

## An Experimental Study of Nonlinear Viscoelastic Bushing Model for Axial Mode

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A bushing is a device used in automotive suspension systems to cushion the force transmitted from the wheel to the frame of the vehicle. A bushing is essentially a hollow cylinder which is bonded to a solid metal shaft at its inner surface and a metal sleeve at its outer surface. The shaft is connected to the suspension and the sleeve is connected to the frame. The cylinder provides the cushion when it deforms due to relative motion between the shaft and sleeve. The relation between the force applied to the shaft or sleeve and its deformation is nonlinear and exhibits features of viscoelasticity. An explicit force-displacement relation has been introduced for multi-body dynamics simulations. The relation is expressed in terms of a force relaxation function and a method of determination by experiments on bushings has been developed. Solutions allow for comparison between the force-displacement behavior by experiments and that predicted by the proposed method. It is shown that the predictions by the proposed force-displacement relation are in very good agreement with the experimental results.

**Key Words :** Nonlinear Viscoelasticity, Bushing, Force Relaxation Function

### 1. Introduction

Bushings are structural elements which are used in suspension systems of automobiles and other vehicles. A bushing is, in effect, a hollow cylinder contained between an outer steel cylindrical sleeve and an inner steel cylindrical rod. The steel sleeve and rod are connected to the components in the automotive suspension system and are used to transfer forces from the wheel to the chassis.

Bushings are used to reduce shocks and vibrations in this connection. Because those are connected to different parts of the suspension system, the sleeve and rod undergo relative displacements and rotations about axes both parallel and perpendicular to their common centerline. It is the relative displacement that allows for the transmission of force through a bushing.

In analyzing suspension systems that contain bushings, engineers use multi-body dynamics computer codes. In the multi-body dynamics codes, the bushings are typically modeled by using a system of force-displacement relations. Thus, determining a correct force-displacement relation becomes an important task in analyzing a bushing. However, a bushing material is viscoelastic and nonlinear, hence, it causes the relation be-

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tween the nonlinear and time dependent corresponding forces and displacements as well as causing coupling between the different modes of response. This viscoelasticity cannot be neglected because it is a mechanism of energy dissipation that is essential for accurate force calculations.

Very few studies of the development of force-displacement relations for bushings have been found in the open literature. Adkins and Gent (1954) developed force-displacement relations for radial, axial, torsional, and conical modes of deformation of bonded cylindrical bushings. Since their results are obtained using the linear elastic theory, the relations do not account for nonlinear, time-dependent effects, or coupling between modes.

Morman et al. (1981) modeled a bushing material as a nonlinear viscoelastic solid. They assumed an appropriate constitutive equation and presented a finite element method for analyzing the response for small amplitude oscillations superposed on large equilibrium deformations. Although their method is important for studying many aspects of bushing response, it does not account for transient response. Thus, it is of limited use in a multi-body dynamics code and both of these works suffer from the disadvantages associated with defining a boundary value problem.

Kim and Youn (2001) proposed a viscoelastic constitutive equation for rubber under small oscillatory load superimposed on large static deformation. The model was extended to a generalized viscoelastic constitutive equation including widely-used Morman's model. Static deformation correction factor was introduced to consider the influence of pre-strain on the relaxation function.

Wineman et al. (1998) suggested a force-displacement relation for single mode bushing response that combines nonlinear dependence on displacement with the time dependence of viscoelasticity. In particular, force is given by a nonlinear single integral expression in terms of a force relaxation function, a bushing property, which represents the force when the bushing is subjected to a step displacement history. They described a process for determining the force

relaxation function for a bushing by mechanical testing and data processing and demonstrated this process for radial deformations of bushing.

