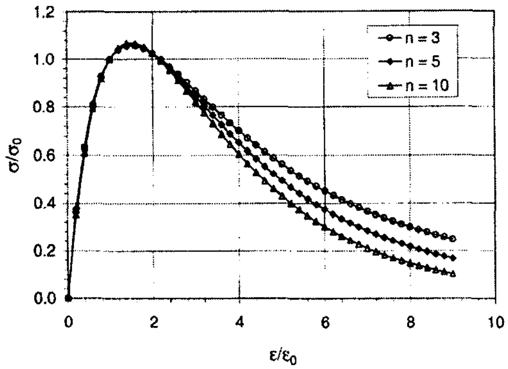

Fig. 1 Uniaxial power law softening stress relation for the strain softening expo n=3, 5, 10 and the constant ç=1.5

In this work, the attention is focused on the crack-tip field in the stage of strain softening where the plastic strain $\bar{\varepsilon}_p$ is large enough. In this case, the plastic potential function in (6) can be simplified as

$$F(\sigma_e,\bar{\varepsilon}_p)=\sigma_e-C(\bar{\varepsilon}_p)^{-n} \tag{10}$$

where the constant $C=E[(1+n\eta)\varepsilon_0]^{(1+n)}$. From (9) and (10), one obtains the plastic flow factor in the damage region at the crack tip as follows

$$\lambda=-\frac{3}{2nC^2}\left(\frac{C}{\sigma_e}\right)^{(2+\frac{1}{n})}\dot{\bar{\varepsilon}}_e \tag{11}$$

3. Asymptotic Equations and Continuous Solutions

3.1 Kinematical Quantities

It is assumed that the stress compon-ents near a dynamic crack tip possess the following logarithmic singularity

$$\sigma_{\alpha\beta}(r,\theta)=\left(\ln\frac{R}{r}\right)^{\delta}\sum_{m=0}^{\infty}\sigma_{\alpha\beta}^{(m)}(\theta)\left(\ln\frac{R}{r}\right)^{-m} \tag{12}$$

where is an undetermined exponent of stress singularity, R is a constant with the physical dimension of length, which may be used as a measure of the plastic zone size.

If we only consider the dominant term of the singular stress expansion (12), from the steady-state condition $d(\ )/dt=-V\partial(\ )/\partial x$ [7] and the motion equations (1), then the particle velocity near the crack tip must be taken the asymptotic expansions as

$$v_\alpha(r,\theta)=\left(\ln\frac{R}{r}\right)^{\delta}\left[A_\alpha\ln\frac{R}{r}+B_\alpha(\theta)\right]+o\left[\left(\ln\frac{R}{r}\right)^{\delta}\right] \tag{13}$$

where $A_1\geq 0$ is an unknown constant, $A_2\equiv 0$ for mode-I cracks, $B_\alpha(\theta)$ are the angular functions of particle velocity, o[ ] is the infinitesimal order symbol. Following the similar analyses of Leighton et al.[7] for the elastic-perfectly plastic material, our study shows that when $A_1\neq 0$ in (13), one can only construct the asymptotic crack-tip fields with discontinuous stresses and strains as those reported by Gao[16]. As introduced in Section 1, such discontinuous solutions must be ruled out. To seek the continuous asymptotic fields, we will study the asymptotic solutions with $A_1 = 0$ here. Then equation (13) becomes

$$v_\alpha(r,\theta) = \left(\ln\frac{R}{r}\right)^\delta B_\alpha(\theta) + o\left[\left(\ln\frac{R}{r}\right)^\delta\right] \quad (14)$$

