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Structure-Control Combined Optimal Design of 3-D Truss Structure Considering Initial State and Feedback Gain

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ABSTRACT: *This paper proposes an optimum, problematic design for structural and control systems, taking a 3-D truss structure as an example. The structure is subjected to initial static loads and time-varying disturbances. The structure is controlled by a state feedback* H_∞ *controller which suppress the effects of disturbances. The design variables are the cross sectional areas of truss members. The structural objective function is the structural weight. For the control objective, we consider two types of performance indices. The first function represents the effect of the initial loads. The second function is the norm of the feedback gain. These objective functions are in conflict with each other but are transformed into one control objective by the weighting method. The structural objective is treated as the constraint. By introducing the second control objective which considers the magnitude of the feedback gain, we can create a design to model errors.*

1. Introduction

Space structures, are required to be light-weight in order to avoid excessive transportation costs. However, when the structures are lighter, their stiffness becomes small, and even a little disturbance can cause big vibrations. The inner damping of space structures is so small that once such vibrations are caused, they are difficult to suppress. For these cases, designs considering the effect of active vibration control have been started. Typical methods have treated the design of structural and control systems separately. Recently, in the field of large space structures, the necessity of optimizing structural and control systems has led to strict control requirements, as well as increased research (Tada and Park, 2000). Generally, in a combined optimal design, a weighting sum of a structural and control objective functions is adopted as a single objective function that is minimized.

In this paper, we formulate a design problem which determines structural sizes considering control objective functions subject to the constraint of the constant structural weight as one of combined optimal design. We take a 3-D truss structure as an object. For the structure, an FEM model with an H_∞ control system is formed, and used. With H_∞ control, a structure becomes stable. The H_∞ norm of the transfer function from the disturbance inputs to the controlled outputs in the closed-loop system can be kept within a certain value (Kim, 1999; Sanpei and Mita, 1990).

With a control system, a design considering a feedback gain is attempted. This is because costs will increase significantly if a feedback gain becomes too big. Additionally, modeling errors increase greatly with a large feedback gain. Therefore, the performance index considering a feedback gain is also introduced. Then, the control objective is to suppress the controlled output and to consider a feedback gain. The design variables are the cross sectional areas of the truss members and the structural performance index, which is limited. This nonlinear optimization problem is solved with sequential linear programming.

Thus, we create a design considering the effects of the initial state and the disturbance, provided that an H_∞ control is adopted. Under these conditions, we discuss how the minimization of the control objective functions affects the structures through numerical examples, as in the case of a 3-D truss.

2. Problem Statement

2.1 Combined optimization

For a 3-D truss structure, deformed by initial static loads, which begins to and suffer from continuous disturbance, the design problem that obtains gets a certain vibration characteristic and decreases the structural weight by changing cross sectional areas of truss members is treated.

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2.2 Problem formulation

We take a 3-D truss structure as an object. The original FEM spatial model of a DOF structure is expressed by the following equation of n motion.

$$M_s \ddot{q} + D_s \dot{q} + K_s q = L_1 w + L_2 u \tag{1}$$

where M_s, D_s and $K_s \in R^{n \times n}$ are the mass, the damping, and the stiffness matrices. $q \in R^n, w \in R^p$ and $u \in R^d$ are the displacement, the disturbance force, and the control force vectors. p and d are the numbers of the disturbances and the control forces, respectively. The state equation of the model becomes

$$\dot{x} = Ax + B_1 w + B_2 u \tag{2}$$

$$z = Cx + Du \tag{3}$$

$$y = x \tag{4}$$

where $x \in R^n, z \in R^e$ and $y \in R^f$ are the state variable, the controlled output, and the measured output vectors. In this paper, the optimal system is designed under the assumptions that equations. (2) and (3) are both stable and detectable and that the control system is composed of the state feedback (that is, $y = x$). The initial displacement is obtained by the initial load p_0 from the stiffness equation:

$$q_0(a) = K^s(a)^{-1} p_0 \tag{5}$$

The initial displacement is a function of design variables, cross sectional areas a, is the stiffness matrix K_s. With the initial load p_0, the initial state is given by

$$x_0 = \begin{bmatrix} K_s^{-1} p_0 \\ 0 \end{bmatrix} \tag{6}$$

where we assume that the initial velocity is zero.