Lee and Wineman (1999) carried out a study using numerical simulations in order to evaluate how well the force-displacement relation in (Wineman et al., 1998) represented the response of a bushing. They used a constitutive equation developed by McGuirt and Lianis (1970) from experiments on SBR together with the governing field equation of nonlinear viscoelasticity and carried out numerical simulation of the mechanical testing program discussed in (Wineman et al., 1998) using this numerically generated data. They determined an expression for the force relaxation function. Then, they compared predictions of the force-displacement model proposed in (Wineman et al., 1998) with exact predictions of the McGuirt and Lianis model. The comparison was very satisfactory. Thus, the study in (Lee and Wineman, 1999) provided support for the use of force-displacement relation suggested in (Wineman et al., 1998).

The purpose of the present work is to present the results of an experimental program for determining the force relaxation property for a bushing in axial mode. The nonlinear viscoelastic force-displacement relation and method of determining the force relaxation property are outlined in section 2, in order to make this paper as self contained as possible. Experimental results are presented in section 3. The relaxation property is determined in section 4 and its predictive quality is presented in section 5. Section 6 contains final comments.

## **2. Nonlinear Viscoelastic Force-Displacement Relation**

We consider a force-displacement relation for the bushing which combines nonlinear dependence on displacement and time effects of viscoelasticity. The simplest relation which meets these conditions appears to be that introduced by Pipkin and Rogers (1968) for the nonlinear viscoelastic response of polymers. In the current context, it has the form

$$F(t) = R(\Delta(t), 0) + \int_0^t \frac{\partial R(\Delta(s), t-s)}{\partial(t-s)} ds \quad (1)$$

$F(t)$  is the function of time and represents the force at current time  $t$  and  $s$  denotes time and has the unit of sec.  $R(\Delta, t)$  is a property of the bushing and represents the force at time  $t$  due to a step axial displacement of amount  $\Delta$  applied at time zero. That is,  $R(\Delta, t)$  can be interpreted as the axial displacement-dependent force relaxation function. It is reasonable to assume that  $R(0^-, t) = 0$  and that  $R(\Delta, t)$  is a monotonically decreasing function of time  $t$ . A typical force relaxation function is shown in Fig. 1.

By applying integration by parts to Eq. (1), the Pipkin-Rogers model can be rewritten as follows :

$$F(t) = R(\Delta(0), t) + \int_0^t \frac{\partial R(\Delta(s), t-s)}{\partial \Delta(s)} \frac{d\Delta(s)}{ds} ds \quad (2)$$

Ideally,  $R(\Delta, t)$  can be determined by subjecting the bushing to a deformation in which the outer sleeve undergoes a step axial displacement with respect to the inner rod. This ideal process cannot be realized due to the inertia of the testing apparatus. A method which can be used to determine  $R(\Delta, t)$  is outlined here.

Consider a displacement which increases at a constant rate to displacement  $\Delta_i$  in time interval  $T_j^*$ , and is then held fixed. It is described by

$$\Delta(t) = \begin{cases} \frac{\Delta_i}{T_j^*} t, & 0 \leq t \leq T_j^* \\ = \Delta_i, & T_j^* \leq t \end{cases} \quad (3)$$

where  $T_j^*$  is defined as the rise time, which is the

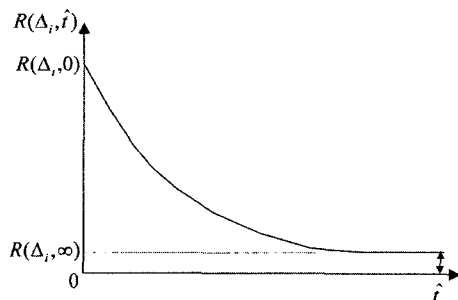


Fig. 1 Displacement-dependent force relaxation function for  $\Delta(t) = \Delta_i$

time where the displacement rate in displacement vs. time function changes from constant to zero.

