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A Development of the Test for Mathematical Creative Problem Solving Ability *[1]

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The purpose of this study is to develop a test, which can be used in creative problem solving ability in mathematics of the mathematically gifted and the regular students. This test tool is composed of three categories; fluency (number of responses), flexibility (number of different kinds of responses), and originality (degree of uniqueness of responses) which are the factors of the creativity. After applying to 462 middle school students, this test was analyzed into item analysis. As a results of item analysis, it turned out to be meaningful (reliability: 0.80, validity: item 1(1.05), item 2(1.10), item 3(0.85), item 4(0.90), item 5(1.08), item difficulty: item 1(–0.22), item 2(–0.41), item 3(0.23), item 4(0.40), item 5(–0.01), item discriminating power: item 1(0.73), item 2(0.73), item 3(0.67), item 4(0.51), item 5(0.56)), over the level of a standard basis. This means that the test tool was useful in the test process of creative problem solving ability in mathematics

Key words: mathematical development, problem solving, creative problems.
ZDM classification: D50
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1. INTRODUCTION

School Education has long focused on problem solving (*cf.* Dillon 1982; Ramirez 2002). Cognitivism and Constructivism, both of which have been providing a basic framework for school education, stressing the importance of improving students' problem solving ability. Cognitivists insist that school education should concentrate on enhancing students' ability to solve structured problems.

On the other hand, constructivists and problem-based learning theorists advocate the use of unstructured problems in learning, emphasizing that the process of solving an unstructured problem is different from that of solving a structured problem. They also hold that the process of solving an unstructured problem should be preceded by the process of redefining or restructuring it (*cf.* Reiter-Palmon, Mumford, Boes & Runco 1997). However, they argue that problem solving is more important than any other part of learning and the process of structuring a problem is simply a process of solving it. Although constructivists have been attempting to link the process of structuring an unstructured problem to creativity, they still fail to broaden their interest and make a comprehensive study of problem finding ability. Runco & Chand (1995) explain that finding a problem is the starting point and the key to producing creative products. In current years, finding a problem has sometimes been considered as a creative process in itself (*cf.* Dillon 1988; Voss & Means 1989).

NCTM (2000) Standards suggests that, in order to prepare for the 21st century, today's students should identify themselves with the ability to use mathematical knowledge for problem solving, with the ability to communicate mathematically, and with the ability to reason mathematically and a mathematical propensity. It also states that students need to be provided with challenging problems that can stimulate students to develop diverse and sound ways of mathematical thinking and to think creatively. It adds that guiding students to solve a problem using several methods and strategies help students develop and extend their mathematical thinking. Creative thinking ability and expressive ability in the field of mathematics can be measured by 'open-ended' or 'open-response' problems and questions that require more than one answer.

Based on previous studies as described above, this study aims to examine and analyze how differently gifted and regular second-grade middle-school students respond to open-ended problems, which can be used as essential vehicles to measure mathematical creativity. In order to investigate how creatively they solve problems, they were presented with problems that have several answers that can be solved with original and unique ideas.

1.1. The concept of "open problem"

One of the aim of the PME discussion group was to find answers to the question, " What are 'open-ended problems'?". This was becase the group of open-ended problems does not seem to be well defined. In the course of the discussion, several types of problems were put forward: Investigations, problem posing, real-life situations, projects, problem fields (or problem sequences), problems without question, and problem variations ("what-if" method). Examples of these groups of problems can be found in the papers published on this subject (*cf.* Nohda 1995; Silver 1995; Stacey 1995). A relationship between ill-structured problems and problem finding was found in Voss (1990). Indeed, "problem finding, that is, how individuals formulate and identify a problem in itself is an ill-structured problem" (*cf.* Voss 1990, p.12).

The concept "open problem" can be explained as follows: We will begin by its opposite, supposing that if its starting situation and goal situation are closed, *i.e.* exactly explained, a problem is closed. If the starting situation and / or the goal situation are open, *i.e.* they are not closed, we have an open problem (Table 1). Problems dealt with in school mathematics are usually closed problems (or more generally closed tasks) that leave not much room for creative thinking (*cf.* Pehkonen 1995a).

Table 1: The classification of problems according to their starting and goal situations

goal situation / starting situation	CLOSED (*i.e.* exactly explained)	OPEN
CLOSED (*i.e.* exactly explained)	closed problems	open-ended problems, real-life situations investigations, problem fields problem variations
OPEN	real-life situations problem variations	real-life situations, problem variations projects, problem posing

1.2. A prior study and relation to mathematical creativity

Creativity is similar to problem posing in its multiplicity in nature. Psychologists identified what as a special construct other than intelligence. Problem posing, or problem finding, has long been viewed as a characteristic of creative activity or exceptional talent in many fields of human endeavor. For example, Getzels & Csikszentmihalyi (1976) studied artistic creativity and characterized problem finding as a center of creative artistic experience. The notions of fluency, flexibility and novelty were adapted and applied in the domain of mathematics by Balka (1974), who asked subjects to pose mathematical problems that could be answered on the basis of information provided in a set of stories about real world situations. Problem posing, along with problem solving, is central to the discipline of mathematics and the nature of mathematical thinking (*cf.* Silver 1994). When mathematicians engage in the intellectual work of the discipline, it can be argued that the self-directed posing of problems to be solved is an important characteristic (*cf.* Pólya 1954).

