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Combined Optimal Design of Flexible Beam with Sliding Mode Control System

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ABSTRACT: *In order to achieve the desired lightweight and robust design of a structure, it is preferable to design a structure and its control system, simultaneously, which is termed the combined optimal design. A constant-cross-sectional area cantilever beam was chosen as the optimum design method. An initial load and a time-varying disturbance were applied at the free end of the beam. Sliding mode control was selected, due to its insensitivity to the disturbance, compared with other modes. It is known that the sliding mode control is robust to the disturbance and is uncertain, only if a matching condition is met, after giving a switching hyper plane. In this study, the optimum method was used for the design of the switching hyper plane, and the objective function of the optimum switching hyper plane was assumed to be the objective of the control system. The total weight of the structure was treated as a constraint, and the cross sectional areas of the beam were considered as design variables, the result being a nonlinear programming problem. To solve it, the sequential linear programming method was applied. As a result of the optimum design, the effect of attenuating vibrations has been substantially improved. Moreover, the lightweight design of the structure became possible as a result of the relationship of the weight of the structure to the control objective function.*

1. Introduction

To design a flexible structure, in which lightweight is required, it is important to consider the structure and control system, mutually, when we create the design, as their dynamic characteristics shall have an influence on each other. In connection with this, the sliding mode control theory, which removes disturbance and uncertainty in the input space of the system, has been focused on recently. Sliding mode control is a control method employed to stabilize the system by restricting state trajectory of the target system on the switching plane, and then moving the state to a point of equilibrium to generate sliding mode in the switching plane, with switching operation input from both sides of switching plane, which is set to the stated space (Hung, 1993; Harashima, 1985; Park, 1996). Onoda and Haftka(1987) minimized the linear sum of the structural weight with control energy, under the restriction of vibration energy. Khot(1998) minimized the structural weight under the restrictions of characteristic vibration numbers, damping coefficient of the closed-loop system for flexible structure.

In this paper, we have chosen beam structure to be the design objective. We presented the optimal design method of structure,which is applied sliding mode control, to repress the initial external force and vibration, which is from a continuous disturbance. At first, we set the structure as the control objective,which is lower, dimensionization by mode conversion after modeling cantilever beam, design objective, by using finite element method. In designing the sliding mode control system, we designed a control value that prevents chattering, by setting up the boundary layer using the concluding sliding mode control method. We also designed the optimal switching plane, which minimizes state fluctuation, by using a binominal objective function of the state. We then dealt with the combined optimal design problem, which means a non-linear programming problem, in order to minimize the objective function of the control system. The control system objective function is by specifying that binominal objective functions of state as objective function of optimal design problem. The design variables are cross sectional areas of beam structure upper limit values of structure weight are taken

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2. System Modeling

The equation of motion of n degree of freedom structure system is expressed as follow

$$M\ddot{q}+D\dot{q}+Kq=L(w+u) \quad (1)$$

Each factor, $M\in R^{n\times n}, D\in R^{n\times n}, K\in R^{n\times n}$ represents mass, damping, and stiffness matrices, here

$q \in R^n, w \in R^m, u \in R^m$ meant displacement, disturbance input, and control input vector. L is the arrangement matrix of disturbance input and of control input, We supposed that a matching condition, when the arrangement matrix of the control input and the disturbance input are the same, is satisfied in this research. M, K is computed using the finite element method, and the structural damping characteristic is supposed by Rayleigh Damping.

Next, determine the lower dimensionization system with mode coordinate conversion, we changed displacement vector q to mode coordinate vector ξ as follows:

$$q = \Phi\xi \tag{2}$$

$\Phi \in R^{n \times r}$ is the characteristic mode determinant of the system, here, and r is the adopted mode number. By defining the following state variable:

$$x = \begin{bmatrix} \xi \\ \dot{\xi} \end{bmatrix} \tag{3}$$

Eq. (1) is converted to the state equation as follows in Eq. (4)

$$\dot{x} = Ax + B(w + u) \tag{4}$$

$$A = \begin{bmatrix} 0 & I \\ -\Omega^2 & -\Lambda \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ \Phi^T L \end{bmatrix} \tag{5}$$

$$\Omega^2 = \Phi^T K \Phi = diag(\Omega_1^2, \Omega_2^2, \cdots \Omega_r^2) \tag{6}$$
$$\Lambda = \Phi^T D \Phi$$

And we also formalized Φ to make $\Phi^T M \Phi = I$ be formed.

