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An Investigation of the Selection Process of Mathematically Gifted Students[1]

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The purpose of this paper is to review the gifted education from a reflective perspective. Especially, this research touches upon the issues of selection process from a critical point of view. Most of the problems presented in the mathematics competition or in the programs for preparing such competitions share the similar characteristic: the circumstances that are given for questions are too artificial and complicated; problem solving processes are superficially and fragmentally related to mathematical knowledge; and the previous experience with the problem very much decides whether a student can solve the problem and the speed of problem solving. In contrast, the problems for selecting students for Gifted Education Center clearly show what the related mathematical knowledge is and what kind of mathematical thinking ability these problems intend to assess. Accordingly, the process of solving these problems can be considered an important criterion of a student's mathematical ability. In addition, these kinds of problems can encourage students to keep further interest, and can be used as tasks for mathematical investigation later. We hope that this paper will initiate further discussions on issues derived from the mathematically gifted student selection process.

한글 초록(Abstract in Korea): 최근 들어 영재교육에 관한 논의가 갑자기 활발하게 이루어지고 있다. 소란스럽게 확산된 대부분의 교육 운동이 그러했듯이 영재교육도 짧은 번영 후 길고 신랄한 비판의 운명에 처하는 것은 아닌지 걱정스럽다. 부모들의 이상적인 교육 열기는 자녀를 지명도 있

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는 영재센터에서 교육시키고 싶은 열망으로 이어지고, 이에 따라 영재교육에 대한 수요가 급증하고 있다. 뿐만 아니라 정책적으로도 영재교육을 장려하기 때문에, 대학의 영재센터를 중심으로 운영되던 영재교육이 이제는 각 초중등학교 단위에서도 실시하기에 이르렀다. 이와 같이 영재 교육이 성급하고 무분별하게 확산되고 있는 이 시점에서 영재 교육에 대한 반성적 성찰이 필요하다. 영재교육은 크게 선발, 교육, 평가의 세 가지 요소를 중심으로 이루어지는데, 그 중에서 이 글은 영재 선발과 평가의 과정을 비판적인 관점에서 점검하고자 한다. 경시대회나 영재 선발을 위한 준비 기관에서 제공하는 문제들은 우리의 분석에 따르면 수학적으로 또 교육적으로 그리 바람직하지 않은 경우가 적지 않았다. 우선 문제 상황이 지나치게 인위적이고 복잡하며, 수학적 지식과는 피상적으로 그리고 단편적으로만 연결되어 있는 경우가 많다. 또한 해결과정이 조잡하고, 수학보다는 임시방편적인 방법에 의존하였으며, 이전에 문제를 해결한 경험에 따라 해결 여부나 속도가 크게 좌우되는 경향이 있다. 청주교육대학교의 영재 선발은 이러한 전철을 밟지 않기 위해 노력해 왔다. 본 고에서는 그러한 노력의 일부를 소개하였으며, 여기서 소개한 영재 판별 문항이 최선의 것은 아니지만 앞의 부적합한 문항들과 질적으로 다르다고 할 수 있다. 영재교육 후의 재평가 역시 영재 선발이나 교육 못하지 않게 중요하다. 청주교육대학교의 영재 프로그램에서는 교육 내용을 단순하게 확인하는 것이 아니라 얼마나 교육 내용을 이해하고 확장적으로 적용하였는가를 평가하는 문제를 개발하여 활용해 왔다. 본 고가 영재 선발이 내포하는 근본적이면서도 심각한 문제들을 제기하여 자기 성찰의 기회를 갖는 시작점이 되기를 바란다.

Key words: gifted education programs, gifted student selection process.
ZDM classification: C33, C32
MSC2000 classification: 97C30

I. INTRODUCTION

In Korea, the gifted education appears to gain much popularity lately that it has become a 'hot issue' in the Korean education circle. Unusual enthusiasm of Korean parents for education led them to take their children to a reputable center for gifted education programs, which enormously increases the demand for gifted education. Gifted education programs were primarily operated by gifted education centers run by universities. Now many elementary and secondary schools have started launching gifted educational programs because government's policies encourage gifted education. At this juncture where gifted education has been rampantly expanding without thorough planning, it is necessary to review the gifted education from a reflective perspective.