In this paper, as we develop a combined optimal design of structural and control systems, performance indices of respective systems must be established. The structural objective function is the structural weight. As the control objective, we consider two types of performance indices. The first function, J_1, represents the effect of the initial loads, and the second, J_2, is the norm of the feedback gain. These objective functions are explained in the next section.

2.3 Multi objective problem

The objective functions J_1 and J_2 may be in conflict with each other. Here, the weighting method and the ε-constraint method, which are used to obtain the Pareto optimal solutions for multi objective problems(Tada and Park, 1999) are explained.

[Weighting Method]

m objective functions $f_i(x)\,(i = 1, \cdots, m)$ to be considered are transformed into a weighting sum as a single objective function and them minimized.

$$\min_x = \sum_{i=1}^{m} w_i f_i(x) \tag{7}$$

$$\sum_{i=1}^{m} w_i = 1 \tag{8}$$

$$x \in X$$

where x, X and $w_i(i = 1, \cdots, m)$ are design variables, a set of feasible solutions, and weighting. In this paper, two kinds of control objective functions are transformed into a single function by this method.

[ε-constraint Method]

One objective functions $f_j(x)\,(j = 1, \cdots, r)$ is minimized under the condition that others are limited to less than certain values.

$$\min_x f_1(x) \tag{9}$$

$$\text{s.t.} \quad f_j(x) \le \varepsilon_j\; j = 2, \cdots, r \tag{10}$$

$$x \in X$$

The ε-constraint method is used or the structual weight and the control objective function. The control objective function is adopted as $f_1(x)$ in Eq. (9), and the structural objective function is adopted as $f_2(x)$ in Eq. (10). If f_1 and f_2 are in conflict with each other, the constraint in Eq. (10) becomes an equality constraint with the structural weight as a constant.

In short, the two control objective functions J_1 and J_2 are transformed into one control objective, J_1, by the weighting method. The problem of two objectives, structural and control, is transformed into a single objective function by the ε-constraint method. Then, for the controlled system, the objective function is the control objective function $J(a)$. The constraint that the structural weight $W(a)$ is equal to a specified value W_c is given.

$$\min_a J(a) = sJ_1 + tJ_2(a) \qquad (s+t=1) \tag{11}$$

$$\text{s.t.} \quad W(a) = \sum_{i=1}^{n} \rho_i l_i a_i = W_c \tag{12}$$

where ρ_i, l_i and a_i are the density, length, and cross sectional area of the i-th member.

This control objective function $J(a)$ is minimized by changing cross-sectional area a_i. Because the two control objective functions J_1 and J_2 are in conflict, it is impossible to optimize them simultaneously, and the Pareto optima are obtained. Individual designers choose their own solution.

2.4. Sequential linear programming

If an objective function is nonlinear, it is solved with sequential linear programming. In this method, objective functions and constraints are linearized at a certain point. The original nonlinear programming problem is approximated by a linear programming one, and its optimum solution is then obtained. By iterating this linearization, we can search for the optimum solution of the original problem.

3. Design of Control System

In this paper, the control system is designed with the H_∞ control to suppress the effect of the disturbance. As the control objective, we consider two types of performance indices.

3.1 H_∞ Control Problem

3.1.1 General H_∞ control problem

Consider a linear time-invariant plant G that maps disturbances inputs w and controls inputs u to controlled outputs z and measured outputs y (Fig.1).

That is
$$\begin{bmatrix} z(s) \\ y(s) \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} w(s) \\ u(s) \end{bmatrix} \tag{13}$$

where s stands for the Laplace variables.

