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Two-dimensional Coupled Moisture and Heat Flow Model and Sensitivity Analysis

이차원 복합적 습기와 열흐름의 분석모델과 민감도 분석

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요 지

포장 시스템 내에서의 습기흐름과 열흐름은 상호간에 복합적인 작용을 하는 과정들로 인식되어 왔다. 습기의 흐름과 열흐름에 기인한 포장 내에서의 습기와 온도의 분포는 계절적으로 변화할 뿐만 아니라 수직 그리고 수평적으로도 변화한다. 이 논문은 불포화토에서의 이차원 복합적인 습기와 열흐름에 대해서 유한요소법을 사용한 분석모델을 제시한다. 모델을 검증하기 위해 모델에 의한 분석결과는 Canada Alberta에 소재한 GEO-SLOPE사에 의해 개발된 소프트웨어인 GEO-SLOPE의해 분석된 결과와 비교하였다. 그리고 모델에서 사용된 입력데이터가 모델분석에 미치는 영향을 알아보기 위해 ASTM 방법에 의한 민감도 분석을 수행하였다.

Abstract

Moisture flow and heat flow within pavement systems have been recognized as coupled processes with complex interactions between them. The distribution of moisture and temperature within pavement due to the moisture flow and heat flow varies not only seasonally but also vertically and horizontally. This paper presents an analysis model by the finite element method for the two-dimensional coupled moisture and heat flow in unsaturated soils. To test the model the analysis result by the model is compared with the analysis result by the software, GEO-SLOPE developed by GEO-SLOPE International Ltd. in Alberta, Canada. And a sensitivity analysis using ASTM method is performed to identify how model inputs affect the modeling analysis.

Keywords : Heat flow, Moisture flow, Pavement, Sensitivity analysis, Soil-water characteristic curve, Unsaturated soil

1. Introduction

The distribution of moisture and temperature within pavement systems that are usually in unsaturated state varies seasonally depending on environmental conditions. The variations of moisture and temperature i.e. the moisture flow and heat flow are not independent processes but dependent processes affecting each other. Therefore, mathematical models that fully couple moisture and heat flow are required to model moisture and temperature variation within unsaturated soils.

There have been many attempts to analyze the coupled heat and moisture flow by many researchers such as Harlan (1973), Guymon and Luthin (1974), Jame and Norum (1980), Guymon et al. (1981, 1993), and Newman and Wilson (1997). These are the attempts for one-dimensional coupled moisture and heat flow analysis. Currently, there is no model for two-dimensional coupled moisture and heat flow analysis. However, it is considered that two-dimensional analysis may be

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needed more than one-dimensional analysis in practice, because water content and temperature might change horizontally and vertically. If there are some drainage structures within pavement, a hydraulic gradient will occur and thus cause a horizontal moisture flow which in turn causes heat flow by advection in horizontal direction. In this case, water content and temperature will change horizontally as well as vertically. Therefore, it may cause considerable errors to use the calculation results by one-dimensional analysis for the entire pavement section. The present model described in this paper allows for a fully coupled analysis of two-dimensional moisture flow and heat flow in unsaturated soils.

2. Model

The coupled moisture and heat flow analysis consists of complicated processes as shown in the following sections. As the mathematical equations describing the coupled moisture and heat flow are very complicated, the solution is obtained by a numerical method. Figure 1 shows the structure of the model. The model consists of four components which are command window, input, output and the component for analysis of coupled heat and moisture flow.