Substitution of (14) into (3) obtains the incompressibility condition which is expressed by the angular functions of particle velocity

$$B_2'(\theta) = B_1'(\theta)\tan\theta \quad (15)$$

where the prime '′' stands for $\partial/\partial\theta$. From (2), (14) and (15), it follows that

$$\dot{\varepsilon}_{11} = -\dot{\varepsilon}_{22} = -\frac{1}{r}\left(\ln\frac{R}{r}\right)^\delta \cos\theta B_2'(\theta) + o\left[\frac{1}{r}\left(\ln\frac{R}{r}\right)^\delta\right]$$
$$\dot{\varepsilon}_{12} = \frac{1}{2r\sin\theta}\left(\ln\frac{R}{r}\right)^\delta \cos 2\theta B_2'(\theta) + o\left[\frac{1}{r}\left(\ln\frac{R}{r}\right)^\delta\right] \quad (16)$$

The above equations show that the strain components $\varepsilon_{\alpha\beta}$ are of the order of the logarithmic singularity like $(\ln(R/r))^\delta$.

3.2 Dynamic Quantities

From (12), the asymptotic form of the effective stress σ_e is given by

$$\sigma_e(r,\theta) = \left(\ln\frac{R}{r}\right)^\delta \sigma_e^{(0)}(\theta) + \left(\ln\frac{R}{r}\right)^{\delta-1} \sigma_e^{(1)}(\theta) + o\left[\left(\ln\frac{R}{r}\right)^{\delta-1}\right] \quad (17)$$

Under the steady-state condition, from (12) and (17), we have the stress rate components and the effective stress rate, respectively

$$\dot{\sigma}_{\alpha\beta}(r,\theta) = \frac{V}{r}\left(\ln\frac{R}{r}\right)^\delta \sin\theta\sigma_{\alpha\beta}^{(0)\prime}(\theta) + o\left[\frac{1}{r}\left(\ln\frac{R}{r}\right)^\delta\right] \quad (18)$$

$$\dot{\sigma}_e(r,\theta) = \frac{V}{r}\left(\ln\frac{R}{r}\right)^\delta \left[\sin\theta\sigma_e^{(0)\prime}(\theta) + \left(\ln\frac{R}{r}\right)^{-1} \times \left(\sin\theta\sigma_e^{(1)\prime}(\theta) + \delta\cos\theta\sigma_e^{(0)}(\theta)\right)\right] + o\left[\frac{1}{r}\left(\ln\frac{R}{r}\right)^{\delta-1}\right] \quad (19)$$

Inserting equations (17) and (19) into (11), we obtain

$$\lambda = -\frac{3}{2nr}VC^{\frac{1}{n}}\left(\ln\frac{R}{r}\right)^{-(1+\frac{1}{n})\delta}\left[\sigma_e^{(0)}(\theta) + \left(\ln\frac{R}{r}\right)^{-1}\sigma_e^{(1)}(\theta)\right]^{-2-\frac{1}{n}}$$
$$\times\left[\sin\theta\sigma_e^{(0)\prime}(\theta) + \left(\ln\frac{R}{r}\right)^{-1}\left(\sin\theta\sigma_e^{(1)\prime}(\theta) + \delta\cos\theta\sigma_e^{(0)}(\theta)\right)\right] + \text{L} \quad (20)$$

where the dots denote the neglected higher terms. On the other hand, equations (12), (16), (18) and (4) yield $\lambda \sim r^{-1}$. Then one can have

$$\sigma_e^{(0)\prime}(\theta) = 0 \quad \text{or} \quad \sigma_e^{(0)}(\theta) = \sqrt{3}K, \quad \delta = -\frac{n}{n+1} \quad (21)$$

in which the second equation is the constraint condition of plastic deformation in the power-law softening material. K is a free constant that cannot be determined from the asymptotic solution.