The suboptimal H_∞ control problem of a parameter γ consists of finding a controller $K(s)$ such that:

- the closed-loop system is internally stable.
- the H_∞ norm of $T_{zw}(s)$ is strictly less than γ

$$\| T_{zw} \|_\infty < \gamma \tag{14}$$

where $T_{zw}(s)$ is the transfer function from the disturbance input w to the controlled output z in the closed-loop system obtained by applying the static state feedback to the system. γ is a prescribed positive number(Kim, 1999; Tada and Park, 1999).

For a rational, transfer function matrix $T(s)$, then

$$\| T(s) \|_\infty = \sup_\omega \sigma_{\max}(T(j\omega)) \qquad \forall \omega \in R \tag{15}$$

where the maximum singular value of T is denoted by $\sigma_{\max}(T)$.

Though Eq. (14) is given in a frequency domain, it is also expressed in a time domain using the Parseval equation. When z and w are expressed in a time domain,

$$\int_0^\infty \{z^T(t) z(t)\} dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} \{z^T(-j\omega) z(j\omega)\} dw \tag{16}$$

$$\int_0^\infty \{w^T(t) w(t)\} dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} \{w^T(-j\omega) w(j\omega)\} dw \tag{17}$$

Eq. (14) is denoted as follows.

$$T^T(-j\omega) T(j\omega) < \gamma^2 I \qquad \forall \omega \in R \tag{18}$$

Then, using Eqs. (16), (17), (18) and $z(s) = T(s)w(s)$, for all square integrable disturbances w, Eq. (14), which is expressed in a frequency domain is equivalent to the following equation in a time domain.

$$\int_0^\infty \{ z^T(t) z(t) \} dt < \gamma^2 \int_0^\infty \{ w^T(t) w(t) \} dt \tag{19}$$

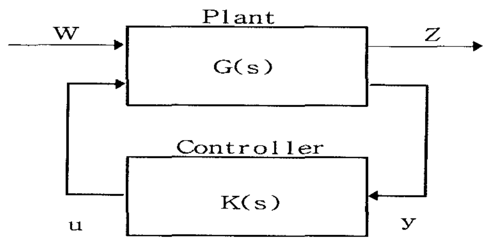


Fig. 1 H_∞ control system

3.1.2 H_∞ Control Problem for State Feedback

For a 3-D truss structure design subjected to initial loads and continuous disturbances, we must consider both the effects of the initial state and the disturbance as the system characteristic. Therefore, H_∞ control is used in the design of the control system.

Consider the linear system

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u \\ z &= Cx + Du \\ y &= x \end{aligned} \tag{20}$$

where $x \in R^n$, $w \in R^d$ and $u \in R^p$ are the state variable, the disturbance, and the control force vectors. $z \in R^e$ and $y \in R^q$ are the controlled output and the measured output vectors, respectively. We assume that the matrices of the system of Eq. (20) satisfy the following corresponding assumptions.

- (A, B_2) is able to be stabilized.
- (C, A) is detectable.
- D^T [C D] $=$ [0 I].

One of the controllers for the state feedback is given by

$$u = -B_2^T Px \tag{21}$$

$$= -Kx \tag{22}$$

In the above, P, which is solved by the following Riccati equation is a semi-positive matrix and a stable solution.

$$A^T P + PA + \frac{1}{\gamma^2} PB_1 B_1^T P - PB_2 B_2^T P + C^T C = 0 \tag{23}$$

where a stable solution means a solution which makes $A + \gamma^{-2} PBB_1^T P - B_2 B_2^T P$ a stable matrix. If the solution is not stable, we must increase γ and redesign.