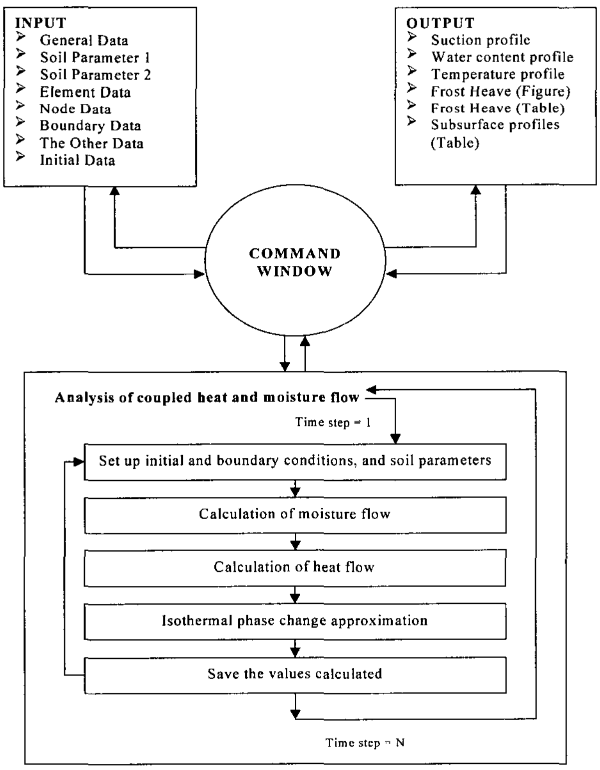


Fig. 1. Structure of model

The program language used to develop the model is MATLAB which is a high-performance language for technical computing. The types of elements which can be used in the model are the linear triangle with three nodes and the linear rectangle with four nodes.

The analysis procedure of the model is shown in the lower part of Figure 1. First, initial and boundary conditions and soil parameters are set up. Second, moisture flow is analyzed and third, heat flow is analyzed. The results analyzed by moisture flow and heat flow are then corrected by an isothermal phase change approximation (the mathematical calculation process for the change of variables due to water phase change, water to ice or ice to water). Finally, the values calculated are saved. This procedure continues until the end of each time step.

2.1 Mathematical Equations

Two partial differential equations are used in the model. They are the moisture flow equation which is an application of mass conservation and the heat flow equation which is an application of the principle of energy conservation. The following equation is the two-dimensional moisture flow equation in an unsaturated soil.

$$\frac{\partial}{\partial x}\left(k_x(\Psi)\frac{\partial \Psi}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y(\Psi)\left(\frac{\partial \Psi}{\partial y}+1\right)\right)$$
$$=c(\Psi)\frac{\partial \Psi}{\partial t}+\frac{\rho_i}{\rho_w}\frac{\partial \theta_i}{\partial t} \tag{1}$$

where $k_x(\Psi)$ and $k_y(\Psi)$ are the hydraulic conduc tivities, Ψ is the suction, $c(\Psi)$ is specific moisture capacity(the slope of the relationship curve between volumetric water content and suction), ρ_i is the density of ice, ρ_w is the density of liquid water, and θ_i is the volumetric ice content. The specific moisture capacity is defined as

$$c(\Psi)=\frac{d\theta}{d\Psi} \tag{2}$$

In the equation (1) the specific moisture capacity $c(\Psi)$ and the hydraulic conductivities ($k_x(\Psi)$ and $k_y(\Psi)$) are not constants but variables depending on suction value.

The specific moisture capacity can be determined from the soil-water characteristic curve and the hydraulic conductivities can be determined from the permeability function which shows suction versus hydraulic conductivity relationship. In the model, two sets of equations were used. One is soil-water characteristic curve by Gardner (1958) and permeability function by Guymon et al. (1993). The other set is soil-water characteristic curve by Fredlund and Xing (1994) and permeability function by Fredlund et al. (1994). Solving a problem, one set of them can be selected by users.

The following equation is the two-dimensional heat flow equation.

$$\frac{\partial}{\partial x}\left(K_T\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_T\frac{\partial T}{\partial y}\right)-v_x C_w\frac{\partial T}{\partial x}-v_y C_w\frac{\partial T}{\partial y}$$
$$=C_m\frac{\partial T}{\partial t}-L\frac{\rho_i}{\rho_w}\frac{\partial \theta_i}{\partial t} \quad (3)$$

where K_T is thermal conductivity of material, T is temperature, v_x and v_y are the velocities of the water flow in x and y direction, C_w is the heat capacity of water(the amount of heat energy required to raise the temperature of a unit water volume by one degree), C_m is the volumetric heat capacity of material(the amount of heat energy required to raise the temperature of a unit material volume by one degree), L is the latent heat of fusion of water which is the thermal energy required for a change of phase, ρ_i and ρ_w are the densities of the ice and water, and θ_i is the volumetric ice content. The thermal parameters in equation (3) can be computed from DeVries (1966) relationship:

$$C_m=C_w\theta_u+C_i\theta_i+C_s(1-\theta_0) \quad (4)$$

$$K_T=K_w\theta_u+K_i\theta_i+K_s(1-\theta_0) \quad (5)$$

where θ_u is unsaturated volumetric water content, θ_0 is porosity of soil, C_w, C_i, and C_s are volumetric heat capacities of water, ice, and soil, and K_w, K_i, and K_s are thermal conductivities of water, ice, and soil. The velocity of the water flow is calculated from Darcy's law:

$$v_x=-k_x\frac{\partial h}{\partial x}\ ,\qquad v_y=-k_y\frac{\partial h}{\partial y} \quad (6)$$