In fact, problem finding has sometimes been considered as a creative process in itself (*cf.* Dillon 1988; Voss & Means 1989). Studies in mathematical creativity were reviewed (*cf.* Haylock 1987) and one may see problem posing ability as a creative ability. In a recent comprehensive review paper on problem posing, however, Silver (1994) extended the discussion and commented that a general relationship between problem posing and creativity was still unknown. For a more detailed report of the study see Leung and Silver (1997). From the first result, fluency is general in both verbal creativity and problem posing but according to the second, flexibility is not. Finally, the third result suggests that flexibility is specific within arithmetic problem posing. The third result was also obtained in a replicated study in Taiwan ($r = 0.286$; $P < 0.01$; Leung 1995). Studies on the relationship between general creativity and mathematical creativity (*cf.* Evans 1964; Haylock 1978; Lee & Hwang 2003) were reviewed and there was a correlation between general creativity and mathematical creativity. We examined relations between Mathematical Creative Problem Solving Ability Test (MCPSAT: Kim et al. 1997) and Torrance Test of Creative Thinking Figural A (TTCT; adapted for Korea by Kim 1998). The results of the study can be summarized as follows; first, there was a correlation between the originality of general creativity and the three elements-fluency, flexibility, and the total-of mathematical creativity (significant at $p < 0.01$). Second, there was also a correlation between the total of general creativity and the three elements of mathematical creativity (significant at $p < 0.05$) (*cf.* Lee & Hwang 2003). Yoshihiko (1997) think that openness like "open-ended approach" and "from problem to problem" is one aspect of fostering mathematical creativity. Because, "open-ended approach" means end products are open, and "from problem to problem" means ways to develop a problem are open.

In the 1980's, the idea of using some form of open-ended problems in classroom spreaded all over the world, and research on its possibilities was very active in many countries (*cf.* Nohda 1988, 1991; Pehkonen 1995a, 1995b; Silver & Mamona 1989; Williams 1989; Mason 1991; Stacey 1991, 1995; Zimmermann 1991; Clarke & Sullivan 1992; Silver 1993, 1995; Leung 1993; Silver & Cai 1996). Studies are currently under the way on the relationship between mathematical creativity and open-ended problems (*cf.* Kwon et al. 1999; Min 1999; Byun 2001; Moon 2002). This means that the test process of creative problem solving ability in mathematics should be considered with open-ended problems.

1.3. Creativity in school mathematics

Mathematician considered mathematical creativity a major element of mathematical bility and have tried to define it. After searching literature and research on mathematical creativity Aiken (1973) concluded that mathematical creativity is always defined on the basis of process and various products. Judging from literature and studies on mathematical creativity, the nature of creativity can be classified into two perspectives: Firstly, mathematical creativity is regarded as cognitive ability that leads to emphasize creative thinking. Secondly, mathematical creativity is essentially defined with focus on products.

1.4. Creativity as measuring factor

Mathematical creativity and divergent products can be summarized by measuring factors as follows: first, fluency is used as measuring factor (*cf.* Foster 1970; Baur 1971; Maxwell 1974; Dunn 1976). Second, flexibility is used (*cf.* Krutetskii 1976). Third, fluency and originality are used (*cf.* Mainville 1972). Fourth, fluency, flexibility and originality are used (*cf.* Evans 1964; Zosa 1978; Balka 1974; Kim et al. 1997; Song 1998).

2. DESIGN OF ANALYSIS

For this research, we examined and analyzed the responses to open-ended problems of the mathematically gifted and the regular students with three categories; fluency, flexibility, and originality which are the factors of the creativity. Purposes for this study included (a) the reliability, validity, difficulty, relevance and discrimination of open-ended problems; (b) a comparative study on the characteristic of responses to open-ended problems of the mathematically gifted and the regular students.

Because it is unreasonable to generalize test results from a single open-ended problem, more items are required for detailed analysis of test results. In this study, internal validity and difficulty were assessed based on Rasch's 1-parameter item-response model.

2.1. Participants

The subjects of this study are 53 volunteers from the Gifted Education Center of Hanbat National University in Daejeon and 409 students from middle-schools in Daejeon.

2.2. Instrumentation

Five following problems were selected as the open-ended problems for this study.

Problem 1. Sixteen dot problem, a transformed version of the nine dot problem in

Haylock (1984), Kim et al. (1997) and Song(1988).

Problem 2. Regular hexagon problem, a transformed version of the quadrangle problem in Kim et al. (1997).

Problem 3. Water-flask problem in Becker & Shimada (1997).

Problem 4. Marble problem in Becker & Shimada (1997).

Problem 5. Classifying several solid figure problems in Becker & Shimada (1997).

2.3. Design and Procedure

Data collection. Sampling was done in May 2003. Prior to conducting the test, the subjects were instructed by the tester for 5 minutes on how to complete their answer sheets. Subjects were given 50 minutes to present various types of original and unique answers.

Marking method and standard. The method and standard of marking the responses are as follows.

1) All types of responses to items are analyzed and recorded.
2) Same types of responses are selected and classified.
3) Scores are given by categorized responses where score of fluency, flexibility, and originality are analyzed. Each scoring method is suggested below.
   (1) Flexibility: how many types of categorized response a student can made. Students are allowed to write a maximum of 15 answers for one problem thus maximum score of flexibility is 15. For example, if a student's answers are classified into 3 categories of responses, then flexibility score is 3.
   (2) Fluency: how many correct answers exist with a categorized response type. When a student made multiple correct answers in a category, the score can be given to the maximum of 5.
   (3) Originality: how original response an answer is which no other students could think of. That is, originality score reflects the relative rarity of response. Originality is measured as the following procedure.