3.2 Objective function for control system

3.2.1 Objective function considering Initial state, J_1

If the initial state is a certain non-zero state ($x_0 \neq 0$), the following equation is obtained instead of Eq. (19).

$$\int_0^\infty (z^T z)dt \le x_0 P x_0 + \gamma^2 \int_0^\infty (w^T w)dt \tag{24}$$

The first term on the right side represents the effect of the initial state x_0, and the second term represents the disturbance w. If the effect of the initial load in Eq. (24) becomes small, the control force can be decreased. We take the first term as the first control objective function.

$$J_1 = x_0^T P x_0 \tag{25}$$

3.2.2 Objective function considering feedback gain norm, J_2

The norm of the feedback gain is defined(Kimura, 1990) as follows:

$$N = trace(K^T RK) \tag{26}$$

where R is the weighting matrix. If $R = I$, Eq. (26) represents the square-sum of all elements in the gain matrix K. If this norm is decreased, each element in the matrix K may become small. Then, in order to design the control system considering the feedback gain, this performance index is adopted as the second control objective J_2.

$$J_2 = trace(K^T K) \tag{27}$$

4. Numerical Examples

4.1 Design object

We take a 3-D truss structure shown in Fig.2 a numerical example. Considering non-dimensional form, the length of long members is 10, short members $2\sqrt{2}$, density 5.0, and Young's modulus 100. The nodes from 5 to 10 are fixed. The sensors which measure displacements are located at nodes 1, 2, 3, and 4, and the actuators which give control forces are located at node 1 in x, y, and z directions. The damping matrix is assumed by

$$D_s(a) = \alpha M_s(a) + \beta K_s(a) \tag{28}$$

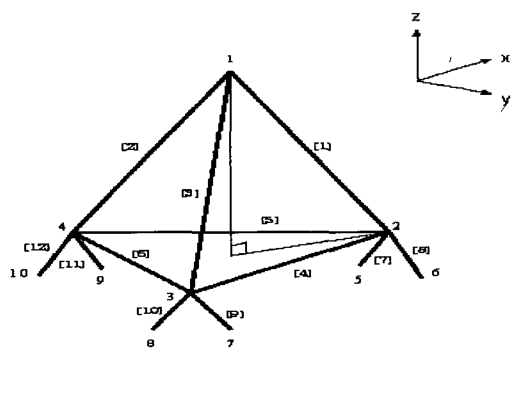
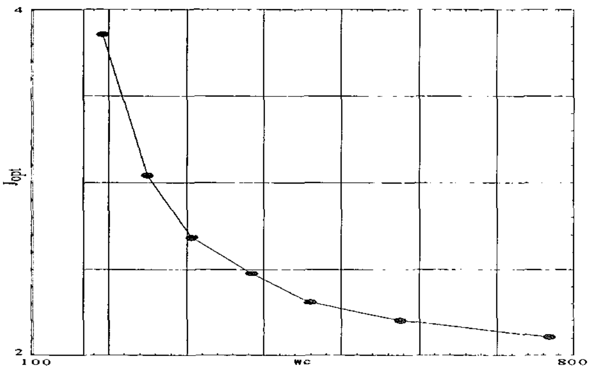


Fig. 2 3-D truss structure

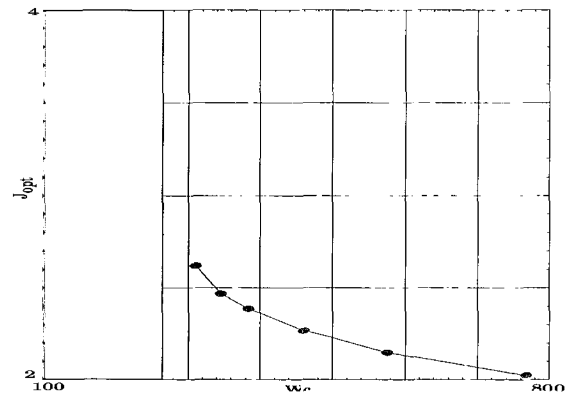
where $\alpha = 0.001$ and $\beta = 0.001$. The initial cross-sectional areas are all constant $(a_{i}=1 : i = 1, \cdots, 12)$, and the weight of this initial truss is adopted as the specified value of the structural weight.