Let this ramp to constant displacement history be denoted by  $\Delta(\Delta_i, T_j^*, t)$ . The force output history corresponding to this ramp to constant displacement history can be expected to increase monotonically until time  $T_j^*$ , and then relax for times  $t > T_j^*$ , when the displacement is held constant. As  $T_j^*$  approaches zero, the ramp to constant displacement control test approaches the step displacement control test. Moreover, as  $T_j^*$  approaches zero, the force response can be expected to approach the displacement-dependent force relaxation function  $R(\Delta_i, t)$ . In order to show this, let Eq. (3) be substituted into Eq. (2), and consider any time  $t \geq T_j^*$ . Since  $R(\Delta(0), t) = R(0, t) = 0$  and  $d\Delta(s)/ds = 0, s \geq T_j^*$ ,

$$F(t) = \int_0^{T_j^*} \frac{\partial R(\Delta(s), t-s)}{\partial \Delta(s)} \frac{d\Delta(s)}{ds} ds \quad (4)$$

This force will be denoted by  $\tilde{R}(\Delta_i, t, T_j^*)$ .

In Eq. (4), let the integration variable be changed from  $s$  to  $\Delta, \Delta = \Delta_i s / T_j^*$ . Also, let  $t = T_j^* + \hat{t}$ , where  $\hat{t}$  denotes a fixed time measured from  $T_j^*$ , when the displacement is held constant. Then, Eq. (4) becomes

$$\tilde{R}(\Delta_i, \hat{t}, T_j^*) = \int_0^{\Delta_i} \frac{\partial R(\Delta, \hat{t} + T_j^* (1 - \frac{\Delta}{\Delta_i}))}{\partial \Delta} d\Delta \quad (5)$$

As  $T_j^*$  approaches zero, Eq. (5) becomes

$$\lim_{T_j^* \rightarrow 0} \tilde{R}(\Delta_i, \hat{t}, T_j^*) = \int_0^{\Delta_i} \frac{\partial R(\Delta, \hat{t})}{\partial \Delta} d\Delta = R(\Delta_i, \hat{t}) \quad (6)$$

Let  $\Delta_i, i = 1, 2, 3, \dots, n_\Delta$  be a set of axial displacements and let  $T_j^*, j = 1, 2, 3, \dots, n_T$ , be the set of rise times at which the ramp to constant displacement history described in Eq. (3) is carried out.

Consider a fixed value of displacement  $\Delta_i$  and let the rise time  $T_j^*$  decrease as  $j$  increases, that is,  $T_{j+1}^* < T_j^*$ . A set of such histories for  $T_j^*, (j = 1, 2, 3)$ , is shown in Fig. 2.

Because of experimental limitations, the limit  $T_j^* \rightarrow 0$  can not be reached. This difficulty is approached as follows. Let  $\tilde{R}_{exp}(\Delta_i, T_j^*, t)$  denote the experimentally determined force history

response at time  $t \geq T_j^*$ . When the slope of ramp to constant displacement history  $\Delta(\Delta_i, T_j^*, t)$  changes from positive to zero at the rise time  $T_j^*$ , the slope of the force-time response changes from positive to negative at time  $T_j^*$ . Let  $\hat{t}$  be a fixed time interval measured from this break in the slope at time  $T_j^*$ , that is,  $t = \hat{t} + T_j^*$ . For the rise times  $T_j^*, j=1, 2, 3, \dots, n_T^*$ ,  $\tilde{R}_{exp}(\Delta_i, T_j^*, T_j^* + \hat{t})$  denotes a set of data points associated with time  $\hat{t}$  and displacement  $\Delta_i$ .

Figure 3 shows plots of the experimentally determined force output  $\tilde{R}_{exp}(\Delta_i, T_j^*, t)$  versus  $t$ , with a fixed value at  $\hat{t}$  for several rise times  $T_j^*$ . Moreover, it shows a plot of  $\tilde{R}_{exp}(\Delta_i, T_j^*, T_j^* + \hat{t})$  versus  $T_j^*$  for three rise times  $T_j^*$ . Let  $\tilde{R}_{fit}(\Delta_i, \hat{t}, T^*)$  denote a function of  $T^*$ , which is fit to this set of data points, so that

$$\tilde{R}_{fit}(\Delta_i, \hat{t}, T_j^*) = \tilde{R}_{exp}(\Delta_i, T_j^*, T_j^* + \hat{t}), \quad (7)$$

