A COMPARATIVE EVALUATION OF THE ESTIMATORS OF THE 2-PARAMETER GENERALIZED PARETO DISTRIBUTION

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Abstract: Parameters and quantiles of the 2-parameter generalized Pareto distribution were estimated using the methods of regular moments, modified moments, probability weighted moments, linear moments, maximum likelihood, and entropy for Monte Carlo-generated samples. The performance of these seven estimators was statistically compared, with the objective of identifying the most robust estimator. It was found that in general the methods of probability-weighted moments and L-moments performed better than the methods of maximum likelihood estimation, moments and entropy, especially for smaller values of the coefficient of variation and probability of exceedance.

Keywords: Computer simulation, entropy, L-moments, maximum likelihood estimation, probability-weighted moments, regular moments.

1. INTRODUCTION

The generalized Pareto distribution (GPD), first proposed by Pareto in 1897 for the shape parameter < 0, is known as the Pareto type II distribution as well as the Pearson type VI distribution. Since Pickands (1975) introduced the term generalized Pareto (GP) distribution, it has been applied to a number of areas, encompassing socio-economic processes, physical and biological phenomena (Saksena and Johnson, 1984), reliability analysis, and analyses of environmental extremes. Davison and Smith (1990) pointed out that the GP distribution might form the basis of a broad modeling approach to high-level exceedances. DuMouchel (1983) applied it to estimate the tail thickness, whereas Davison (1984a, 1984b) modeled contamination due to long-range atmospheric transport of radionuclides. Ochoa et al. (1980) found the Pareto distributions to be more suitable for annual peak flow data than the exponential-tailed distributions which are common to hydrologic frequency analysis. Van Montfort and Witter (1985, 1986, 1991) applied the GP distribution to model the peaks over threshold (POT) streamflows and rainfall series, and Smith (1984, 1987) applied it to analyze flood frequencies.

Similarly, Joe (1987) employed it to estimate quantiles of the maximum of N observations. Wang (1991) applied it to develop a POT model for flood peaks with Poisson arrival time, whereas Rosbjerg, et al. (1992) compared the use of the 2-parameter GP and exponential distributions as distribution models for exceedances with the parent distribution being a generalized GP distribution. In an extreme value analysis of the flow of Burbage Brook, Barrett (1992) used the GP distribution to model the POT flood series with Poisson interarrival times. Davison and Smith (1990) presented a comprehensive analysis of the extremes of data by use of the GP distribution for modeling the sizes and occurrences of exceedances over high thresholds.

The cumulative distribution function (CDF) of the 2-parameter GPD (GPD2) of a random variable X can be expressed as

$$F(x) = 1 - (1 - a\frac{x}{b})^{\frac{1}{a}}, a \neq 0 \quad (1)$$

$$F(x) = 1 - \exp(-\frac{x}{b}), a = 0 \quad (2)$$

The probability density function (PDF) of the GPD2 can be written as

$$f(x) = \frac{1}{b}(1 - a\frac{x}{b})^{\frac{1}{a}-1}, a \neq 0 \quad (3)$$

$$f(x) = \frac{1}{b}\exp(-\frac{x}{b}), a = 0 \quad (4)$$

where a is the scale parameter with $-\infty < a < \infty$, b is the shape parameter with $b > 0$, and the range of x is $0 \leq x \leq \infty$ for $a \leq 0$ and $0 \leq x \leq b/a$ for a > 0.

The 2-parameter GPD specializes into uniform, triangular, exponential and Pareto distributions as special cases. For $a > 0$, the distribution has a heavy Pareto-type upper tail. The case $a = 0$ is the exponential distribution. When a > 0 the distribution has an upper end point at b/a. For $a = 1/2$ and 1 the distribution is triangle and uniform respectively. The shapes of the GPD2 distribution for various values of a and b are illustrated in ures 1a-d.

Methods for estimating the GPD2 parameters, including the regular method of moments (RME), probability-weighted moments (PWM), and the method of maximum likelihood estimation (MLE), were reviewed by Hosking and Wallis (1987). Quandt (1966) used the method of moments (RME), whereas Baxter (1980), and Cook and Mumme (1981) used the method of maximum likelihood estimation (MLE). Van Montfort and Witter (1986) used MLE to fit the GP distribution to represent the Dutch POT rainfall series, and used an empirical correction formula to reduce bias of the scale and shape parameter estimates. Davison and Smith (1990) used MLE, PWM, a graphical method, and the least squares method to estimate the GP distribution parameters. Singh and Guo (1995 a, b, 1997) presented the entropy (ENT)-based parameter estimation method for 2-parameter, 2-parameter generalized Pareto and 3-parameter generalized Pareto distributions. Singh (1998) summarized entropy-based parameter estimation for the Pareto family. Moharram et al. (1993) compared MLE, RME, PWM and least squares method, using Monte Carlo simulation. They found the least squares method to be superior to other methods for shape parameter greater than zero and PWM for shape parameter less than zero.

The maximum likelihood estimators exist in large samples provided that $a < 1$ and are as-

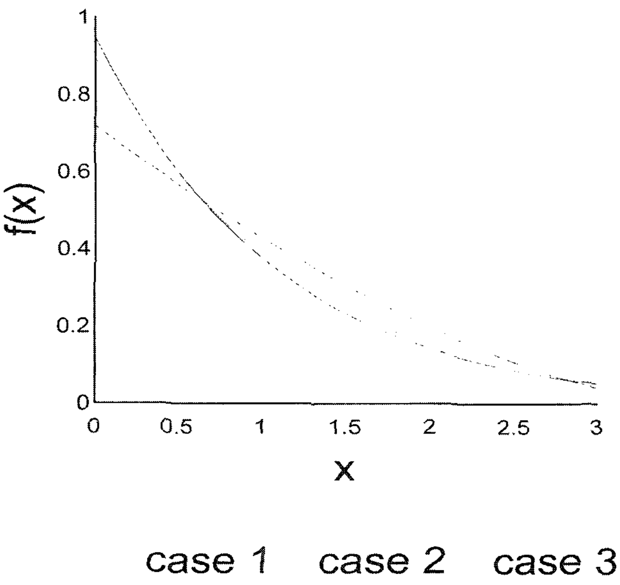

Figure 1a. Probability density function of the three parameter generalized Pareto distribution for population cases(1, 2 and 3).