- The frequency is analyzed in that how many students have given the same type of response categorized in sub-level.
- The percentage of the frequency is calculated that the response type belongs to
- The score is given as below according to the percentage of frequency.
  * 3% above: 0 * 2% above–3% below: 1
  * 1% above–2% below: 2 * 1% below: 3

• The originality score has no upper limit.

This test does not suggest total score due to the following reasons:

If regular score for each item is given, then regular score for 3 sub-ability factors is also fixed that leads to incorrect discrimination of student's ability. For instance, 5 flexibility scores are decided and reference marks are given such as frequency

1–2 score 1, 3–4 score 2, 5–6 score 3, ...

then both students giving 3 responses and 4 responses have the same score. This is true to other ability factors.

Data analyses. A reference table for scoring the responses was developed by selecting and classifying all relevant responses to each item according to their types and identifying the frequency of each type. In order to evaluate item-internal consistence reliability and discrimination, Cronbach α. was calculated using SPSS 10.0K. Internal validity and difficulty were calculated using BIGSTEPS (*cf.* Livacre & Wright 1994, 2003) based on Rasch's 1-parameter item-response model.

3. RESULTS AND CONCLUSIONS

3.1. Analysis of quality of test instruments

Item-internal Consistence Reliability. To evaluate the reliability of the test, Cronbach α was calculated, which indicates item-internal consistence reliability. Cronbach α was .80, suggesting that the test is fairly reliable.

Internal Validity by Item Relevance Index. The internal validity of each test item was calculated using BIGSTEPS, a computer program designed to measure parameter values and conduct item analysis based on Rasch's 1-parameter response model. The analysis model used in this study was the Partial Credit model. Every item relevance index was less than 1.2, which implies that all items are relevant for an analysis model.

Table 2: Open-ended Item relevance indexes

Item	1	2	3	4	5	Total
Infit	1.05	1.10	0.85	0.90	1.08	1.00
Outfit	1.01	1.02	0.83	0.90	1.05	0.96

Difficulty. Item difficulty refers to the degree of difficulty of an item. In this study, item difficulty was calculated based on Rasch's 1-parameter item response model. The item difficulty of 0.0 means "average". A higher positive number indicates a higher difficulty. On a "difficulty" scale, the differences in difficulty between items are evenly distributed as far as the logit score does not exceed 0.6. Every item reliability index was higher than 0.80, which implies that the used items are well separated and highly relevant for discriminating between students on the basis of creative problem solving ability.

Table 3: Open-ended item difficulties

Item	1	2	3	4	5	Total
Difficulties	–.22	–.41	.23	.40	–.0.1	0.00

Discrimination. The discrimination of each open-ended item was analyzed by point-biserial correlation. Point-biserial correlation represents the correlation between the score for a single item and the total score for the remaining items. An item with a negative value is not suitable for discriminating between high and low ability students. Most of the items with point-biserial correlation of less than 0 are not relevant for discriminating between students because they allow students to get good marks easily based on their previous knowledge. However, considering that there was no item calculated as less than 0, all items seem to be more or less relevant for discriminating between students on the basis of mathematical creativity.

Table 4: Open-ended item discriminations

Item	1	2	3	4	5	Total
Discriminations	.73	.73	.67	.51	.56	1.00

3.2. Analysis of answers by type

Considering that there can be multiple numbers and types of answers to each item, all possible responses were selected and classified by their types and the frequency of each type was measured. In order to develop a reference table for scoring the responses, a table of response types was firstly prepared by analyzing the number of responses and the number of response types, then a criteria for giving marks for originality was established by identifying the frequency of each type and its mathematical utility. The detailed analysis of items are showed in appendix 2.

Totally 462 students participated in this test. While numbers with parenthesis show percentage in proportion to 462. The frequency is calculated under the same standard between of gifted students and regular students. Fluency, flexibility, and originality scores are resulted from the same reference table.

3.3. Open-ended test results

The following table shows results of the test by items. Each score for fluency, flexibility, and originality is resulted from minimal and maximal score of each item.

Correlation means Pearson's correlation coefficient of each item score and total score. Almost all items show the correlation of 0.69–0.74. Scores represented in coefficient are marks of fluency, flexibility, originality, and minimum/maximum of total score. For example, fluency score ranges from minimum score of 0 to maximum score of 40.

As items 1 and 2 have a maximal fluency limit of 15 responses, they are familiar items to students. However fewer the maximal flexibility score, more difficult items that give various types of ideas the students were offered. Items 3 to 5 have fewer maximal scores in both fluency and flexibility, which indicates items 3 and 5 are difficult items to stimulate various ideas. This is either of the two: the nature of the question itself makes difficulty in suggesting various responses or various types of responses are possible, but students have difficulty thinking of it. Some items have the latter case.

The number shown in the note is a ratio of students who got at least one point higher score in each item. The lower ratio, more difficult response to give, and vice versa.