Substitution of (12), (17) and the second equation of (21) into (7) yields the relation of the stress angular functions in (12) as follows

$$\frac{1}{4}\left(\sigma_{11}^{(0)}(\theta) - \sigma_{22}^{(0)}(\theta)\right)^2 + \left(\sigma_{12}^{(0)}(\theta)\right)^2 = K^2 \quad (22)$$

This equation is similar to the von Mises yield condition of incompressible elastic-perfectly plastic materials. Therefore, we may introduce a stress function $\psi(\theta)$ in such a manner that

$$\begin{aligned}
\sigma_{11}^{(0)}(\theta) &= \sigma_h^{(0)}(\theta) - K\cos(\psi(\theta) - 2\theta) \\
\sigma_{22}^{(0)}(\theta) &= \sigma_h^{(0)}(\theta) + K\cos(\psi(\theta) - 2\theta) \\
\sigma_{12}^{(0)}(\theta) &= K\sin(\psi(\theta) - 2\theta)
\end{aligned} \tag{23}$$

where $\sigma_h^{(0)}(\theta) = \left(\sigma_{11}^{(0)}(\theta) + \sigma_{22}^{(0)}(\theta)\right)/2$ is the mean or hydrostatic stress.

3.3 Governing Equations

Substituting (12), (14) and (23) into the motion equations (1), and substituting (12), (16), (18) and (23) into the constitutive equations (4), after some manipulations, we obtain the final governing equations of the angular functions of the dominant asymptotic fields in the plastic regions as follows

$$\begin{aligned}
&(\psi'(\theta) - 2)(\cos^2\psi - M^2\sin^2\theta) = 0 \\
&\bar{\lambda} = \frac{1}{2}(\psi'(\theta) - 2)\sin\theta\tan\psi \\
&\sigma_h^{(0)\prime}(\theta) = K(\psi'(\theta) - 2)\sin\psi \\
&\bar{B}_1'(\theta) = M^{-2}(\psi'(\theta) - 2)\cos\psi\cot\theta \\
&\bar{B}_2'(\theta) = M^{-2}(\psi'(\theta) - 2)\cos\psi
\end{aligned} \tag{24}$$

where $M = V(\rho/\mu)^{1/2}$ is the Mach number, the dimensionless parameter $\bar{\lambda}(\theta) = \mu r\lambda(r,\theta)/V$ and $\bar{B}_\alpha(\theta) = \mu B_\alpha(\theta)/KV$.

For the mode-I crack, the boundary conditions which is associated with the ordinary differential equations (24) are

$$\psi(0) = 0,\ \psi(\pi) = \pi,\ B_2(0) = 0,\ \sigma_h^{(0)}(\pi) = K \tag{25}$$

3.4 Fully Continuous Solutions

It is observed that the above governing equations (24) and the boundary conditions (25) of the dominant asymptotic fields near a dynamic mode-I crack tip in power-law strain softening incompressible materials are exactly the same as those of cor-responding crack-tip fields in the elastic-perfectly plastic incompressible material[7]. Moreover, the further study has shown that the possible elastic unloading condition for the two kinds of materials is also same. The only difference for the two cases is that K in the stress fields (23) is a free constant relating to the strain softening exponent n in the present work, but it is the shear yield stress for the perfectly-plastic materials. Therefore, the angular variations of the field quantities in this paper are identical with those for elastic-perfectly plastic materials which have been con-sidered in detail by Leighton et al[7]. As a con-sequence, the dynamic crack-tip field for the strain softening material also consists of a uniform sector in $0 \le \theta \le \theta_1^*$, a non-uniform centered fan sector in $\theta_1^* \le \theta \le \theta_2^*$, and another uniform sector in $\theta_2^* \le \theta \le \pi$. Moreover, the stress function $\psi(\theta)$ and the angular mean stress $\sigma_h^{(0)}(\theta)$ in the angular stress components (19) are given by

$$\psi(\theta) = \begin{cases} 2\theta, & \text{if } 0 \le \theta \le \theta_1^* \\ \cos^{-1}(-M\sin\theta), & \text{if } \theta_1^* \le \theta \le \theta_2^* \\ 2\theta - \pi, & \text{if } \theta_2^* \le \theta \le \pi \end{cases} \tag{26}$$