$T_j^*, j=1, 2, 3, \dots, n_T^*$

$\tilde{R}_{fit}(\Delta_i, \hat{t}, T^*)$  is used to extrapolate the data for  $T_j^*, j=1, 2, 3, \dots, n_T^*$  to  $T_j^*=0$ . In particular, it is assumed that

$$\lim_{T^* \rightarrow 0} \tilde{R}_{fit}(\Delta_i, \hat{t}, T^*) = \tilde{R}_{fit}(\Delta_i, \hat{t}, 0) = \lim_{T_j^* \rightarrow 0} \tilde{R}_{exp}(\Delta_i, T_j^*, T_j^* + \hat{t}) = R(\Delta_i, \hat{t}) \quad (8)$$

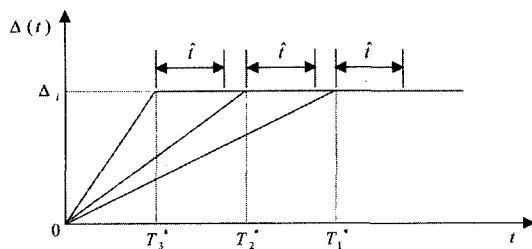


Fig. 2 Ramp to constant displacement history (input)

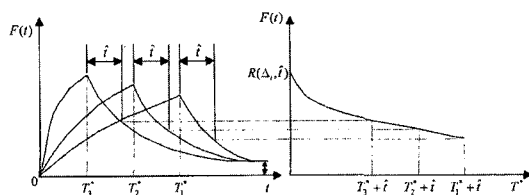


Fig. 3 Force output for ramp to constant displacement control tests

A choice for  $\tilde{R}_{fit}(\Delta_i, \hat{t}, T^*)$  was determined by considering the extrapolation. For a given displacement  $\Delta_i$  and a fixed time interval  $\hat{t}$ ,  $\tilde{R}_{fit}(\Delta_i, \hat{t}, T^*)$  is defined by the relation

$$\tilde{R}_{fit}(\Delta_i, \hat{t}, T^*) = C_1 + C_2 \left( \frac{1 - e^{-(T^*/\tau)}}{(T^*/\tau)} \right) \quad (9)$$

$C_1, C_2$ , and  $\tau$  are constants which are found by minimizing

$$\sum_{j=1}^{n_T^*} \left( \left( C_1 + C_2 \left( \frac{1 - e^{-(T_j^*/\tau)}}{(T_j^*/\tau)} \right) \right) - \tilde{R}_{exp}(\Delta_i, T_j^*, T_j^* + \hat{t}) \right)^2 \quad (10)$$

By substituting Eq. (9) into Eq. (8) and using the result  $\lim_{T^* \rightarrow 0} [1 - e^{-(T^*/\tau)} / (T^*/\tau)] = 1$ , it is found that

$$R(\Delta_i, \hat{t}) = C_1 + C_2 \quad (11)$$

The process indicated in Eqs. (10) and (11) can be carried out on any specified set of displacement  $\Delta_i$  and times  $\hat{t}$ . This process gives  $R(\Delta_i, \hat{t})$ , the force at time  $\hat{t}$  due to an axial displacement  $\Delta_i$ . By repeating this process for a range of displacement  $\Delta_i$  and time  $\hat{t}$ , the relaxation function can be determined as Fig. 1.

### 3. Experimental Results

In this paper, only axial mode deformations were considered. Reference and current configurations in axial mode are shown in Fig. 4.