Table 5: Scores by items

(N=462)

Items	Fluency			Flexibility			Originality			Total			M			SD			Correlation	Note		
	G	R	T	G	R	T	G	R	T	G	R	T	G	R	T	G	R	T	T	G	R	T
1	0-12	0-15	0-15	0-6	0-6	0-6	0-15	0-3	0-15	0-32	0-20	0-32	11.77	4.92	5.71	5.93	4.71	5.33	.69	94.3	76.0	78.1
2	0-15	0-15	0-15	0-6	0-5	0-6	0	0	0	0-20	0-20	0-20	12.72	6.99	7.65	5.28	5.50	5.76	.69	96.2	78.2	80.3
3	0-7	0-4	0-7	0-6	0-4	0-6	0-8	0-8	0-8	0-21	0-14	0-21	5.38	1.57	2.00	4.86	2.65	3.22	.72	73.6	36.4	40.7
4	0-5	0-4	0-5	0-4	0-4	0-4	0-9	0-9	0-9	0-18	0-14	0-18	2.94	1.14	1.34	3.53	2.43	2.64	.75	62.3	24.7	29.0
5	0-6	0-7	0-7	0-4	0-5	0-5	0-6	0-9	0-9	0-12	0-18	0-18	5.02	3.70	3.85	2.97	3.47	3.44	.74	96.2	71.4	74.2
Total	3-40	0-38	0-40	2-19	0-18	0-19	0-16	0-23	0-23	5-74	0-68	0-74	37.83	18.31	20.55	12.71	11.93	13.53	.75	100	97.1	97.4
M	23.00	12.02	12.39	11.28	6.11	6.70	4.26	3.55	1.46	37.83	18.31	20.55										
SD	7.57	7.34	8.29	3.31	3.42	3.78	2.38	1.19	2.77	12.71	11.93	13.53										

**, $p < 0.01$

* correlation is a coefficient of correlation with total score.

* 'note' is a ratio of students having over score 1.

* denote by G: gifted students, R: regular students, T: Total

The results of analysis of the differences between students' responses are as follows:

1) **Item 1.** Responses are categorized largely into four types:

   (1) Using a single basic figure.
   (2) Using more than two figures.
   (3) Using the median point.
   (4) Using curved lines.

Gifted students gave an even distribution of response types, showing an extraordinary ability in giving response using curves.

2) **Item 2.** Responses are categorized largely into five types as showen in Appendix 2.

Responses were so numerous that the score for originality was 0 (cf. every relative frequency of response type to item 2 exceeds 0.03). Item 2 was the easiest to solve, and its difficulty was –0.41. Students showed various of point of views, but most of their expressions were not advanced level. In these problems, the approach were already given to the students and they did not need to devise their own. Item 2 is narrowed simply to identifying some figures that have a certain property.

3) **Item 3**. Responses are categorized lloughly into six types:

(1) Linking each dots.
(2) Using a diagonal line.
(3) Using an inner dot.
(4) Using circle.
(5) Using square.
(6) Others.

As the difficulty of item 3 was 0.23, gifted students gave more active response than regular students did. Most of the gifted students scored for originality. Regular students lacked ability to express their originality clearly.

4) **Item 4**. It was the most difficult one to solve.

Few of regular students could solve item 4, though students with higher scores were more or less responsive to it. Item 4 was found to be very difficult for the subjects. However, the discrimination power of item 4 was at the acceptable level. In these problems, the approach is already given to the students and they did not need to devise their own.

5) **Item 5**. It is simply narrowed to identifying some figures that have certain property.

A large number of students ansewered to item 5, which was answered in more various ways than items 3 and 4 were.

3.4. Analysis of scores by answers

The frequency is calculated under same standard with a difference of gifted students and regular students. Fluency, flexibility, and originality scores are resulted from the same reference table (*cf.* Table 6 and 7).

Table 6: Reference table for total and frequency of fluency, flexibility and originality

Scores	fluency					
	Gifted students			Regular students		
	frequency	percent	accumulation percent	frequency	percent	accumulation percent
.00				12	2.9	2.9
1.00				11	2.7	5.6
2.00				23	5.6	11.2
3.00	1	1.9	1.9	16	3.9	15.2
4.00				20	4.9	20.0
5.00				31	7.6	27.6
6.00				21	5.1	32.8
7.00				19	4.6	37.4
8.00				22	5.4	42.8
9.00				21	5.1	47.9
10.00	1	1.9	3.8	22	5.4	53.3
11.00	1	1.9	5.7	19	4.6	57.9
12.00				18	4.4	62.3
13.00	3	5.7	11.3	23	5.6	68.0
14.00	2	3.8	15.1	14	3.4	71.4
15.00	2	3.8	18.9	12	2.9	74.3
16.00	2	3.8	22.6	11	2.7	77.0
17.00				15	3.7	80.7
18.00				8	2.0	82.6
19.00	3	5.7	28.3	13	3.2	85.8
20.00	6	11.3	39.6	11	2.7	88.5
21.00	2	3.8	43.4	6	1.5	90.0
22.00	1	1.9	45.3	8	2.0	91.9
23.00	2	3.8	49.1	6	1.5	93.4
24.00	4	7.5	56.6	5	1.2	94.6
25.00	3	5.7	62.3	6	1.5	96.1
26.00	4	7.5	69.8	5	1.2	97.3
27.00	1	1.9	71.7			
28.00	4	7.5	79.2	1	.2	97.6
29.00	3	5.7	84.9	2	.5	98.0
30.00	1	1.9	86.8	4	1.0	99.0
31.00				1	.2	99.3
32.00	2	3.8	90.6	2	.5	99.8
33.00						
34.00	1	1.9	92.5			
35.00	1	1.9	94.3			
36.00	1	1.9	96.2			
37.00						
38.00				1	.2	100.0
39.00	1	1.9	98.1			
40.00	1	1.9	100.0			