3. Sliding Mode Control

Sliding plane of Eq. (4) is established as follows(Nonami, 1994):

$$\sigma = Sx \tag{7}$$

Also, supposed as follows:

(A,B): controllable, rank B=m

det(SB)≠0, rank S=m

$$B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \qquad det\, B_2 \neq 0$$

When sliding mode happens,

$$\sigma = 0,\ \dot{\sigma} = S\dot{x} = SAx(t) + SBu(t) = 0 \tag{8}$$

Sector equivalence control input is given as follows from Eq. (8):

$$u_{eq} = -(SB)^{-1} SAx \tag{9}$$

It is possible to extrapolate the control input by using the last sliding mode control method as follows:

$$u = u_l + u_{nl} = -k_l x - k_{nl} \frac{\sigma}{\|\sigma\|} \tag{10}$$

Here, we think that the control input is consistent with the sector feedback control clause u_l and independent clause of non-linear control clause. k_{nl} is the scholar function, which is calculated by design. In the case that the state trajectory exists beyond $\sigma_i = 0$ area, k_{nl} takes charge role to regulate the speed of the switching plane. From Lyanpunov function, u_l is defined as follows:

$$u_l = u_{eq} = -(SB)^{-1} SAx \tag{11}$$

Control value is designed as follows in order to suppress chattering .

$$u = \begin{cases} u_l - k_{nl} \frac{\sigma}{\|\sigma\|} & \sigma > \delta \\ u_l - k_{nl} \frac{\sigma}{\delta} & -\delta < \sigma < \delta \end{cases} \tag{12}$$

δ is the thickness of the boundary layer plane surroundings, which is set to approximate discontinuity input to continuous function. The canonical system, as shown below, is derived from the canonical system conversion determinant, which is used for lower dimensionization.

$$T = \begin{bmatrix} I_{n-m} & -B_1 B_2^{-1} \\ 0 & I_m \end{bmatrix} \tag{13}$$

$$\begin{cases} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ B_2 \end{bmatrix} u \\ \sigma = \begin{bmatrix} \bar{S} & I_m \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \end{cases} \tag{14}$$

Here, $ST^{-1}=\begin{bmatrix} \bar{S} & I_m \end{bmatrix}$, $x_2=\bar{S}x_1$ forms from, $\sigma=0$, x_2 is meant as input from x_1 of sub system. In other words, Eq. (14) shows lower dimensionization as follows:

$$\begin{cases} \dot{x}_1=(A_{11}-A_{12}\bar{S})x_1 \\ \sigma=0 \end{cases} \tag{15}$$

(A_{11}, A_{12}) is temporary since (A, B) is set to be temporary. In this case, the dynamic characteristic for sliding mode within $\sigma=0$ is decided by determinant S. To obtain the optimal switching plane, which means to have less state fluctuation after sliding mode on system of Eq. (14), we introduced the following objective function:

$$J_{con}=\int_{ts}^{t} X^T Q X dt \tag{16}$$

$$Q=\begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix}, Q_{21}^T=Q_{12}, Q>0 \tag{17}$$

ts states the time while $X=[x_1^T \; x_2^T]$ becomes sliding mode. Eq. (16) is converted to as follows by Eq. (14), (17):

$$J_{con}=\int_{ts}^{t}(x_1^T Q_{11} x_1+2x_1^T Q_{12} x_2+x_2^T Q_{22} x_2)dt \tag{18}$$

$$PA_{11}^*+A_{11}^{*T}P-PA_{12}Q_{22}^{-1}A_{12}^TP+Q_{11}^*=0 \tag{19}$$

$$Q_{11}^*=Q_{11}-Q_{12}Q_{22}^{-1}Q_{12}^T \tag{20}$$

$$A_{11}^*=A_{11}-A_{12}Q_{22}^{-1}Q_{12}^T \tag{21}$$

$$\sigma=SX=[A_{12}^TP+Q_{12}^T \quad Q_{22}]X \tag{22}$$

If we generate sliding mode, and define the optimal switching plane as follows in Eq. (22) with a single answer P of Riccati equation., it is possible to compose the control system that minimize J_{con}. Fig 1 shows the Block diagram of the system.