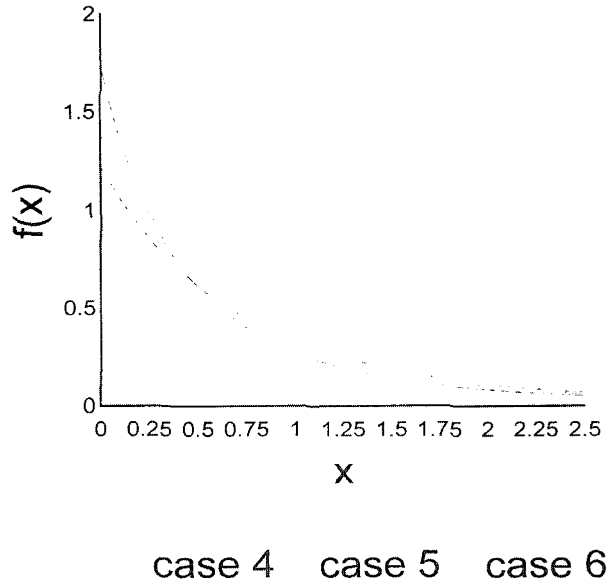

Figure 1b. Probability density function of the three parameter generalized Pareto distribution for population cases(4, 5 and 6).

ymptotically normal and efficient if $a < 1/2$ (Smith, 1985). The variance of the distribution exists only for $a > -1/2$ and skewness for $a > -1/3$. The main conclusions reached by Hosking and Wallis (1987) were that MLE, although asymptotically the most efficient method, does not clearly display its efficiency even in samples as large as 500, that the method of moments is generally reliable except when $a < -0.2$ and that PWM estimation may be recommended if it seems likely that $a < 0$, particularly if it is important that the estimated extreme quantiles should have low bias or that the asymptotic theory should give a good approximation to the standard errors of the estimates.

Although, it would appear in the presence of the above cited literature and particularly in light of Hosking and Wallis (1987) that a new study on parameter and quantile estimation of the 2-parameter generalized Pareto (GPD2) distribution was not warranted. However, a closer scrutiny suggests otherwise. For a given sample data set, the ranges of the distribution parameters are not known in advance. Rather what is known is its mean, variance and skewness, and other moment properties. Therefore, It would be more logical to classify the best estimators in terms of these readily derivable data characteristics than to classify them in terms of a priori unknown parameters. While selecting the range of these characteristics (coefficient of variation in this case), all estimators should satisfy the familiar properties of consistency, asymptotic normality and efficiency. This approach was followed in this study. For theoretical and practical reasons, this study was restricted within the $-1/2 < a < 1/2$ range.

The objective of this paper is to statistically compare the performance of five estimators and variants of two of them, including the methods of regular moments, two modified moments, probability weighted moments, L-moments, maximum likelihood estimation and entropy, using Monte Carlo simulated data, and statistically evaluate the performance of these estimators. These methods were evaluated based on their relative performance in terms of robustness, variability and bias of large quantile estimates from varying sample sizes.

2. PARAMETER ESTIMATORS

2.1 Regular Moment Estimator (RME)

The moment estimation equations of GPD2 were derived by Hosking and Wallis (1987). Note that $E\,(1-a(x-c)/b)^r = 1/(1+ar)$ if $1+ra>0$. The rth moment of X exists if $a>-1/r$. Provided that they exist, the moment estimators are

$$\bar{x} = \frac{b}{1+a} \quad (5)$$

$$S^2 = \frac{b}{(1+a)^2(1+2a)} \quad (6)$$

where $\bar{x}$ and s^2 are the mean and variance, respectively. Equations (5) and (6) yield explicit relations for estimation of a and b as

$$a = \frac{1}{2}\left(\frac{\bar{x}^2}{S^2} - 1\right) \quad (7)$$

$$b = \frac{1}{2}\bar{x}\left(\frac{\bar{x}^2}{S^2} + 1\right) \quad (8)$$

2.2 Modified Moment Estimator (MME)

The regular moment estimator is valid only when $a>-2$. This somehow limits the practical application. Qaundt(1966) suggested two modified versions of the RME which involve restructuring equation (3) in terms some known property of the data. These two modified methods of moments are briefly outlined below.

2.2.1 Modified Moment Estimator (MME1)

In this modification, equation (4) is replaced by $E[F(x_1)]=F(x_1)$, which yields

$$1 - a\frac{x_1}{b} = \left(\frac{n}{n+1}\right)^a \tag{9}$$

By substituting the value of b from equation (5), one gets

$$\frac{x_1}{\bar{x}} = \frac{1+a}{a}\left[1 - \left(\frac{n}{n+1}\right)^n\right] \tag{10}$$

2.2.2 Modified Moment Estimator (MME2)

In this modification , equation (4) is replaced by $E[x_1]=x_1$. CDF of x_1 is given by

$$Fx_1(x) = 1 - [1 - F(x)]^n = 1 - \left[1 - a\frac{x}{b}\right]^{\frac{n}{a}} \tag{11}$$

and its PDF is

$$f_{x_1}(x) = \frac{n}{b}\left(1 - a\frac{x}{b}\right)^{\frac{n}{a} - 1} \tag{12}$$

Thus,

$$E(x_1) = x_1 = \frac{b}{n+a} \tag{13}$$

Eliminating b using equation (5), one gets

$$\frac{x_1}{\bar{x}} = \frac{1+a}{n+a} \tag{14}$$

2.3 Probability Weighted Moment Estimator

The probability-weighted moments (PWM) estimation equations of GPD2 were given by Hosking and Wallis (1987) as

$$a = \frac{W_0 - 4W1}{2W1 - W_0} \tag{15}$$

$$b = -\frac{2W_0W1}{2W1 - W_0} \tag{16}$$

where $W_r = M_{10r} = E\{x(F)[1-F(x)]^r\}$

2.4 Method of L-Moments (MLM)

The method of L-moments is a modification of the method of probability weighted moments developed by Greenwood et al (1979). Following Hosking and Wallis (1997) L-moments of GPG2 can be described as (a>-1):

$$\lambda_1 = W_0 \tag{17}$$

$$\lambda_2 = W_0 - 2W_1 \tag{18}$$

The shape parameter (a) and the scale parameter (b) can be defined as

$$a = \frac{\lambda_1}{\lambda_2} - 2 \tag{19}$$

$$b = (1 + a)\,\lambda_1 \tag{20}$$

Substituting the values of the L-moments, one gets

$$a = \frac{W_0 - 4W_1}{2W_1 - W_0} \tag{21}$$

$$b = -\frac{2W_0W1}{2W1 - W_0} \tag{22}$$

Equations (21) and (22) are same as equations (15) and (16) of PWM.