and

$$\sigma_h^{(0)}(\theta)=\begin{cases} K\begin{bmatrix}1+M\sin\theta_1^*-2E(\theta_1^*;M)\\ -M\sin\theta_2^*-2E(\theta_2^*;M)\end{bmatrix}, & \text{if } 0\le\theta\le\theta_1^* \\ K\begin{bmatrix}1+M\sin\theta-2E(\theta;M)\\ -M\sin\theta_2^*-2E(\theta_2^*;M)\end{bmatrix}, & \text{if } \theta_1^*\le\theta\le\theta_2^* \\ K, & \text{if } \theta_2^*\le\theta\le\pi \end{cases} \quad (27)$$

where $E(\theta;M)$ is the elliptic integral of the second kind. θ_1^* and θ_2^* are the sector transition angles defined by

$$\theta_1^*=\sin^{-1}\left(\frac{M+\sqrt{8+M^2}}{4}\right)$$
$$\theta_2^*=\pi-\sin^{-1}\left(\frac{-M+\sqrt{8+M^2}}{4}\right) \quad (28)$$

From (26), (27) and (23), the angular distributions of the stress function and the stress components are shown in Fig. 2 and Fig. 3, respectively.

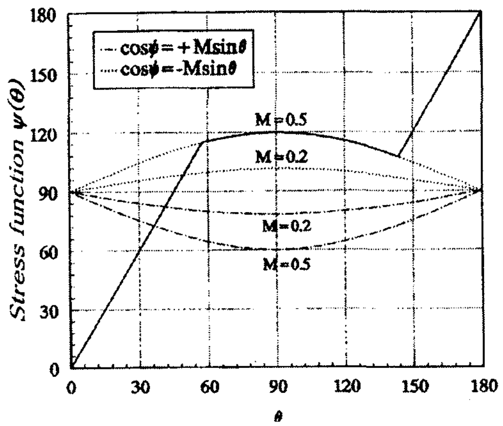

Fig. 2 Angular variation of stress function for M=0.2 and 0.5

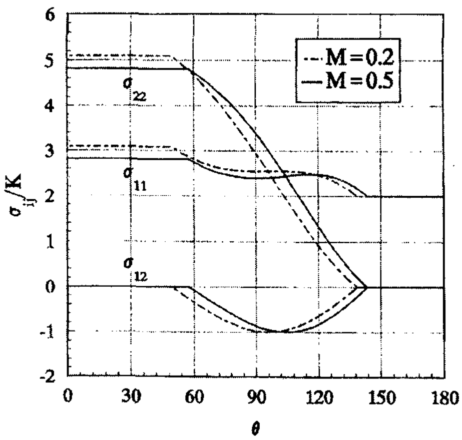

Fig. 3 Angular distributions of stress components for M=0.2 and 0.5

These results are the plastic solution around the entire dynamic crack tip without elastic unloading. The parameter K in Fig. 3 is a constant relating to the strain softening exponent n that cannot be determined from the asymptotic solution, but it can be determined by matching the asymptotic solution with the far-field solution such as the finite element numerical solution. Fig. 3 shows that the tensile stress ahead of the moving crack tip decreases with the increasing crack speed M.

4. Conclusions

An elastic-plastic asymptotic analysis has been carried out for the steady-state dynamic mode-I crack propagation in an incompressible power-law strain softening material under plane strain conditions. The deformation of the material obeys the J_2 flow theory of plasticity.

(1) For the power-law strain softening material, when the stresses and

strains have the same logarithmic singularity as

$$\sigma_{\alpha\beta} \sim \left(\ln\frac{R}{r}\right)^{\frac{1}{n-1}} \text{ and } \varepsilon_{\alpha\beta} \sim \left(\ln\frac{R}{r}\right)^{\frac{1}{n-1}}.$$

one can construct the fully continuous asymptotic crack-tip fields.

(2) The structures of the dominant asymptotic fields for mode-I crack in power-law softening materials are exactly the same as those in elastic-perfectly plastic materials, that is to say, the angular variations of field quantities are identical, the elastic unloading does not appear behind the crack-tip, the crack tip is enclosed entirely by plastic sectors.