Scores	flexibility					
	Gifted students			Regular students		
	frequency	percent	accumulation percent	frequency	percent	accumulation percent
.00				12	2.9	2.9
1.00				21	5.1	8.1
2.00	1	1.9	1.9	29	7.1	15.2
3.00				39	9.5	24.7
4.00	1	1.9	3.8	39	9.5	34.2
5.00				47	11.5	45.7
6.00				47	11.5	57.2
7.00	4	7.5	11.3	48	11.7	68.9
8.00	4	7.5	18.9	28	6.8	75.8
9.00	3	5.7	24.5	29	7.1	82.9
10.00	10	18.9	43.4	26	6.4	89.2
11.00	5	9.4	52.8	13	3.2	92.4
12.00	8	15.1	67.9	17	4.2	96.6
13.00	3	5.7	73.6	6	1.5	98.0
14.00	7	13.2	86.8	1	.2	98.3
15.00	1	1.9	88.7	4	1.0	99.3
16.00	3	5.7	94.3	2	.5	99.8
17.00	1	1.9	96.2			
18.00	1	1.9	98.1			
19.00	1	1.9	100.0			

Scores	originality					
	Gifted students			Regular students		
	frequency	percent	accumulation percent	frequency	percent	accumulation percent
.00	20	37.7	37.7	286	69.9	69.9
1.00	2	3.8	41.5	17	4.2	74.1
2.00	3	5.7	47.2	9	2.2	76.3
3.00	12	22.6	69.8	50	12.2	88.5
4.00				12	2.9	91.4
5.00	2	3.8	73.6	4	1.0	92.4
6.00	4	7.5	81.1	16	3.9	96.3
7.00	1	1.9	83.0	9	2.2	98.5
8.00	1	1.9	84.9	2	.5	99.0
9.00	2	3.8	88.7	1	.2	99.3
10.00	1	1.9	90.6			
11.00	1	1.9	92.5	1	.2	99.5
12.00	2	3.8	96.2			
16.00	2	3.8	100.0			
23.00				1	.2	100.0

Table 7: Reference table for standard of frequency

Scores	fluency			flexibility			originality			Total		
	frequency	percent	accumulation percent	frequency	percent	accumulation percent	frequency	percent	accumulation percent	frequency	percent	accumulation percent
.00	12	2.6	2.6	12	2.6	2.6	306	66.2	66.2	12	2.6	2.6
1.00	11	2.4	5.0	21	4.5	7.1	19	4.1	70.3			
2.00	23	5.0	10.0	30	6.5	13.6	12	2.6	72.9	11	2.4	5.0
3.00	17	3.7	13.6	39	8.4	22.1	62	13.4	86.4	8	1.7	6.7
4.00	20	4.3	18.0	40	8.7	30.7	12	2.6	89.0	14	3.0	9.7
5.00	31	6.7	24.7	47	10.2	40.9	6	1.3	90.3	12	2.6	12.3
6.00	21	4.5	29.2	47	10.2	51.1	20	4.3	94.6	8	1.7	14.1
7.00	19	4.1	33.3	52	11.3	62.3	10	2.2	96.8	15	3.2	17.3
8.00	22	4.8	38.1	32	6.9	69.3	3	.6	97.4	16	3.5	20.8
9.00	21	4.5	42.6	32	6.9	76.2	3	.6	98.1	17	3.7	24.5
10.00	23	5.0	47.6	36	7.8	84.0	1	.2	98.3	16	3.5	27.9
11.00	20	4.3	51.9	18	3.9	87.9	2	.4	98.7	5	1.1	29.0
12.00	18	3.9	55.8	25	5.4	93.3	2	.4	99.1	10	2.2	31.2
13.00	26	5.6	61.5	9	1.9	95.2				15	3.2	34.4
14.00	16	3.5	64.9	8	1.7	97.0				16	3.5	37.9
15.00	14	3.0	68.0	5	1.1	98.1	1	.2	99.4	16	3.5	41.3
16.00	13	2.8	70.8	5	1.1	99.1	2	.4	99.8	18	3.9	45.2
17.00	15	3.2	74.0	1	.2	99.4				9	1.9	47.2
18.00	8	1.7	75.8	2	.4	99.8				18	3.9	51.1
19.00	16	3.5	79.2	1	.2	100.0				12	2.6	53.7
20.00	17	3.7	82.9							12	2.6	56.3
21.00	8	1.7	84.6							11	2.4	58.7
22.00	9	1.9	86.6							17	3.7	62.3
23.00	8	1.7	88.3				1	.2	100.0	12	2.6	64.9
24.00	9	1.9	90.3							6	1.3	66.2
25.00	9	1.9	92.2							9	1.9	68.2
26.00	9	1.9	94.2							7	1.5	69.7
27.00	1	.2	94.4							11	2.4	72.1
28.00	5	1.1	95.5							6	1.3	73.4
29.00	5	1.1	96.5							4	.9	74.2
30.00	5	1.1	97.6							9	1.9	76.2

Tabel 1 (continued.)

31.00	1	.2	97.8							8	1.7	77.9
32.00	4	.9	98.7							10	2.2	80.1
33.00										10	2.2	82.3
34.00	1	.2	98.9							9	1.9	84.2
35.00	1	.2	99.1							7	1.5	85.7
36.00	1	.2	99.4							2	.4	86.1
37.00	1	.2	99.6							3	.6	86.8
38.00	1	.2	99.8							9	1.9	88.7
39.00	1	.2	100.0							4	.9	89.6
40.00										6	1.3	90.9
41.00										7	1.5	92.4
42.00										4	.9	93.3
43.00										6	1.3	94.6
44.00										4	.9	95.5
45.00										2	.4	95.9
46.00										2	.4	96.3
47.00										2	.4	96.8
48.00										1	.2	97.0
49.00												
50.00												
51.00										2	.4	97.4
52.00										2	.4	97.8
53.00										1	.2	98.1
54.00										2	.4	98.5
55.00										1	.2	98.7
56.00												
57.00										1	.2	98.9
58.00												
59.00												
60.00										2	.4	99.4
65.00										1	.2	99.6
68.00										1	.2	99.8
74.00										1	.2	100.0

3.5. Conclusions

Five open-ended problems were designed to evaluate students' creative problem solving ability in terms of fluency, flexibility and originality, which are the sub-elements of creativity. For this study, a total of 462 gifted and average second-grade middle school students were tested and then each item's reliability, validity, difficulty, discrimination, etc. were analyzed based on item-response theory. The results are as follows:

First, the coefficient of item-internal consistency reliability (Cronbach α) was 0.80.