Experiments were carried out on an MTS 858 (2.5ton) testing machine and bushings were provided by a commercial company. The bushing was fixed at its outer radius and the inner rod

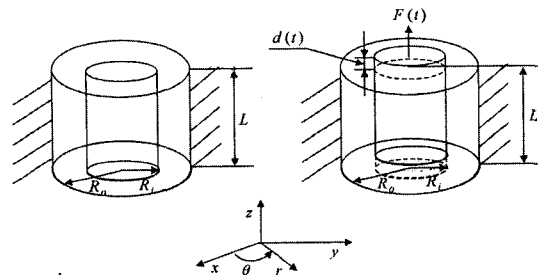


Fig. 4 Reference and current configurations in axial mode

**Table 1** Given displacement histories for experiment

$d_i$	$T_j^*$	$T_1^*$	$T_2^*$	$T_3^*$	$T_4^*$	$T_5^*$	$T_6^*$
$d_1=4$ mm		8 sec.	6 sec.	4 sec.	2 sec.	1 sec.	0.5 sec.
$d_2=5$ mm		8 sec.	6 sec.	4 sec.	2 sec.	1 sec.	0.5 sec.
$d_3=6$ mm		8 sec.	6 sec.	4 sec.	2 sec.	1 sec.	0.5 sec.
$d_4=7$ mm		8 sec.	6 sec.	4 sec.	2 sec.	1 sec.	0.5 sec.
$d_5=8$ mm		8 sec.	6 sec.	4 sec.	2 sec.	1 sec.	0.5 sec.

was subjected to a displacement  $d(t)$  as indicated in Fig. 4. In particular, the ramp to constant displacement histories were as follows :

$$d(t) = \frac{d_i}{T_j^*} t, \quad 0 \leq t \leq T_j^* \quad (12)$$

$$= d_i, \quad T_j^* \leq t \leq 120 \text{ sec.}$$

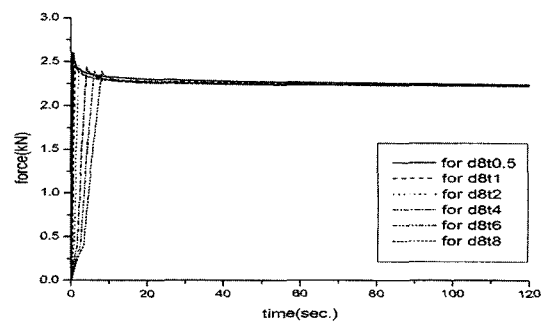
$$i=1, 2, 3, 4, 5, \quad j=1, 2, 3, 4, 5, 6$$

The set of rise times  $T_j^*$  and displacements  $d_i$  are shown in Table 1.

While factors such as aging, amplitude and frequency of loading, and temperature do affect conditioning and degradation of the material over time, they were not considered in this study. It can be considered in the future study.

The ideal type of bushing test is step displacement control test. However, due to inertia of the testing equipment, a true step displacement control test is not physically possible. Therefore, a ramp to constant displacement was used instead of a step displacement. The test controller was programmed to increase displacement at a constant rate during a period from time zero to a rise time  $T^*$  and hold it fixed at  $d_i$  after the rise time  $T^*$ . As the rise time  $T^*$  of the ramp for each test decreases, the ramp to constant displacement control test approaches a step displacement control test. Thirty ramp to constant displacement control tests in Table 1 were performed until  $t=120$  sec. A typical force output for  $d_i=8$  mm is shown in Fig. 5.

For each of the ramp to constant displacement control tests, the peak force occurs when the prescribed displacement changes from increasing to being held constant and the peak load is the greatest at the shortest rise time. The force relaxes to an equilibrium during the period in which the



**Fig. 5** Force output for  $d_i=8$  mm (from experiment)

displacement remains constant. This response is found in typical viscoelastic materials.

#### 4. Mathematical Representation of $R(d, t)$ for the Pipkin-Rogers Model

In this section, we consider a relation in which the force is expressed explicitly in terms of the displacement history, namely, the Pipkin-Rogers model presented in section 2. For present purposes, we use the form of the Pipkin-Rogers model in Eq. (2). Using the force extrapolation method described in section 2 for each of  $d_i=4, 5, 6, 7, 8$  mm, the constant values of the force relaxation functions for  $0 \leq t \leq 100$  sec. are obtained. The extrapolation process used was carried for  $0 \leq t \leq 60$  sec. because the experiments showed the force was constant for  $t \geq 60$  sec. They are shown in Fig. 6.