(3) The present asymptotic stress field includes a free constant K relating to the strain softening exponent n which cannot be determined from the asymptotic solution, but it can be determined by matching the asymptotic solution with the far-field solution.

(4) The solution reported in the paper represents a possible mathematical structure of the damage near-tip fields for a dynamically propagating crack. Whether they can be achieved in reality or not must be examined against full-field solutions for properly posed boundary-initial value problems obtain-ed with, for example, the finite element methods. Such full-field numerical solutions for the problem considered are not available yet.

References

[1] Rosakis, A.J. and Ravichandran, G. Dynamic failure mechanics. International Journal of Solids and Structures 37, pp. 331-348. 2000.

[2] Gao, Y. C. and Nemat-Nasser, S. Dynamic fields near a crack tip growing in an elastic-perfectly-plastic solid. Mechanics of Materials 2, pp. 47-60. 1983a.

[3] Gao, Y. C. and Nemat-Nasser S. Near-tip dynamic fields for a crack advancing in a power-law elastic-plastic material : Modes I, II, III. Mechanics of Materials 2, pp. 305-317. 1983b.

[4] Gao, Y.C. and Nemat-Nasser, S. Mode II dynamic fields near a crack tip growing in an elastic-perfectly-plastic solid. Journal of the Mechanics and Physics of Solids 32, pp. 1-19. 1984

[5] Nemat-Nasser, S. and GAO, Y.C. Discontinuities in elastic-plastic solids. Mechanics of Materials 7, pp. 215-229. 1988.

[6] Nemat-Nasser, S. and Obata, M. Some basic issues in dynamic crack-growth in elastic plastic solids. International Journal of Fracture 42, pp. 287-300. 1990.

[7] Leighton, J. T., Champion, C. R. and Freund, L. B. Asymptotic analysis of steady dynamic crack growth in an elastic-plastic material. Journal of the Mechanics and Physics of Solids 35, pp. 541-563. 1987.

[8] Drugan, W.J. and Shen, Y. Restrictions on dynamically propagating surfaces of strong discontinuity in elastic-plastic solids. Journal of the Mechanics and Physics of Solids 35, pp. 771-787. 1987

[9] Varias, A. G. and Shih, C. F. Dynamic steady crack growth in elastic-plastic solids propagation of strong dis-continuities. Journal of the Mechanics and Physics of Solids 42, pp. 1817-1848. 1994

[10] Huang, Y., Zhu, X. K. and Hwang, K. C. On the possibility of strong discon-tinuity for dynamic crack propagating in compressible elastic perfectly-plastic material. Acta Mechanica Solida Sinica 8, pp. 188-193. 1995

[11] Zhu, X.K. and Hwang, K.C. Dynamic asymptotic fields near a crack tip growing in a power-law hardening compressible material. Mechanics of Materials 24, pp. 213-220. 1996

[12] Zhang, L., Zhu, X.K. and Hwang, K.C. Near-tip fields of steady dynamic crack growth in a compressible elastic-plastic material. Journal of the Mechanics and Physics of Solids 45, pp. 989-1005. 1997.

[13] Zhu, X.K., Hwang, K.C. and Liu, G.T. Near-tip fields for a dynamic crack propagating in a power-law elastic-plastic material. International Journal of Fracture 85, pp. 201-209. 1997.

[14] Drugan, W.J. Thermodynamic equi-valence of steady-state shocks and smooth wave in general media; applications to elastic-plastic shocks and dynamic fracture. Journal of the Mechanics and Physics of Solids 46, pp. 313-336. 1998.

[15] Xu, Y. and Saigal, S. An element free Galerkin analysis of steady dynamic growth of a model I crack in elastic-plastic materials. International Journal of Solids and Structures 36, pp. 1045-1079. 1999.

[16] Gao, Y. C. The asymptotic solution to the dynamic crack-tip field in a strain-damage material. International Journal of Engineering Science 24, pp. 1045-1055. 1986.