2.5 Maximum Likelihood Estimator

The log-likelihood function for GPD2 given by equation (3) can be written as

$$L = -n \ln b + \left(\frac{1}{a} - 1\right) \sum_{i=1}^{n} \ln\left(1 - a\frac{x_i}{b}\right) \tag{23}$$

Differentiating the maximum likelihood function L with respect to a and b, two parameter equations are obtained for estimation as:

$$\sum_{i=1}^{N} \left[\frac{x_i/b}{1 - a x_i/b}\right] = \frac{n}{1-a} \tag{24}$$

$$\sum_{i=1}^{N} \left[\ln\left(1 - a\frac{x_i}{b}\right)\right] = -na \tag{25}$$

Equations (24) and (25) are solved iteratively. To find the local maximum of log L, a numerical method is required. Due to the very sensitive nature of the log function of the Pareto distribution, the rate of failure given by equation (23) is quite high. As seen from the graphs in Figures 2-a and b and 3-a and b, the local maximum, even if it exists, is sometimes prone to be skipped over, i. E., the solution of equations (24) and (25) is not found. This perhaps was the problem encountered by Hosking and Wallis (1987) when the solution was not found in too many of their cases. In this study, a little laborious but more reliable approach involving two steps was used. First, a feasible range for a wide ranging set of parameters of a and b was marked. In the second stage, solution set was refined using the bisection method.

2.6 Entropy Estimator

The entropy estimation equations of GPD2 are given as

$$E\left[\ln\left(1 - a\frac{x_i}{b}\right)\right] = -a \tag{26}$$

$$E\left[\frac{x_i/b}{1 - a x_i/b}\right] = \frac{1}{1-a} \tag{27}$$

For a finite data set equations (26) and (27) are similar to equations (24) and (25) respectively.

3. EXPERIMENTAL DESIGN

3.1 Monte Carlo Simulations

The inverse of equation (1) is

$$x(F) = \left(\frac{b}{a}\right)(1 - F^{a}),\ a \neq 0 \tag{28}$$

and that of equation (2) is

$$x = -b\ln(1 - F), a = 0 \tag{29}$$

where x (F) denotes the quantile of the cumulative probability P or 1 - F(x).

To assess the performance of the parameter estimation methods outlined above, Monte Carlo sampling experiments were conducted. Eight GPD2 population cases, listed in Table 1, were considered for eight values of the coefficient of variation (CV). These CV values are not exhaustive but do span the range normally encountered in hydrology and environmental and water resources. For each population case, 1000 random samples of size 10, 20, 50, 100, 200, and 500 were generated, and then parameters and quantiles were estimated.

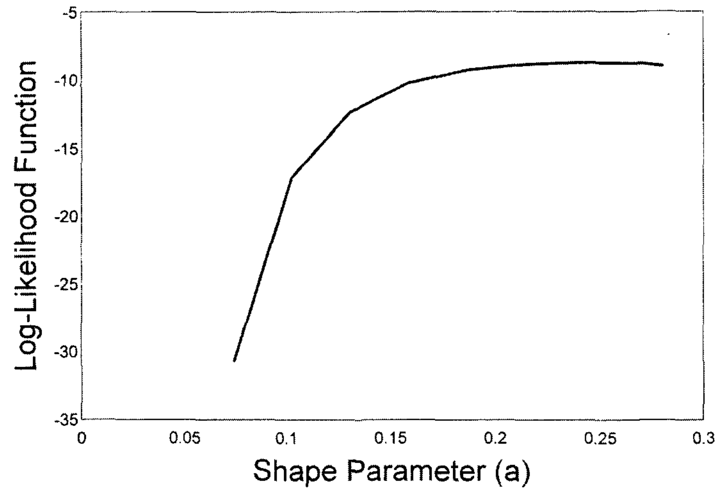

Figure 2a. Variation of log-likelihood function with parameter a: a > 0.

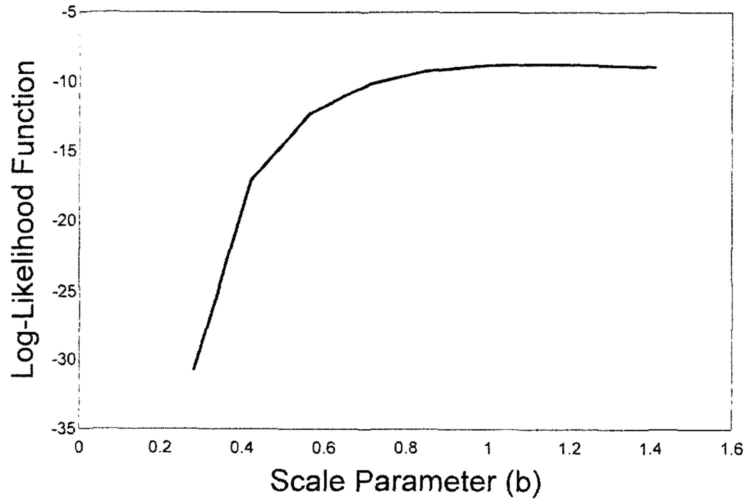

Figure 2b. Variation of log-likelihood function with parameter b: a > 0.

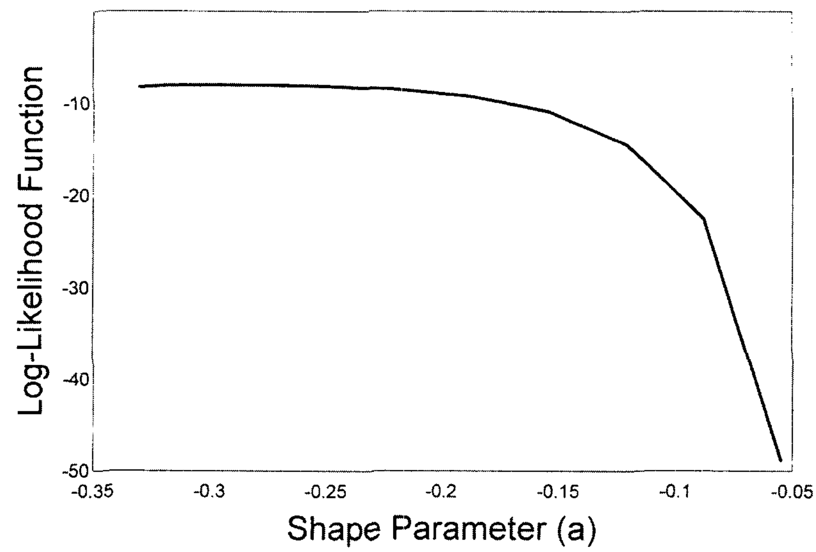

Figure 3a. Variation of log-likelihood function with parameter a: a < 0.