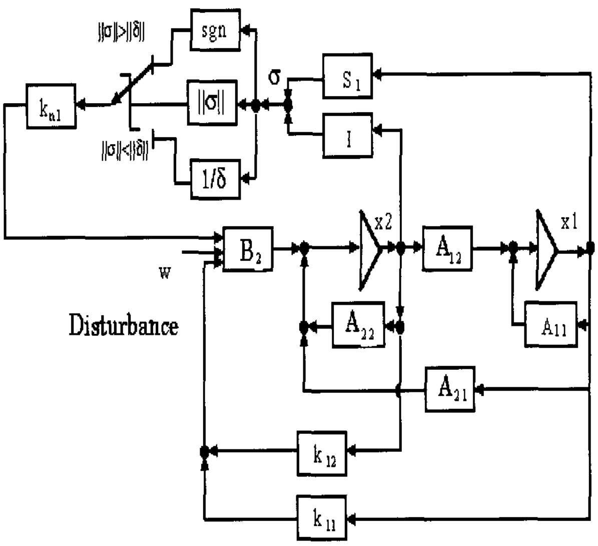

Fig. 1 Block diagram of sliding mode control system

4. Optimal Design of the Structure

The minimum value of the control system objective function of Eq. (18) is expressed as follows(Park and Choi, 1996; Kim, 1999; Kim et al. 2001):

$$J=\min J_{con}=x_1(0)^T P x_1(0) \tag{23}$$

$x_1(0)$ is initial state of x_1, and $x_1(0)$, $x_2(0)$ is as follows in by Eq. (3) and (13).

$$\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix}=Tx(0)=T\begin{bmatrix} \xi(0) \\ 0 \end{bmatrix} \tag{24}$$

The following Eq. formed by both Eq. (2) and the condition $\Phi^T M \Phi=I$.

$$\xi(0)=\Phi^T M(a) q(0) \tag{25}$$

In this paper, the changed state, due to an initial external force, is considered as the initial state. Therefore, when inserting Eq. (25) into Eq. (24), the following is formalized

$$x_1(0)=[I_{z\times z} \; O_{z\times(2r-z)}]T\Phi^T M(a)\begin{bmatrix} K(a)^{-1}F_0 \\ 0 \end{bmatrix} \tag{26}$$

$K(a)$ is a stiffness determinant, $M(a)$ is a mass determinant, a is cross sectional area, F_0 is the initial external force, T is the system canonical conversion determinant, Φ is the characteristic mode determinant of the system, r is the adopted mode number, and z is number the of state including x_1.

From now on, minimizing the objective function of the control system, after establishing the upper limit value in structure weight. The design problem is familiarized as follows:

$$\min_{a} \quad J(a) = x_1(0)P\,x_1(0) \tag{27}$$

$$st. \quad W(a) \sum_{i=1}^{n} p_i\, l_i\, a_i \le W_c \tag{28}$$

$$a_i \ge \underline{a}_i, \qquad i=1,\cdots,n \tag{29}$$

$$PA_{11}^{*} + A_{11}^{T}P - PA_{12}Q_{22}^{-1}A_{12}^{T}P + Q_{11}^{*} = 0 \tag{30}$$

Each factor of ρ_i, l_i, a_i is the low limit value of density, length, sectional area of element; W_c is a weight regulation requirement which is supposed to be the initial weight. Sequential linear programming is used for optimization technique of the optimal design problem about the variable of the structure system. The design problem, which is linearlized by Sequential linear programming, in the optional design point as follows:

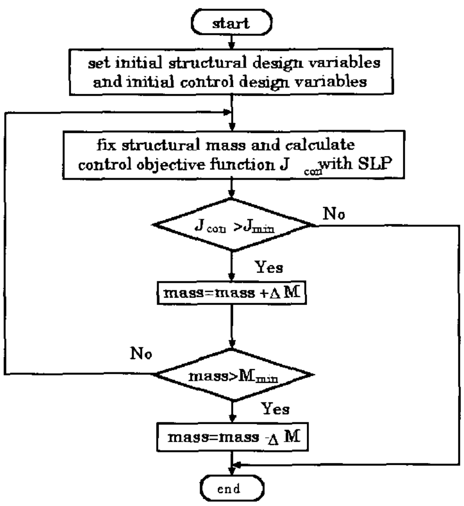

Fig. 2 Computation flow chart

$$\min_{a} \quad \nabla J(a)\triangle(a) \tag{31}$$

$$s.t.\ \nabla W(\triangle a) \le 0 \tag{32}$$

$$|\triangle a_i| \le \varepsilon_i,\ i=1,\cdots,n \tag{33}$$

$$a_i \ge \underline{a}_i, \qquad i=i=1,\cdots,n \tag{34}$$

ε_i is move limit here. Fig. 2 is showing the flow chart of optimal design problem.

5. Numerical Simulation

Cantilever beam structure is the objective, which is shown in Fig. 3, in this research. Each [1], [2], [3], [4] represents the element numbers, and each number 1, 2, 3, 4, 5 represents the node numbers.

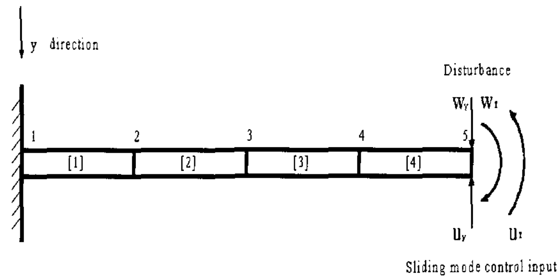

Fig. 3 Cantilever beam

We simply describe how to define $M(a), K(a)$ by finite element method. Hereby we defined l, length of each element , stiffness matrix k of one dimensional element of the cross sectional area a and mass determinant m as follows:

$$k = \frac{E_l a}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$m = \frac{\rho a l}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

E_l is a module of direct elasticity, and ρ is density. k, m is a determinant of part coordinate system. Therefore, it is changed to be the determinant of the whole coordinate system to coordinate conversion. We shall obtain the stiffness matrix, $K(a)$, the mass matrix, $M(a)$ for the whole structure, using the method that adds the whole coordinate stiffness matrix and the mass matrix of each element.

Structure of Fig. 3 is said to have the physical amount of Table 1. Each physical amount is dimensionless. The damping characteristic is supposed by Rayleigh Damping

$$D(a) = \alpha M(a) + \beta K(a) \tag{35}$$

It is $\alpha = 0.01$, $\beta = 0.003$ here.

The cross sectional area (early sectional area) for early structure is fixed by $a_i = 10$, and because node number 1 is restricted, degree of freedom is 0. When we think that initial external force, and serial indeterminancy disturbance act for transverse displacement of node 5, it is better that scale extent of this disturbance is supposed by within 10, which is known from the experience. We operated control input and disturbance on the same direction, since the matching condition is met.

Table 1 Data of cantilever beam

Shape of Section	Circle
Elastic Modulus	20N/m2
Total Beam Length	100m
Mass Density	1kg/m3
Number of Elements	4
Number of Nodes per Element	2
Number of D.O.F. per Node	2

5.1 Case 1

Controlled value is sliding mode from Eq. (12). *S*, which is designed by optimum plane design as follows.

$$S = \begin{bmatrix} 0.8452 & 0.4793 & -0.2428 & -0.4298 & 1 & 0 \\ 0.4799 & -0.3534 & 0.8867 & -0.9853 & 0 & 1 \end{bmatrix}$$

Here, the bigger k_{nl} of Eq. (10) is, the faster state trajectory of the structure is restricted into the switching plane. However, if it is too big, there will be problems which chattering or with the control input growth. Also, if k_{nl} is too small, it is impossible to materialize actuality equipment, because switching time of control input is too short. Therefore we fixed k_{nl} by 50.