Note that the plots in Fig. 6 were produced by specifying data points and letting the computer graphics program produce a smooth curve. Because the axial force is odd in the axial displacement, the polynomial contains only odd powers

of displacement  $d$ . Thus,  $R(d, t)$  was represented as follows :

$$R(d, t) = dG_1(t) + d^3G_3(t) \quad (13)$$

Since  $G_1(t)$  and  $G_3(t)$  are the functions of time and the coefficients of  $d$  and  $d^3$  respectively, Eq. (13) shows that  $R(d, t)$  can be expressed explicitly by time and displacement.

Let  $d^{(\gamma)}$  be the value of a step displacement, i.e.,  $d^{(1)}=4$  mm,  $d^{(2)}=5$  mm,  $d^{(3)}=6$  mm,  $d^{(4)}=7$  mm,  $d^{(5)}=8$  mm. Then, the displacement-dependent force relaxation functions for  $d^{(\gamma)}$  represent a data set which can be used for finding  $G_1(t_a)$  and  $G_3(t_a)$ ,  $t_a=(a-1)/10$  sec.,  $a=1, 2, 3, \dots, 601$ . and shown in Fig. 6. Since time interval is defined as  $\Delta t=0.1$  sec., the number of data point is 601.

Then,

$$R(d^{(\gamma)}, t_a) = d^{(\gamma)}G_1(t_a) + (d^{(\gamma)})^3G_3(t_a) \quad (14)$$

where  $t_a=(a-1)/10$  sec.,  $a=1, 2, 3, \dots, 601$ ,  $\gamma=1, 2, 3, 4, 5$ .

Then,  $G_1(t_a)$  and  $G_3(t_a)$  were determined so as to minimize the least-squares error. The least-squares error is defined as follows :

$$E(t_a) = \sum_{\gamma=1}^5 (R(d^{(\gamma)}, t_a) - d^{(\gamma)}G_1(t_a) - (d^{(\gamma)})^3G_3(t_a))^2 \quad (15)$$

where  $t_a=(a-1)/10$  sec.,  $a=1, 2, 3, \dots, 601$ .

The coefficients,  $G_1(t_a)$  and  $G_3(t_a)$  are obtained at a set of times  $t_a$  for  $t_a=(a-1)/10$  sec.,

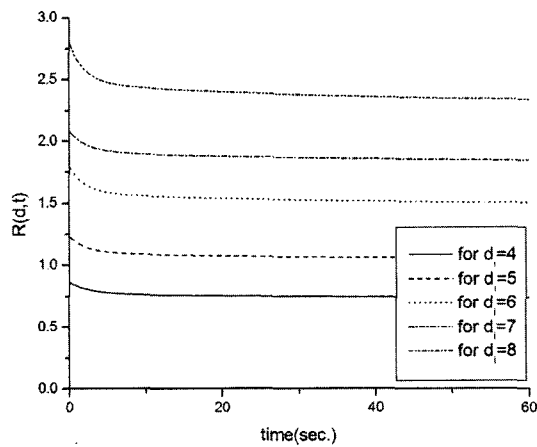


Fig. 6 Displacement-dependent force relaxation function for  $d_i=4, 5, 6, 7, 8$  mm

$a=1, 2, 3, \dots, 601$ .  $G_1(t_a)$  and  $G_3(t_a)$  are shown in Fig. 7.

Note that the plots were produced by specifying data points and letting the computer graphics program produce a smooth curve. These data can be represented by sums of exponential functions,

$$\tilde{G}_i(t) = C_{i1} + C_{i2}e^{-t/\tau_{i2}} + C_{i3}e^{-t/\tau_{i3}} \quad (16)$$

where  $i=1, 3$ . The parameters ( $C_{i\alpha}, \tau_{i\beta}, \alpha=1, 2, 3, \beta=2, 3$ ) were found by using the nonlinear least-squares method and are shown in Table 2.