The last terms of equations (1) and (3) are sink or source terms due to the change of ice phase. These terms are approximated by an isothermal phase change process presented by Hromadka et al. (1981). Then, dropping off the last terms of equations (1) and (3) becomes

$$\frac{\partial}{\partial x}\left(k_x(\Psi)\frac{\partial \Psi}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y(\Psi)\left(\frac{\partial \Psi}{\partial y}+1\right)\right)$$
$$=c(\Psi)\frac{\partial \Psi}{\partial t} \quad (7)$$

$$\frac{\partial}{\partial x}\left(K_T\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_T\frac{\partial T}{\partial y}\right)-v_x C_w\frac{\partial T}{\partial x}$$
$$-v_y C_w\frac{\partial T}{\partial y}=C_m\frac{\partial T}{\partial t} \quad (8)$$

2.2 Numerical Equations

Equation (7) can be expressed as the following numerical equation which was derived by the finite element method.

$$([C(\Psi)]+\omega\triangle t[K(\Psi)])\{\Psi\}_{t+\triangle t}$$
$$=([C(\Psi)]-(1-\omega)\triangle t[K(\Psi)])\{\Psi\}_t$$
$$+\triangle t((1-\omega)\{F\}_t+\omega\{F\}_{t+\triangle t}) \quad (9)$$

where ω is the value depending on the numerical method, $\triangle t$ is time interval and vector $\{F\}$ is the specified rate of the moisture flow. Matrices $[C(\Psi)]$ and $[K(\Psi)]$ are the global matrices which are obtained by combining the following element matrices $[C^{(e)}(\Psi)]$ and $[K^{(e)}(\Psi)]$.

$$[K^{(e)}(\Psi)]$$
$$=\int\int_A\begin{bmatrix}\frac{\partial N_1}{\partial x} & \frac{\partial N_1}{\partial y}\\ \vdots & \vdots\\ \frac{\partial N_n}{\partial x} & \frac{\partial N_n}{\partial y}\end{bmatrix}\begin{bmatrix}k_x(\Psi) & 0\\ 0 & k_y(\Psi)\end{bmatrix}\begin{bmatrix}\frac{\partial N_1}{\partial x} & \cdots & \frac{\partial N_n}{\partial x}\\ \frac{\partial N_1}{\partial y} & \cdots & \frac{\partial N_n}{\partial y}\end{bmatrix}dxdy \quad (10)$$

$$[C^{(e)}(\Psi)]=c(\Psi)\frac{A}{n}\begin{bmatrix}1 & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & 1\end{bmatrix} \quad (11)$$

where the values N are the interpolation functions, A is area of element and n is the number of elements.

The numerical equation for equation (8) by the finite element method is

$$([C]+\omega\triangle t[D])\{T\}_{t+\triangle t}=([C]-(1-\omega)\triangle t[D])\{T\}_t$$
$$+\triangle t((1-\omega)\{F\}+\omega\{F\}_{t+\triangle t}) \quad (12)$$

where vector $\{F\}$ is the specified rate of the heat flow. The element matrices $[C^{(e)}]$ and $[D^{(e)}]$ for global matrices $[C]$ and $[D]$ are

$$[C^{(e)}] = C_m \frac{A}{n} \begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix} \tag{13}$$