Gifted education is mainly implemented following three processes: selection, education and evaluation for reselection. In this paper, the issues of selection and evaluation for reselection will be touched upon from a critical point of view.

II. THE CRITICAL REVIEW OF THE SELECTION OF MATHEMATICALLY GIFTED STUDENTS

Who and how to select is the first issue that we encounter when we implement gifted education programs. As a matter of fact, there is no agreed definition about a gifted student and there have been many different opinions presented regarding how to distinguish gifted students (Kim et al. 1996; Na 1998; Song 1998; Song et al. 2001; Cho et al. 1996). Although different studies suggest different criteria, two common criteria they share are 'multiple-step evaluation processes' and 'evaluation based on mathematical creativity or advanced mathematical thinking capability'.

The affiliated Gifted Education Center (GEC) of Chongju National University of Education also adopts such criteria as its basic principles to select gifted students and implement them in its three-step selection process: application screening that includes recommendations from teachers and parents, problem solving tests that assesses students' mathematical creativity and advanced task-solving tests in high difficulty level (Lee 2001).

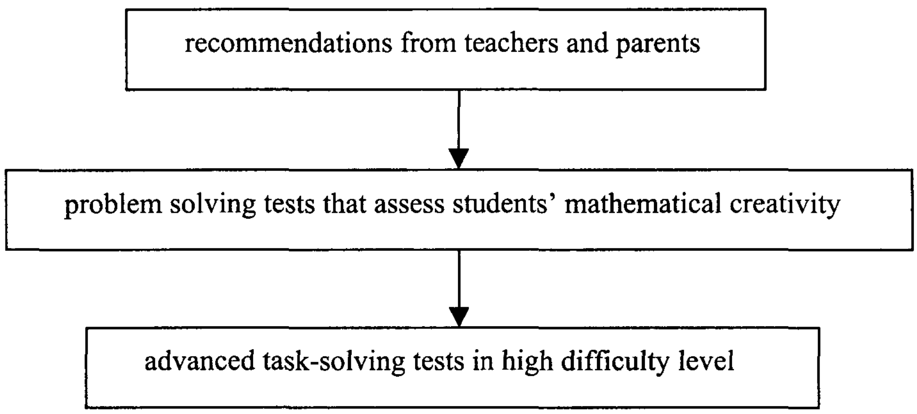


Figure 1. The three-step selection process of gifted students

Among the three-step selection process, the recommendations do not serve as an important criterion for selection. Thus, only a mathematically meaningful set of problems can be a reliable tool to select truly outstanding students with mathematical talent.

1. Inappropriate problems for the selection of mathematically gifted students

Currently, mathematics problems that are used at private gifted education institutions and classes for the preparation of mathematics competitions include many problems that are mathematically and educationally inappropriate. Three examples will be presented here.

Firstly, following problem is created by a reputable institution for preparing mathematics competitions.

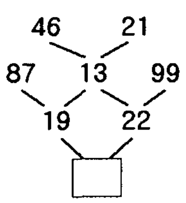
Based on the facts 7★6=4, 8★4=5, 9★8=0, find 6★3.

This problem cannot be solved even after combining all possible formulas with the four arithmetic operations. Since 7+6+4=17, 8+4+5=17, 9+8+0=17, the correct answer is 8 (6★3=8, 6+3+8=17). It is questionable that whether the answers to this problem will vary depending on the level of students' mathematical thinking and whether this problem is appropriate to assess a student's mathematical ability.

A person with basic knowledge of mathematics would think that ★ is a symbol of a binomial operation newly defined using the four arithmetic operations. Based on such an assumption, s/he would try to attempt an answer using the first two numbers. However, the solution to the problems reveals that the third number should be considered. This is different from the typical process of mathematical thinking. This is why it can be more difficult for a person who studied more mathematics. In addition, a student should learn something mathematically significant after solving a problem, which this problem does not accomplish. If the negative numbers are not considered, this problem doesn't make any sense for the cases in which the sum of the first two numbers is bigger than 17. For example, we can't calculate 10★10 with the above method.