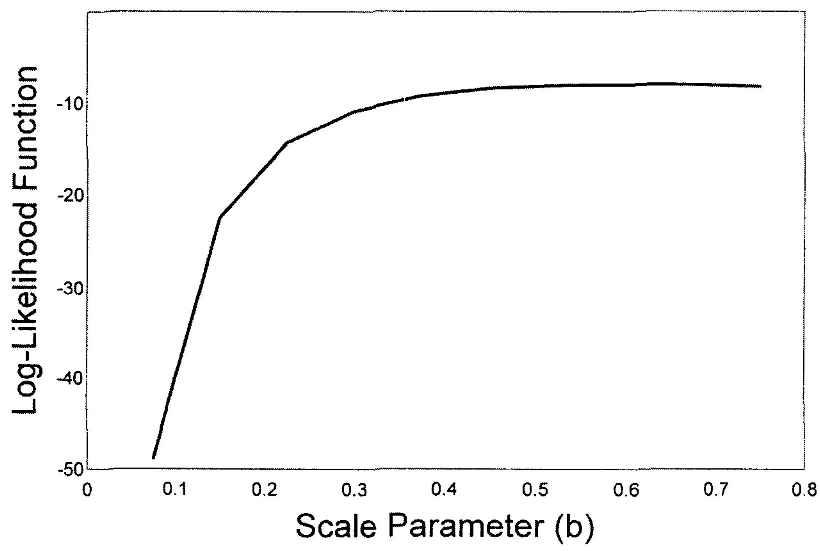

Figure 3b. Variation of log-likelihood function with parameter b: a < 0.

Table 1 GPD2 Population Cases Considered in Sampling Experiment(μ=1)

GPD2 Pupulation	CV	Parameter	
		a	b
case 1	0.75	0.389	1.389
case 2	0.80	0.281	1.281
case 3	0.90	0.117	1.117
case 4	0.95	0.054	1.054
case 5	1.25	-0.180	0.820
case 6	1.50	-0.278	0.722
case 7	2.00	-0.375	0.625
case 8	2.50	-0.420	0.580

3.2 Performance Indices

The performance of parameter estimators was evaluated by using the following performance indices:

Standardized bias,

$$BIAS = \frac{E(\hat{x}) - x}{x} \qquad (30)$$

Standard Error,

$$SE = \frac{S(\hat{x})}{x} \qquad (31)$$

Root Mean Square Error,

$$RMSE = \frac{E\left[(\hat{x} - x)^2\right]^{0.5}}{x} \qquad (32)$$

where $\hat{x}$ is an estimate of x (parameter or quantile), $E[X]$ denotes statistical expectation, and $S(X)$ denotes standard deviation of the respective random variable. $E(\hat{x})$ and $S(\hat{x})$ were calculated as

$$E(\hat{x}) = \frac{1}{N}\sum_{i=1}^{N} \hat{x}_i \qquad (33)$$

$$S(\hat{x}) = \left\{\frac{1}{N-1}\sum_{i=1}^{N}\left[\hat{x}_1 - E(\hat{x})\right]^2\right\}^{0.5} \qquad (34)$$

where the summations are over N estimates of x and N is the number of random samples used in estimation (N = 1000 here). The RMSE can also be expressed as

$$RMSE = \left[\frac{N-1}{N}SE + BIAS^2\right]^{0.5} \qquad (35)$$

These indices were used to measure the variability of parameter and quantile estimates for each simulation. Although they were used to determine the overall "best" parameter estimation method, our particular interest lies in the bias and variability of estimates of quantiles in the extreme tails of the distribution (P = 0.99, 0.999) when the estimates are based on small samples ($N \leq 50$).

Due to the limited number of random number of samples used, the results are not expected to reproduce the true values of BIAS, SE and RMSE, but they do provide a means of comparing the performance of estimation methods used.

3.3 Robustness

The primary objective of this study is to identify a robust estimator for the GPD2. Kuczera (1982a, 1982b) defined a robust estimator as the one that is resistant and efficient over a wide range of population fluctuations. If an estimator performs steadily without undue deterioration in RMSE and BIAS, then it can be expected to perform better than other competing estimators under population conditions different from those on which conclusions were based. Two criteria for identifying a resistant estimator are mini-max and minimum average RMSE (Kuczera, 1982b). Based on the minimum criterion, the preferred estimator is the one whose maximum RMSE for all population cases is minimum. The minimum average criterion is to select the estimator whose RMSE average over the test cases is minimum.

4. RESULTS AND DISCUSSION

The performance of a parameter estimator depends on (1) sample size, *N*, (2) population coefficient of variance, CV, (3) distribution parameter, and (4) the probability of exceedance or the value of the variable exceeded, called quantile. The sample sizes included were 10, 20, 50, 100, 200, and 500; and the coefficients of variation (CV) for each sample size were 0.75, 0.8, 0.9, 0.95, 1.25, 1.5, 2.0, and 2.5. These CV values cover practically most variables of interest in hydrology and environmental and water resources. Most observed data record lengths are within the ranges of sample sizes considered. The probabilities of non-exceedance considered were 0.8, 0.9, 0.95, 0.99, and 0.999. The results of the GPD2 parameter estimation are summarized in Tables 2-5. It should be remarked that the results of the two modifications of the method of regular moments (RME) presented earlier were inferior to those of RME, so for economy of space they were not included in the final comparison. Also, MLE and ENT produced identical results and were therefore clubbed together. Similarly, PWM and MLM were identical in their performance and clubbed together for purposes of comparison.

4.1 BIAS in Parameter Estimates

The results of parameter bias are summarized in Table 2. PWM and MLM performed in a superior manner for both parameters *a* and *b* for sample sizes and all values of CV. RME produced the highest bias in parameter a as well as for CV $\geq$ 0.95, followed by MLE and ENT, regardless of the sample size, but the reverse was true for CV $\leq$ 0.95 and N $\geq$ 50. As the sample size increased beyond 100, the bias in parameters a and b produced by RME, ENT and MLE was not greatly more than that of PWM and MLM if CV was less than 0.95. For CV $\geq$ 0.95 and N $\geq$ 200, MLE, ENT, MLM and PWM became comparable. Thus, in terms of bias, it can be concluded that PWM and MLM are the preferred method. For N $\geq$ 100, any method would be acceptable for CV $\leq$ 0.95, but for CV $\geq$ 2.0, the sample size would have to be larger than 500 for RME and greater than 200 for ENT and MLE.

4.2 RMSE in Parameter Estimates

The results of RMSE in parameters are summarized in Table 3. In general, the RSME of *a* method varied with the parameter to be estimated, the sample size and the coefficient of variation.