$$[D^{(e)}]$$

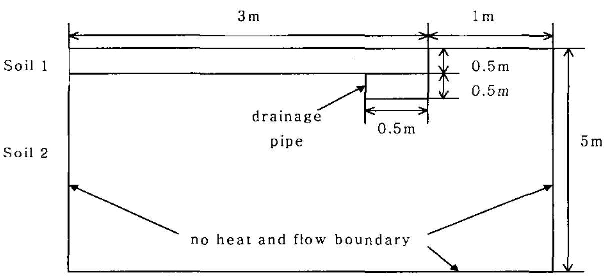
$$= \int\int_A \begin{bmatrix} \frac{\partial N_1}{\partial x} & \frac{\partial N_1}{\partial y} \\ \vdots & \vdots \\ \frac{\partial N_n}{\partial x} & \frac{\partial N_n}{\partial y} \end{bmatrix} \begin{bmatrix} K_T & 0 \\ 0 & K_T \end{bmatrix} \begin{bmatrix} \frac{\partial N_1}{\partial x} & \cdots & \frac{\partial N_n}{\partial x} \\ \frac{\partial N_1}{\partial y} & \cdots & \frac{\partial N_n}{\partial y} \end{bmatrix} dxdy$$

$$= \int\int_A \begin{bmatrix} N_1 & N_1 \\ \vdots & \vdots \\ N_n & N_n \end{bmatrix} \begin{bmatrix} v_x C_w & 0 \\ 0 & v_y C_w \end{bmatrix} \begin{bmatrix} \frac{\partial N_1}{\partial x} & \cdots & \frac{\partial N_n}{\partial x} \\ \frac{\partial N_1}{\partial y} & \cdots & \frac{\partial N_n}{\partial y} \end{bmatrix} dxdy \tag{14}$$

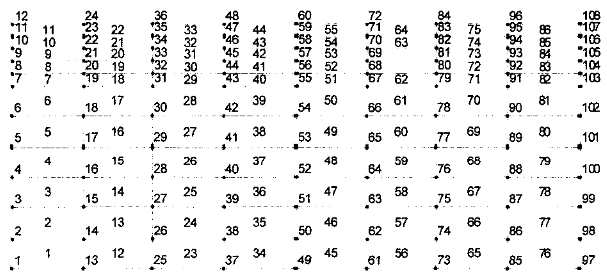
3. Model Verification

The verification of the two-dimensional model presented here was performed indirectly or partly by comparing it with GEO-SLOPE which is a software developed by GEO-SLOPE International Ltd. in Alberta, Canada. GEO-SLOPE consists of five computer programs. SEEP/W and TEMP/W among them were used for the verification. SEEP/W is a seepage analysis program that models both saturated and unsaturated flow. TEMP/W is a program that models the thermal changes in the ground.

Figure 2(a) shows the problem domain for numerical analyses that consists of two kinds of soil and a drainage pipe. The problem domain was divided into 86 elements and has 108 nodes as shown in Figure 2(b). Figure 3 shows the analysis results for water content changes with time by the two models. The figures on the left side are the analysis results by SEEP/W and the figures on the right side are the analysis results by the model presented here. Figure 4 shows the analysis results for temperature changes with time by the two models. The figures on the left side are the analysis results by TEMP/W and the figures on the right side are the analysis results by the model presented here. The analysis results of water content and temperature by the two models at each time show good agreement as shown in Figures 3 and 4.