Second, every item relevance index based on item-response theory was less than 1.2, which implies that all items are relevant for an analysis model. The differences in difficulty among items are evenly distributed as far as the logit score does not exceed 0.6.

Third, every item reliability index was higher than 0.8, which implies that the used items are well separated and highly relevant for discriminating between students on the basis of creative problem solving ability.

Fourth, considering that there was no item calculated as less than 0, all items are seem to be more or less relevant for discriminating between students on the basis of mathematical creativity.

The major targets of these tests were students whose achievement level in mathematics is to top 0–80%. This test was found to be easy for the subjects. However, the discrimination power of the test was at the acceptable level.

The research on the elaboration of creative problem solving in mathematics should be performed in other studies in future.

It is highly recommended that the test for mathematical creative problem solving ability to be introduced in school for mathematics education, since it can stimulate students' mathematical creativity and divergent thinking as well as enhancing their interest in mathematics. Also a teaching based on open-ended learning methods can greatly help students develop their mathematical creativity. This teaching method does not mean simply applying and practicing algorithms presented by the teacher, but encouraging students to challenge new problems and develop flexible thinking and mathematical power.

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Appendix 1

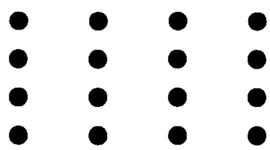
Open-ended Test

Name: ____________, Date of Birth: ______________, Sex: Male__ Female__
School: ____

< Attention >
Please read the following explanation before getting to the questions below.
Every question allows multiple answers. The time given is 50 minutes.

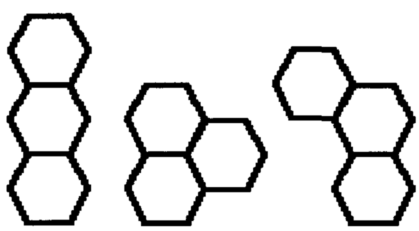
(1) Write a maximum of 15 answers that you think are pertinent to the question.
(2) Give answers that are different from but not similar to one another.
(3) Give answers that can not be easily found.
(4) Present answers in as exact and detailed way as possible.
(5) If you need more space to write, get another answer sheet from the teacher.
■ Do not turn to the next page until instructed by the teacher.

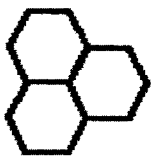
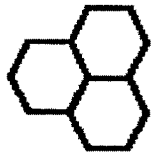
[1] As shown below, there are 16 dots which are arranged with 1cm spacings between them.



Draw lines between the dots to make as many figures as possible with the area of 2 cm^2. (If two or more figures are overlapped when turned around or over, they are considered as identical. No figure should be split in two or have one point in common with another.)

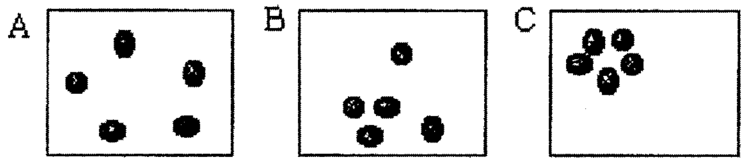
[2] As shown in the Example below, 3 sheets of paper in the shape of a regular hexagon can be joined together along the sides in 3 ways.



 and  are considered as identical.

Then, make all drawings of how to join together 6 sheets of paper in the shape of a regular hexagon along the sides, as in the Example below. (If two or more figures are overlapped when turned around or over, they are considered as identical.)

[3] Three students, A, B, C, each threw five marbles, Which came to rest as shown. In this game, the winner is the student with the smallest scattering of marbles. The degree of scattering seems to decrease in the order A, B, C. Devise as many ways as you can to express numerically the degree of scattering.



[4] A transparent flask in the shape of a right rectangular prism is partially filled with water. When the flask is placed on a table and titled, with one edge of its base being fixed, several geometric shapes of various sizes are formed by the cuboid's faces and the surface of the water. The shapes and sizes may vary according to the degree of tilt or inclination. Try to discover as many invariant relations (rules) concerning these shapes and sizes as possible. Write down all your findings.