As a matter of fact, the above-mentioned operation satisfies commutative law but not associative law. Additionally, this operation does not enable to find 'identity' and 'inverse', which makes it impossible to discuss the further development mathematically. The significance of introducing new binomial operation is to explore its properties, and then compared with those of other number sets. However, this problem does not lead a student to such processes, which makes this problem mathematically meaningless.

The second problem below is also presented by a renowned private gifted education institution.



As an experiment, 16 students participating in GEC program were given this problem, and interestingly their answers showed two extremely different results. Five students solved the problem in less than 10 seconds and all of them had encountered the same problem at a different institution prior. On the other hand, 10 students revealed that the problem was impossible to answer, while one student said that s/he remembered having solved the problem previously but did not recall how to now. Such results show that those who have worked on the problem before were able to solve the problem in a very short time, while the students who had never given the problem were unable to even find a clue to the answer.

The answer to the above question is calculated by first separating second place digit from first place digit of the numbers 19 and 22. This means, the answer 14 can be drawn by calculating 1+9+2+2=14. This problem, like the previous one, presents a binomial operation in a different way, but defined more poorly. To get an answer of 13 out of 46 and 21, and 19 out of 87 and 13 is difficult with natural mathematical thinking. This is a typical kind of problem that 'only those who have previously solved the same problem can solve' again. Whether to solve the problem or not is not decided by mathematical knowledge or thinking but by whether the student has been previously exposed to the same problem.

When the students asked what they thought when given this type of mathematical question, they answered that "calculation is easy but the idea is very strange", "[I think] mathematical scholars are eccentric", and "[I believe that] you can't be a good student in mathematics if you don't have enough luck". This reminds us the statements in the preface of Hiebert (1997), "it really does not make sense. But, we are in math class, so I guess it does in here". It is necessary to experience the unique nature of mathematical thinking. However, this problem does not present the unique nature of mathematics from a positive perspective.

The third example refers to unclear contents in the question being asked despite a lengthy description. The original problem was divided into four parts in the below for the better understanding of the question.

(a) Assuming that only one vehicle per time can be filled at a gas station.
(b) A truck takes 6 minutes, a van takes 4 minutes, while a sedan takes 3 minutes.
(c) Now, a truck, a van and a sedan came simultaneously to fill their gas tanks.
(d) What is the order of gassing up that is least wasteful of time?

The description (a) is relatively plausible. In reality, many vehicles can be filled at the same time using many pumps. However, the assumption is understandable. However,

(b) and (c) are rather artificial. How long it takes to fill a gas tank is determined not by the size of a gas tank but by the amount of money a customer is willing to pay. Additionally, the access road to a gas station is usually one lane, which makes it rare for three different types of vehicles to enter a gas station at the same time. The question described in (d) asks to decide the order of filling up the gas tanks of the three vehicles based on the assumption described in (a), (b) and (c). However, it is difficult to identify what is really asked because no matter what order we decide, it takes the same time to fill the gas tanks of the three vehicles.

The solution is proposed in three separate parts:

1) There are six ways to decide the order of the vehicles to be filled;
2) The smallest sum of minutes to gas up and the minutes of waiting should be the order in which to minimize the time wasted;
3) The order that takes the shortest amount of time to fill the gas tanks is

$$\text{sedan} \Rightarrow \text{van truck.}$$

Here 1) is not directly related to solving this problem. Although the number 2) is the most important key to solving this problem, it is not explicitly reflected in the question. It is not clear that the deduction in 2) is reasonable and it can be arrived through normal mathematical reasoning.