In terms of RMSE of parameter *a*, no method performed uniformly better. If CV $\leq$ 0.95 and *N* $\geq$ 20, RME produced the least RMSE. As the value of CV increased, PWM and MLM per-

Table 2

SAMPLE SIZE	METHOD	Bias in Shape Parameter(a)							
	CV--->	(.75)	(.80)	(.90)	(.95)	(1.25)	(1.5)	(2.0)	(2.5)
10	RME	.393	.557	1.567	3.563	1.481	1.122	.944	.925
	PWM & MLM	.276	.423	1.179	2.681	1.045	.780	.636	.639
	MLE & ENT	.363	.544	1.456	3.239	1.226	.903	.749	.750
20	RME	.162	.247	.770	1.799	.898	.724	.646	.649
	PWM & MLM	.130	.206	.609	1.344	.563	.431	.360	.368
	MLE & ENT	.171	.265	.752	1.624	.660	.499	.424	.432
50	RME	.057	.103	.292	.734	.477	.414	.415	.426
	PWM & MLM	.048	.095	.234	.551	.248	.186	.171	.170
	MLE & ENT	.063	.122	.289	.666	.291	.215	.201	.199
100	RME	.030	.042	.151	.370	.282	.282	.303	.322
	PWM & MLM	.028	.037	.112	.268	.119	.099	.093	.094
	MLE & ENT	.037	.047	.139	.323	.140	.115	.110	.111
200	RME	.015	.020	.075	.191	.168	.187	.228	.247
	PWM & MLM	.014	.016	.054	.140	.059	.048	.052	.051
	MLE & ENT	.018	.021	.067	.169	.069	.056	.061	.060
500	RME	.009	.010	.023	.078	.086	.111	.159	.184
	PWM & MLM	.009	.009	.012	.053	.026	.024	.023	.026
	MLE & ENT	.012	.012	.015	.064	.030	.027	.028	.030
		Bias in Scale Parameter(b)							
10	RME	.142	.145	.174	.191	.285	.366	.485	.553
	PWM & MLM	.108	.116	.135	.147	.181	.213	.256	.286
	MLE & ENT	.142	.149	.166	.177	.212	.247	.301	.336
20	RME	.062	.068	.090	.097	.179	.240	.338	.391
	PWM & MLM	.054	.060	.073	.074	.100	.113	.135	.148
	MLE & ENT	.071	.077	.091	.090	.117	.131	.159	.174
50	RME	.021	.029	.034	.040	.094	.145	.218	.274
	PWM & MLM	.019	.027	.028	.031	.041	.050	.056	.065
	MLE & ENT	.025	.035	.034	.037	.049	.058	.066	.076
100	RME	.011	.011	.018	.019	.058	.099	.168	.213
	PWM & MLM	.011	.010	.014	.014	.021	.026	.033	.036
	MLE & ENT	.014	.013	.017	.017	.025	.030	.039	.042
200	RME	.006	.005	.008	.011	.033	.065	.126	.165
	PWM & MLM	.006	.004	.006	.008	.009	.011	.016	.017
	MLE & ENT	.008	.005	.007	.010	.011	.013	.019	.020
500	RME	.004	.002	.004	.004	.018	.041	.090	.126
	PWM & MLM	.004	.002	.002	.003	.005	.007	.007	.009
	MLE & ENT	.005	.003	.003	.003	.006	.008	.009	.010

Table 3

SAMPLE SIZE	METHOD				RMSE in Shape Parameter(a)				
	CV--->	(.75)	(.80)	(.90)	(.95)	(1.25)	(1.5)	(2.0)	(2.5)
10	RME	1.481	1.812	4.131	8.393	2.462	1.676	1.253	1.182
	PWM & MLM	1.349	1.757	4.060	8.439	2.490	1.654	1.201	1.124
	MLE & ENT	1.589	2.032	4.561	9.328	2.695	1.774	1.303	1.218
20	RME	.818	1.041	2.318	4.813	1.513	1.043	.829	.799
	PWM & MLM	.878	1.153	2.614	5.422	1.632	1.067	.793	.739
	MLE & ENT	1.034	1.333	2.936	5.994	1.766	1.144	.861	.800
50	RME	.467	.580	1.272	2.738	.900	.638	.537	.518
	PWM & MLM	.533	.683	1.520	3.231	.971	.648	.491	.451
	MLE & ENT	.627	.790	1.707	3.571	1.051	.695	.533	.488
100	RME	.319	.395	.868	1.861	.637	.465	.402	.394
	PWM & MLM	.370	.474	1.058	2.233	.672	.450	.357	.324
	MLE & ENT	.436	.548	1.189	2.468	.727	.483	.387	.351
200	RME	.220	.275	.597	1.288	.468	.349	.311	.307
	PWM & MLM	.258	.335	.734	1.550	.472	.324	.262	.240
	MLE & ENT	.304	.387	.825	1.714	.511	.347	.284	.260
500	RME	.138	.172	.370	.808	.323	.248	.230	.232
	PWM & MLM	.163	.211	.458	.975	.295	.206	.174	.163
	MLE & ENT	.192	.244	.514	1.078	.319	.221	.188	.177
					RMSE in Scale Parameter(b)				
10	RME	.617	.594	.612	.594	.652	.721	.881	1.032
	PWM & MLM	.564	.570	.593	.592	.626	.648	.689	.712
	MLE & ENT	.664	.659	.666	.654	.678	.695	.747	.772
20	RME	.366	.356	.366	.358	.398	.452	.564	.655
	PWM & MLM	.373	.372	.388	.380	.397	.409	.424	.438
	MLE & ENT	.440	.430	.436	.420	.430	.439	.460	.474
50	RME	.212	.208	.205	.208	.232	.271	.335	.409
	PWM & MLM	.224	.224	.223	.226	.233	.244	.244	.253
	MLE & ENT	.264	.259	.250	.250	.252	.262	.265	.274
100	RME	.146	.143	.143	.144	.161	.184	.244	.292
	PWM & MLM	.156	.155	.157	.158	.162	.162	.170	.172
	MLE & ENT	.184	.179	.176	.175	.175	.174	.185	.187
200	RME	.101	.100	.098	.100	.114	.131	.176	.215
	PWM & MLM	.109	.109	.108	.109	.112	.115	.117	.122
	MLE & ENT	.128	.127	.122	.120	.122	.123	.127	.132
500	RME	.064	.063	.062	.062	.075	.090	.123	.154
	PWM & MLM	.069	.069	.068	.068	.070	.072	.076	.077
	MLE & ENT	.081	.079	.077	.075	.076	.077	.082	.083