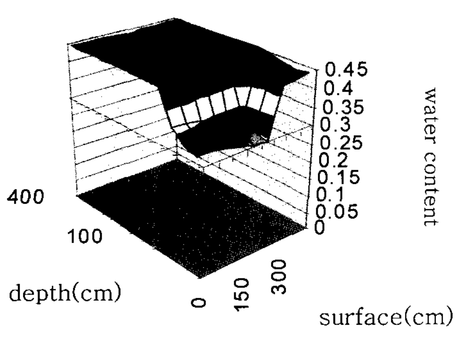


(a) Problem domain

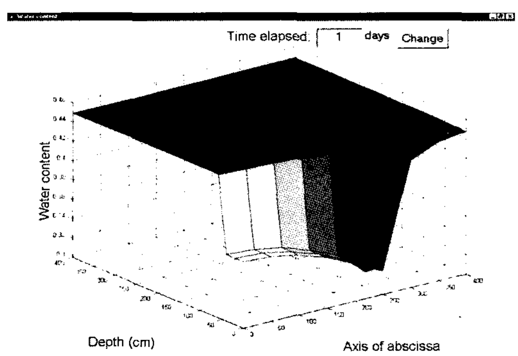


(b) Discretization of problem domain

Fig. 2



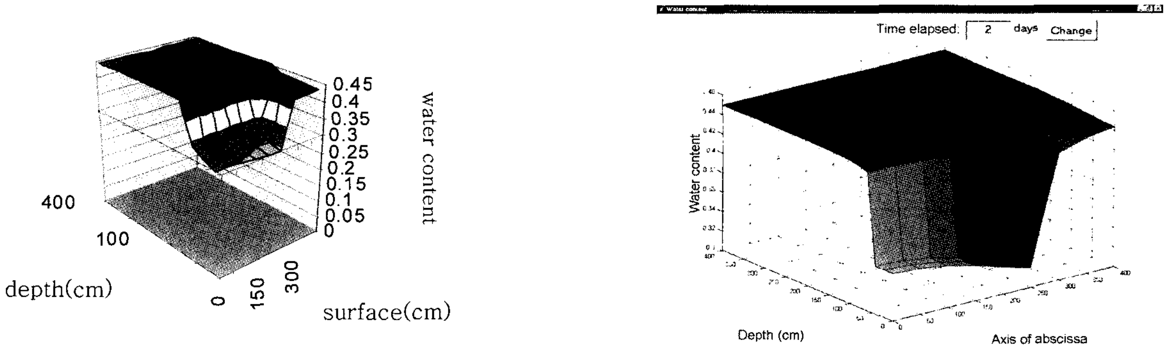
SEEP/W



New model

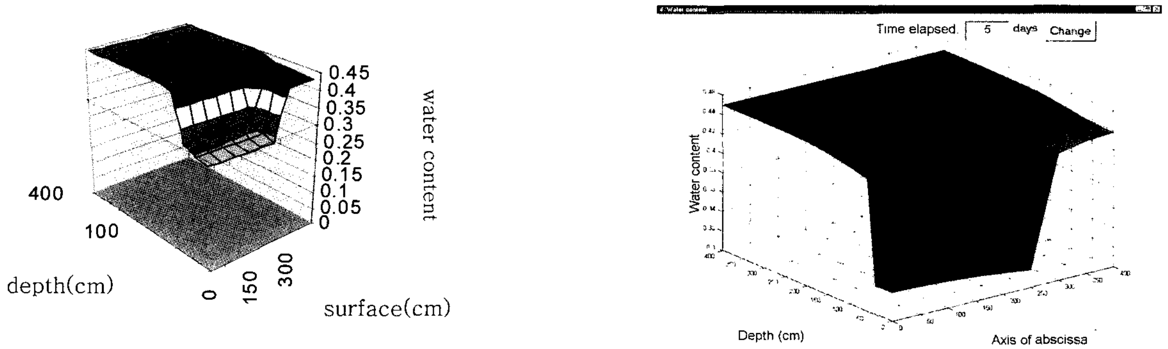
(a) Water content on 1st day

Fig. 3. Comparison of water content changes by two numerical models



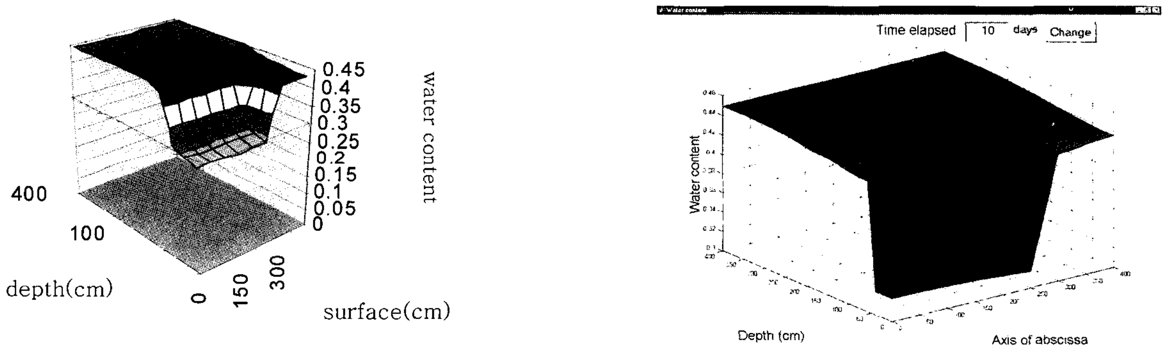
SEEP/W New model

(b) Water content on 2nd day