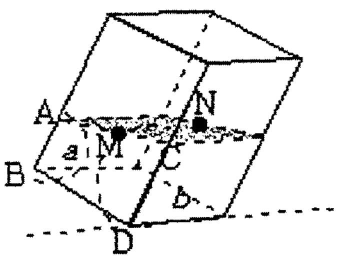


Figure 2.1

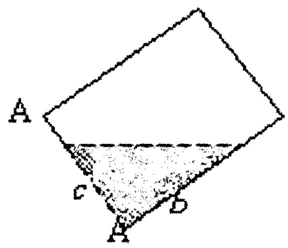
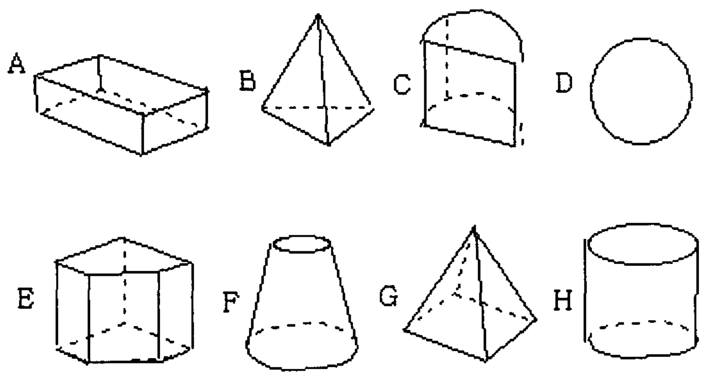


Figure 2.2

[5] Consider the solid figures as shown. Choose one or more figures that share the same characteristics with figure B and write down those characteristics.



[Solutions]

Characteristics	A	B	C	D	E	F	G	H

< Appendix 2 >

Characteristic of Responses to Open-Ended Problems

Table 8: Reference table for scoring Item 1 and the frequency of response type

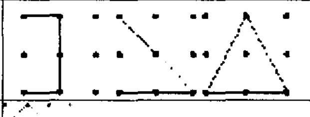
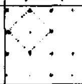
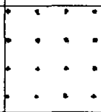
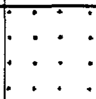
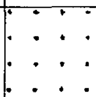
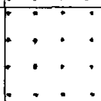
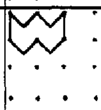
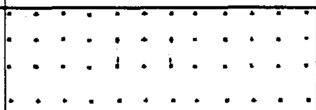
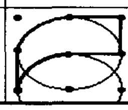
Classification		Students' Observations (Rules)	Originality	Number of Responses		
				Gifted students (N=53)	Regular Students (N=409)	Total (N=462)
Using a single basic figure	line segment		0	151	728	879 (190.3)
	point symmetry		0	86	232	318 (68.8)
	asymmetry		1	6	4	10 (2.2)
Using more than two figures	line segment		0	30	51	81 (17.5)
	point symmetry		0	8	10	18 (3.9)
	asymmetry		0	143	258	401 (87)
Using the middle point	line segment		3	1	3	4 (0.9)
Using Curved lines	line segment		3	3	0	3 (0.7)
	asymmetry		3	2	0	2 (0.4)

Table 9: Reference table for scoring Item 2 and the frequency of response type

Classification		Students' Observations (Rules)	Originality	Number of Responses		
				Gifted students (N=53)	Regular Students (N=409)	Total (N=462)
[illegible]	line segment	[illegible]	0	80	270	350 (76)
	asymmetry	[illegible]	0	37	109	146 (32)
[illegible]	line segment	[illegible]	0	64	196	260 (56)
	point symmetry	[illegible]	0	26	100	126 (27)
	asymmetry	[illegible]	0	74	244	318 (69)
[illegible]	line segment	[illegible]	0	86	418	504 (109)
	point symmetry	[illegible]	0	45	173	218 (47)
	asymmetry	[illegible]	0	63	364	427 (92)
[illegible]	line segment	[illegible]	0	16	51	67 (14.5)
	point symmetry	[illegible]	0	11	52	63 (14)
	asymmetry	[illegible]	0	4	12	16 (3.5)
0		[illegible]	0	15	63	78 (17)

Table 10: Reference table for scoring Item 3 and the frequency of response type

Classification	Students' Observations (Rules)	Originality	Number of Responses		
			Gifted students (*N*=53)	Regular Students (*N*=409)	Total (*N*=462)
Linking each dots	measure the area of pentagon made from linking each dots.	0	34	40	74 (16.0)
	measure the circumference of pentagon made from linking each dots.	0	23	38	61 (13.2)
	measure the circumference of star made from linking every dots.	0	1	6	5(1.1)
	measure and add the distance between dots after linking each dots, and then divide it by the number of lines	2	0	5	5(1.1)
Diagonal Line	measure all lengths of diagonal lines and sort out the longest value.	0	4	10	14(3.0)
	add all lengths of diagonal lines and measure mean value of them.	2	3	2	5(1.1)
	sort out the longest one of 4 lines linking one of 5 dots and the other 4 dots.	1	9	3	12(2.6)
	add the lengths of 4 lines linking one of 5 dots and the other 4 dots.	0	9	6	15(3.3)
	measure it by the total lengths of diagonal lines.	3	0	1	1(0.2)
Inner dot	add the lengths of lines connecting one dot inside pentagon and the other 5 dots.	2	5	3	8(1.7)
	measure mean length of lines connecting one dot inside pentagon and the other 5 dots.	3	3	0	3(0.7)
Using circle	measure the radius(circumference) of the smallest circle including all dots.	3	0	1	1(0.2)
	count the number of marbles outside a circle when the same size of circle is projected on it.	3	0	1	1(0.2)
Using square	measure the area of big square excluding pentagon in it.	2	0	5	5(1.1)
	draw rectangles of same proportion and give point 10, 9, 8 for each rectangle and then add all the points. e. g. 10*1+9*2+8*2 = 44(points)	2	3	2	5(1.1)

Table 10 (continued.)