Table 4

Bias in quantiles

PROBABILITY OF NON-EXCEEDANCE = .800

n	METHOD CV-->	(.75)	(.80)	(.90)	(.95)	(1.25)	(1.5)	(2.0)	(2.5)
10	RME	-.028	-.026	-.017	-.006	.026	.060	.120	.145
	PWM & MLM	-.031	-.032	-.030	-.021	-.017	-.009	.012	.013
	MLE & ENT	-.217	-.193	-.190	-.168	-.099	-.065	-.005	.051
20	RME	-.013	-.010	-.004	-.001	.027	.050	.099	.116
	PWM & MLM	-.016	-.015	-.012	-.010	-.006	-.007	.001	-.003
	MLE & ENT	-.116	-.113	-.099	-.094	-.093	-.065	-.038	.022
50	RME	-.007	-.003	-.001	.001	.016	.038	.069	.098
	PWM & MLM	-.008	-.005	-.004	-.004	-.004	-.001	-.005	-.004
	MLE & ENT	-.046	-.036	-.043	-.042	-.039	-.052	-.028	-.022
100	RME	-.003	-.003	.000	-.001	.012	.027	.061	.081
	PWM & MLM	-.004	-.004	-.002	-.003	-.001	-.001	.000	-.002
	MLE & ENT	-.020	-.023	-.024	-.022	-.027	-.026	-.032	-.017
200	RME	-.001	-.002	-.001	.001	.006	.018	.045	.065
	PWM & MLM	-.001	-.003	-.002	.000	-.002	-.002	-.002	-.004
	MLE & ENT	-.011	-.012	-.011	-.011	-.014	-.018	-.019	-.013
500	RME	.000	-.001	.001	.000	.004	.012	.034	.051
	PWM & MLM	.000	-.001	.000	-.001	.000	.000	-.001	-.002
	MLE & ENT	-.003	-.004	-.008	-.004	-.005	-.005	-.007	-.006

PROBABILITY OF NON-EXCEEDANCE = .900

n	METHOD CV-->	(.75)	(.80)	(.90)	(.95)	(1.25)	(1.5)	(2.0)	(2.5)
10	RME	-.044	-.050	-.054	-.048	-.046	-.031	.004	.014
	PWM & MLM	-.038	-.046	-.052	-.047	-.059	-.060	-.050	-.059
	MLE & ENT	.233	.206	.200	.177	.103	.066	.005	-.052
20	RME	-.023	-.023	-.026	-.026	-.022	-.016	.012	.016
	PWM & MLM	-.021	-.023	-.026	-.025	-.031	-.038	-.037	-.048
	MLE & ENT	.125	.121	.104	.098	.097	.067	.039	-.023
50	RME	-.010	-.010	-.010	-.011	-.013	-.002	.010	.027
	PWM & MLM	-.010	-.010	-.010	-.011	-.016	-.016	-.024	-.025
	MLE & ENT	.049	.039	.045	.044	.041	.053	.028	.023
100	RME	-.005	-.005	-.005	-.006	-.005	-.001	.016	.025
	PWM & MLM	-.005	-.005	-.005	-.006	-.006	-.009	-.010	-.015
	MLE & ENT	.022	.024	.025	.023	.028	.027	.033	.018
200	RME	-.002	-.003	-.003	-.002	-.004	-.001	.011	.021
	PWM & MLM	-.002	-.003	-.003	-.002	-.004	-.006	-.008	-.010
	MLE & ENT	.012	.013	.012	.012	.014	.018	.019	.014
500	RME	-.001	-.001	.000	-.001	-.002	.000	.009	.018
	PWM & MLM	-.001	-.002	.000	-.001	-.001	-.002	-.004	-.005
	MLE & ENT	.003	.004	.008	.005	.005	.005	.007	.006

PROBABILITY OF NON-EXCEEDANCE = .950

n	METHOD CV-->	(.75)	(.80)	(.90)	(.95)	(1.25)	(1.5)	(2.0)	(2.5)
10	RME	-.046	-.059	-.076	-.076	-.104	-.107	-.095	-.098
	PWM & MLM	-.029	-.044	-.058	-.056	-.083	-.092	-.093	-.109
	MLE & ENT	-.250	-.219	-.211	-.186	-.107	-.068	-.005	.053
20	RME	-.024	-.029	-.040	-.043	-.064	-.074	-.066	-.075
	PWM & MLM	-.016	-.021	-.030	-.031	-.045	-.058	-.063	-.081
	MLE & ENT	-.134	-.129	-.110	-.103	-.100	-.069	-.040	.023
50	RME	-.011	-.013	-.016	-.019	-.038	-.040	-.046	-.040
	PWM & MLM	-.007	-.010	-.012	-.014	-.023	-.025	-.037	-.041
	MLE & ENT	-.053	-.041	-.048	-.046	-.042	-.055	-.029	-.023
100	RME	-.006	-.007	-.009	-.011	-.021	-.028	-.028	-.029
	PWM & MLM	-.004	-.005	-.006	-.008	-.010	-.014	-.017	-.024
	MLE & ENT	-.023	-.026	-.027	-.024	-.029	-.027	-.034	-.018
200	RME	-.002	-.004	-.005	-.004	-.014	-.020	-.023	-.022
	PWM & MLM	-.001	-.003	-.004	-.003	-.006	-.008	-.012	-.015
	MLE & ENT	-.013	-.013	-.013	-.013	-.015	-.019	-.020	-.014
500	RME	-.001	-.002	-.001	-.002	-.007	-.011	-.015	-.015
	PWM & MLM	-.001	-.002	.000	-.002	-.002	-.003	-.005	-.007
	MLE & ENT	-.003	-.004	-.009	-.005	-.005	-.005	-.008	-.006

Table 4(Conted.)

PROBABILITY OF NON-EXCEEDANCE = .990

n	METHOD CV-->	(.75)	(.80)	(.90)	(.95)	(1.25)	(1.5)	(2.0)	(2.5)
10	RME	-.019	-.047	-.090	-.104	-.201	-.242	-.277	-.303
	PWM & MLM	.033	.008	-.016	-.024	-.077	-.102	-.124	-.158
	MLE & ENT	.268	.234	.223	.195	.111	.071	.006	-.054
20	RME	-.009	-.021	-.051	-.063	-.137	-.184	-.218	-.252
	PWM & MLM	.020	.010	-.008	-.013	-.038	-.062	-.079	-.111
	MLE & ENT	.143	.137	.116	.109	.104	.071	.041	-.024
50	RME	-.004	-.012	-.021	-.027	-.085	-.115	-.162	-.180
	PWM & MLM	.008	.000	-.003	-.005	-.021	-.023	-.044	-.051
	MLE & ENT	.057	.044	.050	.049	.044	.057	.030	.024
100	RME	-.003	-.005	-.012	-.015	-.051	-.085	-.122	-.146
	PWM & MLM	.003	.002	.000	-.003	-.007	-.014	-.019	-.029
	MLE & ENT	.025	.028	.028	.025	.030	.028	.035	.018
200	RME	-.001	-.003	-.007	-.007	-.032	-.059	-.098	-.119
	PWM & MLM	.002	.001	-.001	-.001	-.004	-.006	-.012	-.017
	MLE & ENT	.013	.014	.013	.013	.015	.020	.020	.014
500	RME	-.001	-.002	-.001	-.004	-.016	-.036	-.072	-.091
	PWM & MLM	.000	.000	.002	-.001	-.002	-.003	-.006	-.008
	MLE & ENT	.004	.004	.009	.005	.005	.005	.008	.006