The issues with the three examples mentioned above can be summarized as follows. Firstly, the circumstances that are given for questions are too artificial and complicated. Secondly, problem solving processes are superficially and fragmentally related to mathematical knowledge. Moreover, problem solving processes are too awkward and rely on a temporal method rather than a genuine mathematical method. Thirdly, the previous experience with the problem very much decides whether a student can solve the problem and the speed of problem solving. Most of the problems presented in the mathematics competition or in the programs for preparing such competitions share the same issues described above.

Therefore, it is sometimes very difficult to identify mathematical significance in those problems. Students given these kinds of questions may be given the wrong conception about mathematics.

2. Appropriate problems for the selection of mathematically gifted students

Considering the aforementioned inappropriate problems, it is most important to develop problems different from those practiced at numerous private institutions that are attracting many students preparing for selection examinations for gifted students. If the students studying at these kinds of institutions are selected and enter GEC, the general public will think that anyone can be trained to become a gifted student, which might trigger competition to spend exorbitant amounts of time and tuition for private education. Next, some efforts made by GEC in regards to the selection process of mathematically gifted students will be introduced. The first example is the GEC's problem asking to see if a student knows the meaning of multiplication algorithm and is able to explain it.

It is said that ancient Egyptians calculated the multiplication of 46×78 using the following method. Please explain briefly why they added only the underlined part in the following calculation.

46 × 78 ⇒ 46 × 1=46	
× 2=92	If you add up the underlined parts
× 4=184	92+184+368+2944=3588,
× 8=368	then the answer is 3588
× 16=736	
× 32=1472	
× 64= 2944	

Among the students who aspire to enter and study at GEC, hardly any of them are unable to multiply two double-digit numbers. Therefore, the above question does not ask how to perform the multiplication but leads a student to think about the main ideas of multiplication algorithm and compare it with the ancient Egyptian method. Some students who had answered correctly all the problems of 'competition style' were unable to understand this basic problem and answered in a completely wrong way (Figure 2).

Figure 2. Examples of student answers who could not understand the multiplication algorithm conceptually

The algorithm of the vertical multiplication of two numbers is based on the understanding of the decimal system and the distributive law, which is listed below:

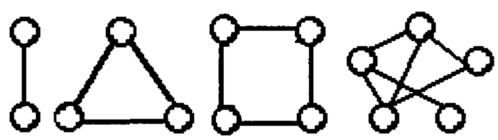
$$\begin{aligned}46 \times 78 &= 46 \times (70+8)\\ &= 46 \times 70 + 46 \times 8\\ &= 3220 + 368\\ &= 3588\end{aligned}$$

Students who understood basic concepts of multiplication and the basic ideas of algorithm presented as the answer in Figure 3.

Figure 3. Examples of students answers who understood the multiplication algorithm conceptually

The second example attempts to diagnose how well s/he utilizes the given conditions to solve a problem.

Here are four diagrams composed of vertices and sides. Please paint the vertices in the way to satisfy the following two conditions. Firstly, the two vertices that are connected should be painted in different colors. Secondly, minimize <u>the number</u> of colors to use.



(1) Please paint the vertices of the four diagrams and explain how many colors you used.
(2) Please find a diagram in which you must use two colors and that is composed of five vertices.
(3) Please find a diagram in which you must use four colors. (You can decide the number of vertices)

Almost all students answered (1) correctly. Out of 67 students who took the test of selecting mathematically gifted students, all but one student used the minimum number of colors necessary. About 54% of the students answered (2) correctly and many of them looked for a solution by modifying presented diagrams as shown in Figure 4.

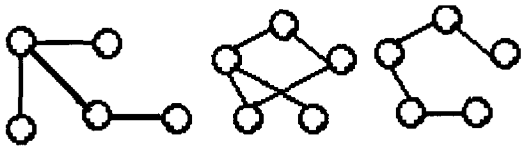


Figure 4. Examples of the correct answers for (2)

However, only six out of 67 students were able to find the correct answer for (3) and all six of them found a diagram with four vertices. Most of the students who could not solve (3) determined too many vertices and tried to connect them, resulting in confusion. Out of these six students, five of them finally entered the GEC and were quick to understand new situations and present new ideas during the learning of GEC program.