Let the relative error of the fit of  $\tilde{G}_i(t)$  to the data represented by  $G_i(t)$  be defined by

$$E_i = \frac{\| \tilde{G}_i(t_a) - G_i(t_a) \|_2}{\| G_i(t_a) \|_2} \times 100(\%) = \frac{\sqrt{\sum_{a=1}^{601} (\tilde{G}_i(t_a) - G_i(t_a))^2}}{\sqrt{\sum_{a=1}^{601} (G_i(t_a))^2}} \times 100(\%) \quad (17)$$

where  $t_a=(a-1)/10$  sec.,  $a=1, 2, 3, \dots, 601$ ,  $i=1, 3$ .

The relative errors are  $E_1=0.01\%$ ,  $E_3=0.05\%$ .

Table 2 Parameters related to displacement-dependent force relaxation function

$i$	$C_{i1}$	$C_{i2}$	$C_{i3}$	$\tau_{i2}$	$\tau_{i3}$
1	1.6222	0.0586	0.1860	33.9885	2.8163
3	2.0179	0.1994	0.2814	38.3531	1.4039

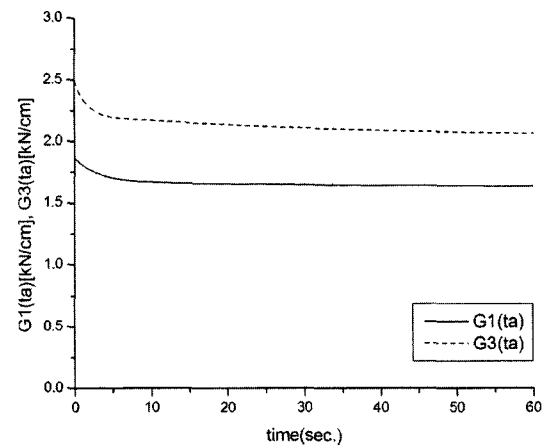


Fig. 7 Coefficients of  $R(d, t)$ :  $t_a=(a-1)/10$  sec.,  $a=1, 2, 3, \dots, 601$ .

Therefore, the data  $(G_i(t_a), i=1, 3, t_a=0, 0.1, 0.2, 0.3, \dots, 60 \text{ sec.})$  can be approximated by  $(\bar{G}_i(t), i=1, 3)$  in Eq. (16) with a relative error below 0.1%. Plots of the points given by the expression in Eq. (16) and the data set are nearly indistinguishable.

Finally, the complete form of the Pipkin-Rogers model for the axial mode obtained by combining Eqs. (2), (13), and (16) is as follows:

$$F(t) = R(d(0), t) + \int_0^t \left\{ \frac{\partial R(d(s), t-s)}{\partial d(s)} \frac{d}{ds}(d(s)) \right\} ds \quad (18)$$

where

$$R(d(s), t) = [C_{11} + C_{12}e^{-t/\tau_{12}} + C_{13}e^{-t/\tau_{13}}]d(s) + [C_{31} + C_{32}e^{-t/\tau_{32}} + C_{33}e^{-t/\tau_{33}}](d(s))^3 \quad (19)$$

where  $C_{i\alpha}, \tau_{i\beta}, i=1, 3, \alpha=1, 2, 3, \beta=2, 3$  are in Table 2.

### 5. Predictive Quality of the Pipkin-Rogers Model

The Pipkin-Rogers model in the previous section was introduced as an approximation to the real bushing model. The purpose of this section is to discuss the predictive capability of the Pipkin-Rogers model to the real model. Both the experimental results and the Pipkin-Rogers model were used to determine the force responses to specified displacement histories for  $0 \leq t \leq 60 \text{ sec.}$  Since both the Pipkin-Rogers outputs and experimental results are vectors, for evaluation purposes, the relative error  $E$  is defined by

$$E = \frac{\|( \text{the Pipkin and Rogers outputs} ) - ( \text{Experimental results} )\|_2}{\|( \text{Experimental results} )\|_2} \times 100(\%) \quad (20)$$

The particular displacement histories for the experiment and the Pipkin-Rogers model are

$$d(t) = \begin{cases} \frac{d_i}{T_j^*} t, & 0 \leq t \leq T_j^* \\ d_i, & T_j^* \leq t \leq 60 \text{ sec.} \end{cases} \quad (21)$$

$$\begin{pmatrix} d_i = 4, 5, 6, 7, 8 \text{ mm} \\ T_j^* = 0.5, 1, 2, 4, 6, 8 \text{ sec.} \end{pmatrix}$$

The force outputs from the Pipkin-Rogers model for  $d_i=8 \text{ mm}$  are shown in Fig. 8.