When you make the boundary layer δ of the switching plane less, control effect improves; if it is too small, it is, yet again, hardly possible to materialize actuality equipment because switching time of non-linear input is becoming shorter. For this reason, we set δ by 0.002. On the assumptions that the force of y direction that is loaded to node 5 of cantilever beam and the size of consecutive disturbances are below 10, the size of initial external force is 1, we presented Fig. 4 showing the transverse displacement of node 5 for each case of LQ control and sliding mode control with out imposing any restrictions on.

5.2 Case 2

Supposing that disturbance and system uncertainty exist, simultaneously we then let system uncertainty, $\triangle B$ be $-0.1\ B_2$ state x_1 sensor be behind time by 0.5, and state x_2 sensor be behind time by 0.05.

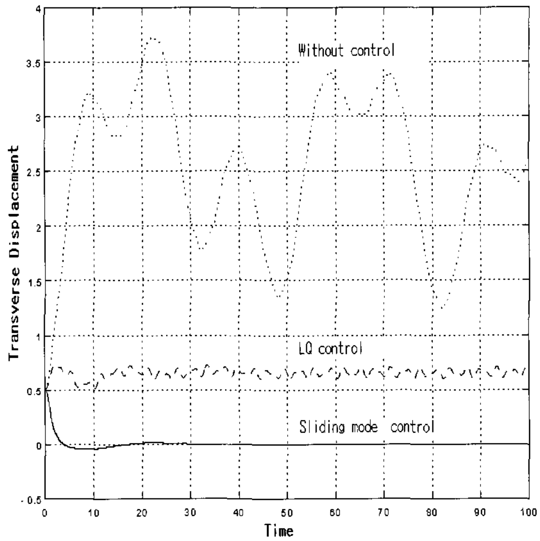

Fig. 4 Transverse displacement of node 5 (case1)

Fig. 5 displays the response compared between LQ control and sliding mode control. LQ control, which has been comparatively stabilized towards transverse displacement of node 5 under the condition that initial external force from Fig. 4 is 1, is expressing unstable response compared to sliding mode control in this case.

On the other hand, we proved sliding mode control is more robust, which has already been inferred by being stabilized on these two cases.

5.3 Combined optimal design

The optimization problem from Eq. (27) to Eq. (30), is set to be $W_c = a_0 L\rho$. The initial cross-sectional area (10) of each element is a_0, L is total beam length, and ρ is mass density. $\underline{a}_i$, the low limit value of the sectional area, is 0.01. We presented Fig. 6 to show the sectional area of each element of beam, the result was achieved by optimal design.

When you look at the result, the cross sectional area of each element of cantilever beam, which is from the cross sectional area of early structure is $a_0 = 10$ is bigger than early sectional area in the case of elements 1 and 4, but smaller than the early sectional area in the case of elements

2 and 3. This means that only by changing the cross sectional area from the optimal design, were we able to achieve the light-weight, which reduced about 4.8% of the total weight. Fig. 7 and Fig. 8 are used to show the response about initial external force and disturbance, compared to early structure by these light-weight structures. Fig. 7 is expressing the response of optimal design construct and transverse displacement response characteristic of early structure, in condition that initial external force and disturbance are imposed. In the case of early structure, it became suitable, after a cycle of vibration, from the beginning to 40 unit time. In the case of optimal design structure, it experienced a cycle of vibration from early to about 90 unit time, but the amplitude is about 1/2 of the early structure. Therefore, it is obvious that the response of the optimal design structure is superior to early structure when initial external force and disturbance are imposed. Also, Fig. 8 is showing the comparison of vibration control effect about each displacement, between early structure and optimal design structure, in case the same initial external force and disturbance loaded equally. Early structure is keeping each displacement of about 0.37, continuously, as well as existing vibration for early 90 unit time.