These two problems were novel to the students because none of them had been exposed to the two questions prior. These two problems contrast with the other examples introduced earlier in that the GEC's problems clearly show what the related mathematical knowledge is and what kind of mathematical thinking ability these problems intend to assess. The structure of these problems is not complicated and the problem solving process is not lengthy. Accordingly, the process of solving this problem can be considered an important criterion of a student's mathematical ability. In addition, these kinds of problems can encourage students to keep further interest, and can be used as tasks for mathematical investigation later.

III. THE EVALUATION FOR THE RESELECTION OF MATHEMATICALLY GIFTED STUDENTS AFTER IMPLEMENTING PROGRAM

One of the units of the GEC program is titled 'Mathematics taught by sand: circumcenter'. This employs three methods of paper folding, experiment using sand, and constructing figures in order to understand the existence of a circumcenter and learn how to find it. Interestingly, in order to prove that a circumcenter exists in every triangle, some students used the properties of congruence of triangle. As shown in Figure 5, another student explained, 'if you spot three vertices on a circle, you can determine all kinds of triangles. Therefore, conversely speaking, there is a circumcenter in every triangle.' In particular, the second explanation attracted much admiration from many students. The student could reach the agreement that according to the location of the points on the circle, any triangle including equilateral, isosceles, right, acute, obtuse triangle can be made. The student formulated a question for himself and the question went through in-depth discussion among students. Later, they confirmed the validity of hypothesis.

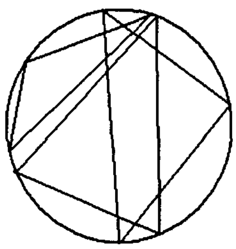


Figure 5. An example of validation

In gifted education, the evaluation for the reselection of mathematically gifted students after implementing program is as important as the initial selection of the gifted students. In the beginning, there are more students on the list as we do not wish to neglect any students with potential. However, after providing education programs, we should reselect students who truly express giftedness. Truly gifted students usually stand out among outstanding students. However, a teacher cannot make a judgment based on his/her own subjective opinion. Therefore, evaluation questions should be developed for the re-selection process. An example of an evaluation question for the above-mentioned topic 'mathematics taught by sand: circumcenter' is as follows.

As we studied during our class, a circumcenter exists for every triangle. Does a circumcenter exist for a quadrilateral as well? Please state what you think first and briefly present the reason.

Nine out of 31 students (about 30%) who studied at GEC gave the correct answer with valid examples and reasoning. Figure 6 is the examples of the correct answers in which students either accurately applies the definition of a circumcenter or appropriately relates their opinion to the discussion performed during the class. Figure 7, however, is the examples of the incorrect answers of the students who are not familiar with mathematical reasoning.

Figure 6. Examples of the correct answers

Figure 7. Examples of the incorrect answers

The following are the conclusions based on the results of evaluation after applying the gifted education programs: firstly, the evaluation problems must be able to assess how much a student understands the content of the studies in the class and how expansively a student can apply what s/he learned, rather than simply confirming what a student learned; secondly, the evaluation questions should be able to judge a student's mathematical reasoning and communication ability.

IV. SUMMARY

The discussion of gifted education has become suddenly popular recently. Such enthusiasm might end up with 'long and bitter criticism on a short-thrived movement' like most of the other education movement that became rapidly popular. To prevent that, we need to ponder upon the issues of gifted education carefully.

In this regard, this paper reviewed examples of undesirable problems that are used as a tool for selecting gifted students. If the given situations of problems are too artificial and mathematically meaningless, selecting gifted children cannot be done in the right manner. Moreover, people might have wrong conception that gifted students can be 'trained' through constant exercises and practices. We are not completely sure whether the examples provided by GEC to select mathematically talented students are ideal or not. However, they are much more sensible than the other inappropriate problems presented earlier. We hope that this paper will shed light on fundamental and serious problems derived from the mathematically gifted student selection process and serve as a starting point for self-reflection on problematic issues.

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