For comparison, the experimental results for  $0 \leq t \leq 60 \text{ sec.}$  at  $d_i=8 \text{ mm}$  shown in Fig. 5, are modified until  $t=60 \text{ sec.}$  and are shown again in Fig. 9.

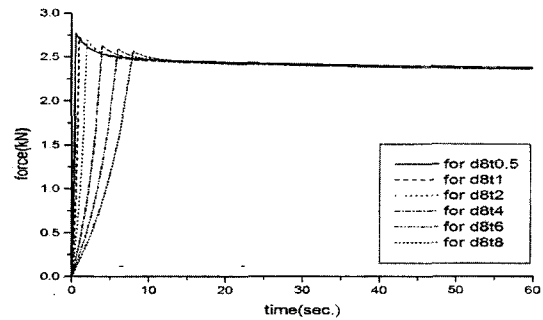


Fig. 8 Force output for  $d_i=8 \text{ mm}$  (from Pipkin-Rogers model)

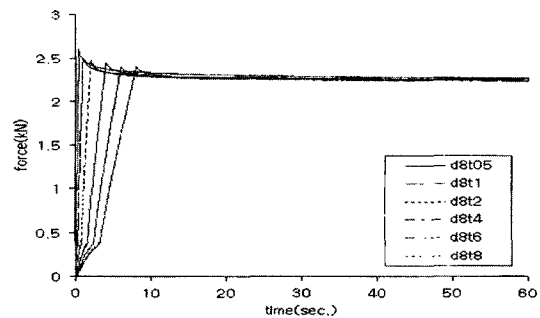


Fig. 9 Force output for  $d_i=8 \text{ mm}$  (from experiment)

Table 3 Relative errors of the Pipkin-Rogers model to the experimental results

time (sec.) displacement (mm)	$T^*=0.5$	$T^*=1$	$T^*=2$	$T^*=4$	$T^*=6$	$T^*=8$
$d_i=4$	9.59%	5.86%	6.9%	7.71%	9.01%	9.07%
$d_i=5$	4.07%	3.15%	3.66%	4.2%	5.02%	5.88%
$d_i=6$	1.54%	1.3%	1.67%	2.47%	3.31%	3.61%
$d_i=7$	4.04%	1.46%	1.5%	1.69%	1.99%	2.3%
$d_i=8$	5.99%	6.24%	6.57%	6.34%	6.89%	6.8%

For all rise times  $T^*$  and  $d_i=4, 5, 6, 7, 8$  mm, the comparison between Pipkin-Rogers outputs and experimental results for  $0 \leq t \leq 60$  sec. are carried out. And, the relative errors of the Pipkin-Rogers model to the experimental results are defined as Eq. (20) and are shown in Table 3.

For the thirty cases considered, the relative errors are below 10% and the Pipkin-Rogers model appears to provide satisfactory agreement with experimental results.

## 6. Summary and Conclusions

The force-displacement relation for axial mode response of a bushing has been studied experimentally. The proposed relation is explicit, but approximate, and is expressed in terms of a force relaxation property determined from experimental results. The force relaxation property in the explicit force-displacement relation was determined by a method which extrapolates results obtained from experiment. However, since this results can be used for the same bushing model in axial mode experiment was carried out, it is not possible to use all of bushing.

However, the success of the present study for axial mode response suggests that the same approach is applied to other bushings and other modes, such as torsional or radial modes. The development of an explicit force-displacement relation for coupled mode response should also be considered. These are treated in separate studies.

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