Others	draw triangles and calculate the largest area of them.	3	0	2	2(0.4)
	measure the deviation and the standard deviation using the coordinates system.	3	2	1	3(0.7)
	draw figures linking dots, then put the same size of figures on A, B, C, and compare the rest they make.	3	0	1	1(0.2)
	put some kinds of objects in figure producing marbles one after another, and find out the number of the objects.	2	5	2	7(1.5)
	measure it by the number of objects between marble and marble.	3	0	1	1(0.2)

Table 11: Reference table for scoring Item 4 and the frequency of response type

Classification	Students' Observations (Rules)	Originality	Number of Responses		
			Gifted students (N=53)	Regular Students (N=409)	Total (N=462)
Constant Sum	$a+b$ is constant	1	8	2	10(2.2)
	The mean value of a and b is constant	3	2	0	2(0.4)
	$b+c$ is constant	2	2	3	5(1.1)
	The sum of the lengths of the edges above the water surface is constant	3	1	1	2(0.4)
Variation	One edge decreases by the amount the other increases	3	0	1	1(0.2)
	When one edge increases, the other decreases	1	0	11	11(2.4)
	The lengths of the edges vary	3	0	1	1(0.2)
	The length of the edge of the water surface becomes greater	3	0	2	2(0.4)
	When one edge becomes 0, the other edge becomes twice its original length	3	1	0	1(0.2)
Slope	when the slope smeller, the area of the water surface becomes smeller	3	0	1	1(0.2)
	$\angle$BDE=$\angle$CAF	3	0	1	1(0.2)
	A transparent flask in the shape of a right rectangular prism is not circle	3	3	0	3(0.7)
	The side plane is a rectangular, when the slope is right angle	3	0	1	1(0.2)
Range	The limit of the length of an edge is 15cm	3	0	1	1(0.2)

Table 11 (continued.)

Shape of water surface	The water surface (upper) and the base are rectangles	3	2	2	4(0.9)
	The water surface is a rectangle or a quadrangle	1	1	12	13(2.8)
	The shape of the side plane changes from trapezoid to triangle	0	8	20	28(6.1)
	The side view is a trapezoid	1	2	5	7(1.5)
	The shape of the water surface changes	3	0	2	2(0.4)
Area	The total area of the side faces does not change	3	2	0	2(0.4)
	The area of the water surface changes	3	2	0	2(0.4)
	The area of the water surface becomes larger	2	0	5	5(1.1)
	The total surface area changes	3	0	1	1(0.2)
Volume	The volume does not changes	0	17	27	44(9.5)
Others	The weight of the water does not change	0	8	22	30(6.5)
	The is a fixed point, when viewed horizontally	1	0	8	8(1.7)
	The form of the water is a quadrangular prism	3	1	1	2(0.4)
	The form of the water changes from a cuboid to a triangular prism	3	2	0	2(0.4)
	The form of the water changes	1	0	7	7(1.5)
	The segment MN is a fixed segment	3	0	2	2(0.4)

Table 12: Reference table for scoring Item 5 and the frequency of response type

Classification	Characteristics / Solids	A	B	C	D	E	F	G	H	Originality	Number of Responses		
											Gifted students (N=53)	Regular Students (N=409)	Total (N=462)
Shape of faces(side and bases)	Having only one base		/					/		0	23	47	70(15.1)
	Side being a triangle		/					/		0	36	148	184(39.8)
	Surface being flat	/	/			/		/		1	3	10	13(2.8)
	Having four faces		/	/						0	11	28	39(8.4)
	Viewed shape from the top being a polygon	/	/			/		/		0	8	8	16(3.5)
	Base is not circle	/	/	/		/		/		3	1	0	1(0.2)
	Base being same to side face	/	/							1	6	0	6(1.3)

Table 12 (continued.)

Number of edges, vertices, faces, angeles and relations among them	Number of edges=(Number of edges of the base)× 2		/					/		3	0	4	4(0.9)
	Edges having straight lines only	/	/					/		1	2	10	12(2.6)
	Having Vertices	/	/			/		/		0	4	49	53(11.5)
	Having Edges	/	/			/		/		3	1	0	1(0.2)
	The length of side edges being the same	/	/	/		/	/	/	/	3	1	0	1(0.2)
	Number of Vertices = (Number of edges of the base) + 1		/					/		3	0	2	2(0.4)
	Number of face = (Number of edges of the base) + 1		/					/		3	0	2	2(0.4)
	Number of edge of the base = Number of side faces		/					/		3	1	2	3(0.7)
	Number of vertices = (Number of edges) × 2/3	/	/							3	1	0	1(0.2)
	Number of vertex of the base is odd		/			/				3	1	0	1(0.2)
Shape of a projection	Shape of shadow being a triangle		/					/		0	2	22	24(5.2)
	Viewed shape from the top being a polygon	/	/			/		/		3	0	4	4(0.9)
Shape of a cross section	Cross section parallel to the base being similar		/				/	/		3	0	2	2(0.4)
	Cross section perpendicular to the base being a rectangle		/				/	/		3	3	0	3(0.7)
	Cross section not parallel to the base being an ellipse		/					/		3	0	2	2(0.4)
	Cross section perpendicular to the base through the vertex being a triangle		/					/		3	0	4	4(0.9)
Pyramid	Pyramid		/					/		0	22	173	195(42.2)
Shape of a development of the solid	Not a solid of revolution	/	/	/		/		/		1	7	5	12(2.6)
Volume	Having volumed	/	/	/	/	/	/	/	/	1	8	4	12(2.6)
Others	Having development figure	/	/	/		/	/	/	/	3	1	0	1(0.2)
	Viewed shape from development figure being a triangle, the number is the same		/					/		3	0	3	3(0.7)