PROBABILITY OF NON-EXCEEDANCE = .999

n	METHOD CV-->	(.75)	(.80)	(.90)	(.95)	(1.25)	(1.5)	(2.0)	(2.5)
10	RME	.060	.021	-.047	-.082	-.277	-.367	-.454	-.502
	PWM & MLM	.203	.182	.180	.160	.096	.063	.005	-.050
	MLE & ENT	-.287	-.249	-.235	-.205	-.115	-.073	-.006	.055
20	RME	.037	.021	-.027	-.049	-.195	-.291	-.378	-.437
	PWM & MLM	.108	.106	.094	.089	.090	.063	.037	-.022
	MLE & ENT	-.154	-.146	-.122	-.114	-.108	-.073	-.042	.024
50	RME	.015	.003	-.009	-.019	-.125	-.195	-.296	-.342
	PWM & MLM	.043	.034	.041	.040	.038	.050	.027	.022
	MLE & ENT	-.061	-.047	-.053	-.051	-.045	-.058	-.030	-.024
100	RME	.007	.004	-.006	-.011	-.075	-.148	-.236	-.289
	PWM & MLM	.019	.021	.023	.021	.026	.025	.032	.017
	MLE & ENT	-.027	-.029	-.030	-.026	-.031	-.029	-.035	-.019
200	RME	.004	.002	-.004	-.005	-.047	-.104	-.194	-.242
	PWM & MLM	.010	.011	.011	.011	.013	.017	.018	.013
	MLE & ENT	-.014	-.015	-.014	-.014	-.016	-.020	-.021	-.014
500	RME	.000	.000	.001	-.003	-.024	-.065	-.146	-.194
	PWM & MLM	.003	.003	.008	.004	.005	.004	.007	.006
	MLE & ENT	-.004	-.005	-.010	-.005	-.005	-.005	-.008	-.006

Table 5

RMSE in quantiles

PROBABILITY OF NON-EXCEEDANCE = .800

n	METHOD CV-->	(.75)	(.80)	(.90)	(.95)	(1.25)	(1.5)	(2.0)	(2.5)
10	RME	.228	.247	.285	.302	.397	.466	.630	.789
	PWM & MLM	.232	.251	.288	.303	.374	.401	.452	.474
	MLE & ENT	.829	.869	1.038	1.062	1.358	1.560	1.730	1.902
20	RME	.164	.180	.206	.215	.271	.326	.429	.509
	PWM & MLM	.166	.181	.208	.216	.258	.282	.305	.315
	MLE & ENT	.521	.580	.668	.711	1.053	1.208	1.358	1.332
50	RME	.105	.114	.130	.137	.170	.206	.257	.323
	PWM & MLM	.106	.115	.131	.138	.164	.180	.191	.199
	MLE & ENT	.283	.315	.397	.438	.653	.828	.917	.999
100	RME	.074	.082	.093	.098	.120	.137	.185	.219
	PWM & MLM	.074	.082	.093	.099	.118	.125	.136	.138
	MLE & ENT	.187	.219	.276	.301	.452	.554	.703	.724
200	RME	.053	.058	.065	.069	.084	.096	.127	.152
	PWM & MLM	.053	.058	.066	.070	.083	.089	.094	.097
	MLE & ENT	.130	.151	.191	.210	.308	.392	.512	.527
500	RME	.034	.036	.041	.043	.053	.060	.078	.098
	PWM & MLM	.034	.037	.042	.043	.053	.056	.060	.062
	MLE & ENT	.080	.095	.119	.131	.191	.239	.314	.361

PROBABILITY OF NON-EXCEEDANCE = .900

n	METHOD CV-->	(.75)	(.80)	(.90)	(.95)	(1.25)	(1.5)	(2.0)	(2.5)
10	RME	.211	.232	.275	.295	.398	.466	.617	.771
	PWM & MLM	.213	.234	.277	.296	.384	.419	.477	.504
	MLE & ENT	.888	.926	1.095	1.116	1.407	1.606	1.771	1.942
20	RME	.145	.165	.196	.210	.277	.333	.430	.505
	PWM & MLM	.148	.168	.198	.211	.269	.302	.331	.343
	MLE & ENT	.559	.618	.705	.747	1.091	1.244	1.390	1.359
50	RME	.091	.102	.124	.133	.177	.215	.264	.325
	PWM & MLM	.094	.104	.126	.134	.174	.195	.213	.224
	MLE & ENT	.303	.335	.419	.460	.677	.852	.939	1.020
100	RME	.063	.072	.087	.095	.126	.146	.191	.220
	PWM & MLM	.065	.074	.089	.096	.124	.138	.155	.158
	MLE & ENT	.200	.233	.292	.317	.468	.570	.719	.739
200	RME	.045	.051	.062	.068	.089	.103	.132	.152
	PWM & MLM	.047	.052	.063	.068	.088	.099	.110	.114
	MLE & ENT	.139	.161	.202	.220	.319	.403	.524	.538
500	RME	.028	.032	.039	.042	.056	.064	.079	.096
	PWM & MLM	.029	.033	.039	.042	.056	.063	.071	.075
	MLE & ENT	.086	.101	.125	.137	.198	.246	.322	.368

PROBABILITY OF NON-EXCEEDANCE = .950

n	METHOD CV-->	(.75)	(.80)	(.90)	(.95)	(1.25)	(1.5)	(2.0)	(2.5)
10	RME	.221	.242	.291	.314	.424	.492	.630	.778
	PWM & MLM	.229	.251	.299	.324	.429	.475	.541	.578
	MLE & ENT	.952	.986	1.155	1.172	1.458	1.654	1.813	1.982
20	RME	.150	.172	.208	.224	.304	.362	.453	.523
	PWM & MLM	.162	.183	.216	.233	.309	.352	.389	.403
	MLE & ENT	.599	.658	.743	.785	1.130	1.281	1.423	1.388
50	RME	.094	.105	.132	.143	.200	.241	.289	.346
	PWM & MLM	.103	.114	.140	.150	.204	.233	.259	.274
	MLE & ENT	.325	.357	.442	.484	.701	.878	.961	1.041
100	RME	.065	.075	.092	.102	.144	.168	.213	.239
	PWM & MLM	.072	.082	.099	.107	.147	.167	.192	.197
	MLE & ENT	.215	.248	.308	.333	.485	.587	.736	.754
200	RME	.046	.052	.066	.073	.104	.123	.152	.169
	PWM & MLM	.052	.058	.070	.077	.104	.121	.141	.145
	MLE & ENT	.149	.172	.213	.232	.331	.415	.537	.549
500	RME	.029	.033	.041	.045	.068	.078	.096	.111
	PWM & MLM	.033	.037	.044	.048	.067	.077	.091	.099
	MLE & ENT	.092	.107	.132	.144	.206	.253	.329	.376

Table 5(Conted.)