4.2 Relation between structural and control systems

First, in the case of the set of weighting for the control object functions, $(s, t) = (0.5, 0.5)$, we perform the optimization of the control objective J for several values of the specified value of the structural weight W_c. Fig.3 shows the relation between the optimum value of the control objective and the specified value of the structural weight in two cases of the H_∞ disturbance attenuation γ. These graphs show the Pareto optimality of the structural objective W and the control one J. It is observed from the comparison of (a) with (b) that the smaller H_∞ norm bound γ, the greater the required structural weight needed, which shows a close relation between the structural design and the control one.



(a) $\gamma = 1.5$



(b) $\gamma = 2.0$

Fig. 3 Pareto optimality of two objectives for structural and control systems

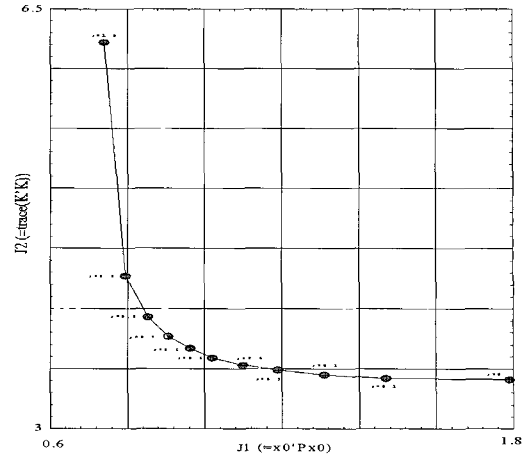
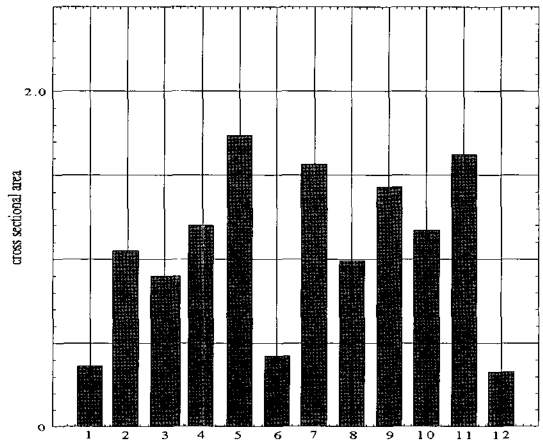
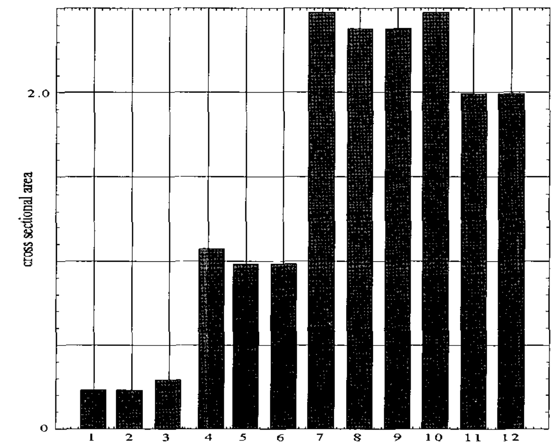


Fig. 4 Pareto optimality of performance indices in control system[



(a) $J = J_1$ $(s = 1.0,\ t = 0)$



(b) $J = J_2$ $(s = 0,\ t = 1.0)$

Fig. 5 Optimum cross sectional area

4.3 Relation between two control objectives

Next, under a certain specified value of the structural weight, we change the weighting of the control objective functions s and t, variously and solve Eqs. (11) and (12), in order to examine the effect of weighting s and t. In Fig.4, the set (J_1, J_2) corresponding to the minimum of J is plotted for several sets of (s, t). Fig.5 shows the optimum cross-sectional areas for two typical cases. It is observed from figures that if we increase the value t and the weighting for J_2, the cross-sectional areas of the supporting part of the truss become large and the difference in the cross sectional area in each stage of the truss becomes small.