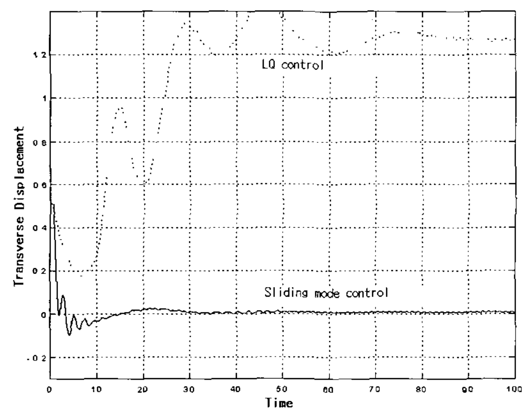

Fig. 5 Transverse displacement of node 5 (case 2)

On the other hand, the optimal design cantilever beam controls vibration from the very beginning, so that it shows very stable vibration levels, in which each displacement converges to almost 0 after 30 unit time. Therefore, the result as shown is Fig. 7 and Fig. 8, displays the fact that optimal design structure is a superior characteristic of angular displacement and controls the vibration of each displacement to early structure, about the external force and disturbance.

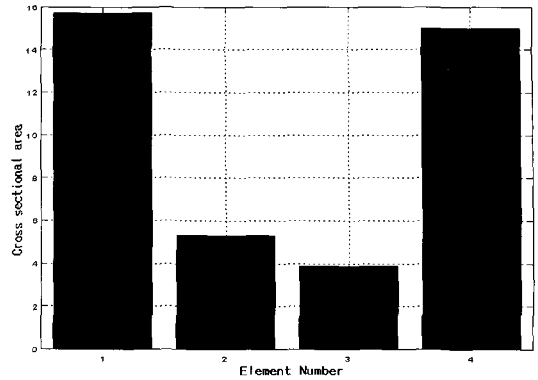

Fig. 6 Distribution of optimal cross sectional area

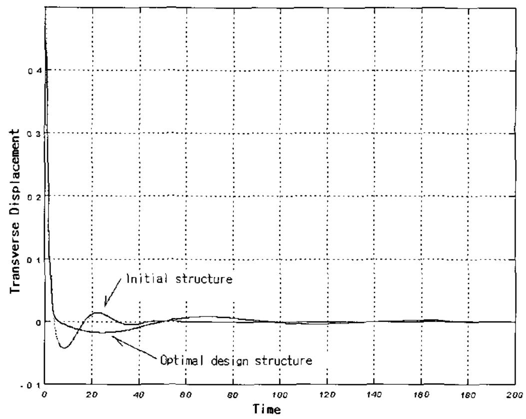

Fig. 7 Transverse displacement of node 5

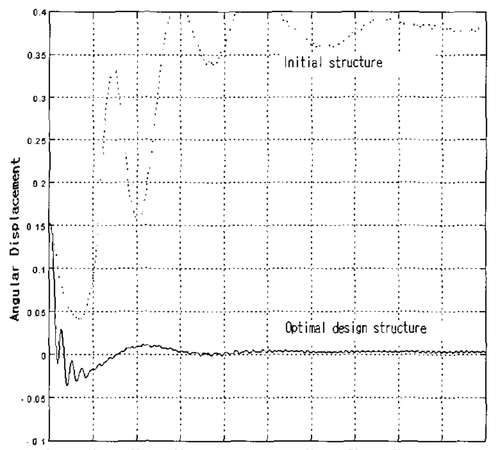

Fig. 8 Angular displacement of node 5

6. Conclusions

This paper proposed the combined optimal design method of structure/control system by using sliding mode control. We verified the effectiveness of combined optimal design with numerical simulation, by designing the switching plane of the sliding mode control system and the cross sectional area of the structure system towards the cantilever beam, which is the objective of this design. We reached the following conclusions:

1) Sliding mode control has very a good quality of robust to transverse displacement of node 5 than LQ control has, when initial external force and disturbance are imposed.

2) The cross sectional area of each element of cantilever beam, using optimal design, is different. In the cases of elements 1 and 4, the design is bigger than in the cases of element 2 and 3 which is a different size. It was possible to achieve a light weight, which reduced the total weight of the early structure by 4.8%.

3) The optimal design structure, which achieved the desired light weight, is superior to early structure in quality of transverse and angular displacement, 0when initial external force and disturbance are imposed.

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