PROBABILITY OF NON-EXCEEDANCE = .990

n	METHOD CV-->	(.75)	(.80)	(.90)	(.95)	(1.25)	(1.5)	(2.0)	(2.5)
	PROBABILITY OF EXCEEDANCE = .990								
n	METHOD								
10	RME	.290	.311	.369	.398	.517	.586	.704	.836
	PWM & MLM	.362	.382	.448	.482	.630	.712	.810	.877
	MLE & ENT	1.021	1.050	1.218	1.232	1.510	1.703	1.856	2.024
20	RME	.206	.229	.273	.295	.402	.463	.545	.606
	PWM & MLM	.260	.285	.328	.351	.484	.555	.621	.632
	MLE & ENT	.642	.701	.784	.825	1.171	1.319	1.457	1.417
50	RME	.131	.143	.181	.198	.285	.334	.383	.434
	PWM & MLM	.163	.177	.216	.233	.328	.385	.431	.462
	MLE & ENT	.348	.380	.466	.508	.726	.904	.984	1.063
100	RME	.092	.103	.128	.143	.215	.249	.299	.324
	PWM & MLM	.114	.128	.154	.166	.236	.278	.332	.342
	MLE & ENT	.230	.264	.324	.350	.502	.605	.754	.770
200	RME	.066	.073	.092	.103	.161	.193	.231	.248
	PWM & MLM	.081	.090	.109	.119	.168	.204	.249	.257
	MLE & ENT	.160	.183	.224	.243	.343	.428	.549	.561
500	RME	.042	.046	.057	.064	.112	.135	.163	.181
	PWM & MLM	.051	.058	.069	.075	.107	.129	.162	.179
	MLE & ENT	.099	.114	.139	.151	.213	.261	.337	.384

PROBABILITY OF NON-EXCEEDANCE = .999

n	METHOD CV-->	(.75)	(.80)	(.90)	(.95)	(1.25)	(1.5)	(2.0)	(2.5)
10	RME	.459	.481	.550	.576	.677	.736	.825	.934
	PWM & MLM	.773	.816	.984	1.010	1.311	1.514	1.690	1.863
	MLE & ENT	1.095	1.119	1.285	1.295	1.565	1.754	1.900	2.066
20	RME	.321	.354	.414	.443	.575	.626	.690	.738
	PWM & MLM	.486	.545	.633	.676	1.016	1.173	1.327	1.304
	MLE & ENT	.689	.747	.827	.867	1.213	1.358	1.492	1.446
50	RME	.195	.215	.277	.311	.440	.491	.531	.578
	PWM & MLM	.264	.295	.377	.417	.631	.804	.896	.979
	MLE & ENT	.374	.405	.492	.534	.753	.931	1.007	1.085
100	RME	.135	.155	.199	.225	.352	.391	.440	.465
	PWM & MLM	.174	.205	.262	.287	.436	.538	.686	.709
	MLE & ENT	.247	.281	.342	.367	.521	.623	.772	.786
200	RME	.095	.110	.143	.162	.273	.321	.361	.380
	PWM & MLM	.121	.142	.181	.200	.298	.380	.500	.516
	MLE & ENT	.171	.195	.237	.256	.355	.441	.563	.573
500	RME	.060	.069	.090	.102	.198	.241	.276	.301
	PWM & MLM	.075	.089	.113	.124	.185	.232	.307	.353
	MLE & ENT	.106	.122	.147	.159	.221	.268	.345	.392

formed better for $N \geq 200$; the performance of ENT and MLE was not greatly inferior.

In terms of RMSE in parameter b, PWM and MLM performed in a uniformly superior manner for CV ≥ 0.95 and for $N \geq 100$. For CV ≤ 0.95 and $N \geq 20$, RMSE produced by RME was the smallest of all the methods. However, overall all methods were somewhat comparable. Thus, one concludes that PWM and MLM are the preferred methods in terms of RMSE of parameters a and b for large values of CV and small sample sizes. For smaller values of CV and large sample sizes, RME would be the preferred method.

4.3 BIAS in Quantiles Estimates

The results of bias in quantile estimation for different probabilities of non-exceedance (P) are summarized in Table 4. Overall, PWM and MLM performed the best in terms of quantile bias for a range of values of the sample size, CV and the probability of non-exceedance (P). The next best method was RME, followed by MLE and ENT. In general, the quantile bias increased with increasing CV and increasing P but decreased with increasing sample size. For large-sample sizes, $N \geq 200$, the quantile bias was small in all cases, regardless of the method. For N ≥ 20, PWM and MLM exhibited uniformly lower bias for all P=s, sample sizes and CV=s. Thus, PWM or MLM would be the preferred method. If the sample size is more than 20 and CV ≤ 0.95, RME yielded small bias and would therefore be acceptable for practical purposes. For larger values of CV, the sample size would have to be much larger, say, greater than 200. ENT and MLE would be acceptable for sample size greater than 200 if P was less than 0.95 and CV was less than 2.5.

4.4 RMSE in Quantiles Estimates

The values of RMSE for different quantiles estimated by GPD2 are summarized in Table 5. RMSE of a given quantile for a given method varied with the sample size and the coefficient of variation. In general, for a given quantile, RMSE increased with CV, regardless of the sample size; of course, it decreased with increasing sample size. Furthermore, RMSE increased with increasing quantile for the same value of CV and sample size.

For P (the probability of non-exceedance) ≥ 0.99, RME produced the least the least value of RMSE regardless of the sample size and the value of CV. The second lowest value of RMSE was produced by MLM and PWN and the highest value was produced by MLE and ENT. However, the reverse was true for P ≤ 0.9. For large sample sizes $N \geq 200$, RME became comparable to MLM and PWM for all values of CV and P. MLE and ENT did not perform in a comparable manner. Thus, it is concluded that for large floods and higher CVs, RME would be the preferred method, whereas, for smaller floods and lower Cvs, MLM and PWM would also be comparable to RME.

5. CONCLUDING REMARKS

The following conclusions are drawn from this study: PWM and MLM performed better than RME, ENT and MLE in general. In terms of parameter bias, PWM and MLM were uniformly better than ENT and MLE. This was also true of the methods for RMSE. In terms of quanile bias, PWM and MLM performed uniformly better. In terms of RMSE, RME performed better for large values of P and CV than did PWM and MLM and was the preferred method. However, for smaller CVs and low values of P, PWM and MLM were comparable.

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