4.4 Comparison of design

In order to show the effect of the gain on the control performance, we pick up two cases (case 1: $s=1.0$, $t=0$, ; case 2: $s=0.9$, $t=0.1$). Although the effect of the initial loads is nearly identical, the magnitude of the gain, J_2, in case 2 where the gain is taken into consideration is much smaller than that in case 1, which ignores the gain as shown in Table 1. $J_{ij} (i = 1,2 ; j = 1,2)$ is the value of the objective function J_i in the case j. In case 2, the H_∞ norm of the complementary sensitivity function, which represents the degree of robust stability in modeling errors, also is comparatively smaller than the H_∞ in the case 1.

Table 1 Comparison of Pareto optimality

	case 1	case 2
$\bar{J}_1 \; (= J_{1j} / J_{11})$	1.00	1.07
$\bar{J}_2 \; (= J_{2j} / J_{21})$	1.00	0.686
H_∞ norm of the complementary sensitivity function	8.59	6.75

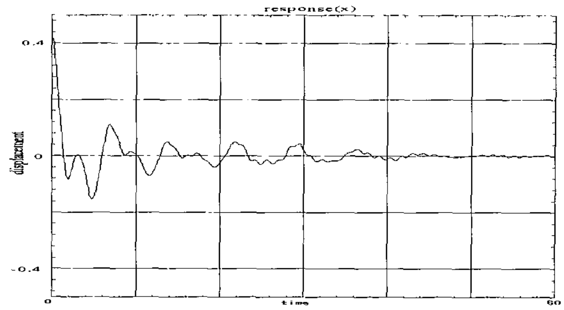
Thus, by introducing the second control objective J_2, which considers the magnitude of the feedback gain, we can perform the design which is robust in modeling errors.

In Fig.6, we show the responses of the displacement in the x-direction at node 1 as examples of simulation in the truss obtained for the two cases. The observed difference in response between the two cases was small.

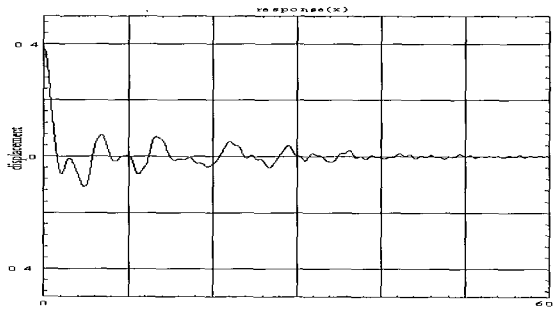
Case 1; without consideration of gain $s=1.0$, $t=0$ $(J=J_1)$

case 2; with consideration of gain,

$s=0.9, t=0$ $(J=0.9J_1+0.1J_2)$



(a) $s = 1.0$, $t = 0$



(b) $s = 1.0$, $t = 0$

Fig. 6 Time responses of displacement at node 1

5. Conclusion

In this paper, we formulated a design problem which that determines structural sizes, considering control objective functions subject to the constraint of the constant structural weight as one of combined optimal design. Two control objective functions were introduced. J_1 represents the effect of the initial loads, and J_2 represents magnitude of the feedback gain. First, two control objective functions were transformed into one control objective function by the weighting method, and the Pareto optimality of the structural and control objectives was shown. Through numerical examples, a close relationship between both systems was represented.

Second, increasing the weighting for the second control objective function under the specified value of the structural weight decreases the difference in the cross sectional area in each stage in the truss. Therefore, it is unlikely that the design dependent only on initial loads if said loads have been disturbed.

Third, the comparison of two cases $(J=J_1$ and $J=0.9J_1+0.1J_2)$ shows that although the effect of the initial loads is almost the same in both cases, the magnitude of the gain in the second case is much smaller. Moreover, in the second case, the H_∞ norm of the complementary sensitivity function, which represents the degree of the robust stability in modeling errors, is also smaller. Thus, by introducing the objective considering the magnitude of the feedback gain, we can create a design unsurpassed in modeling errors.

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