SEEP/W New model

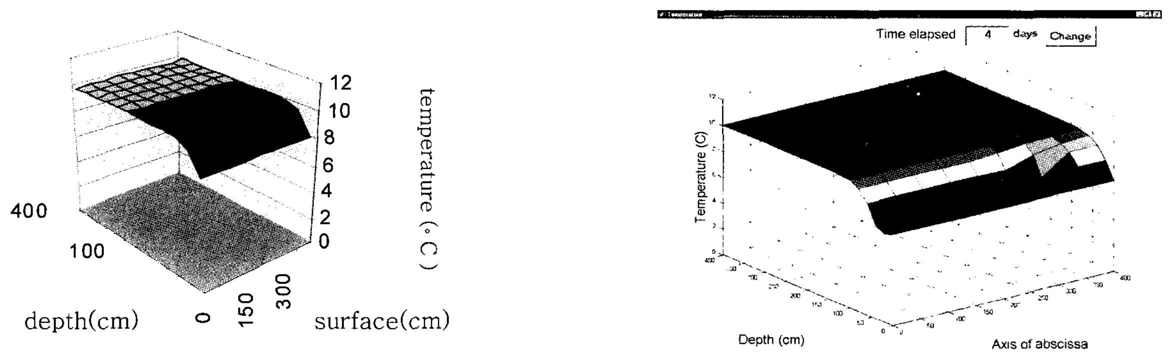
(c) Water content on 5th day



SEEP/W New model

(d) Water content on 10th day

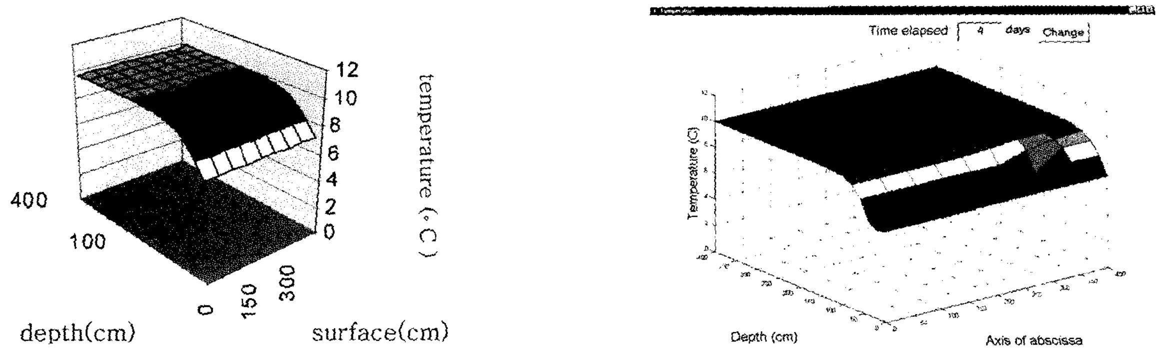
Fig. 3. (Continue) Comparison of water content changes by two numerical models



TEMP/W New model

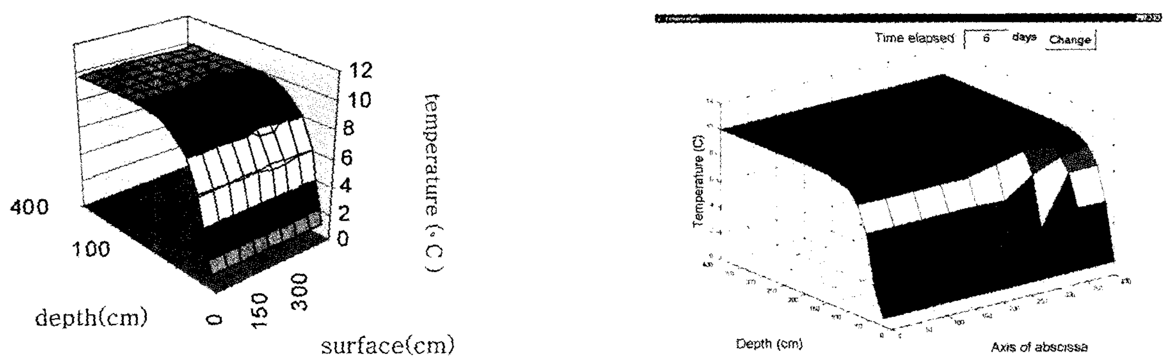
(a) Temperature change on 2nd day

Fig. 4. Comparison of temperature changes by two numerical models



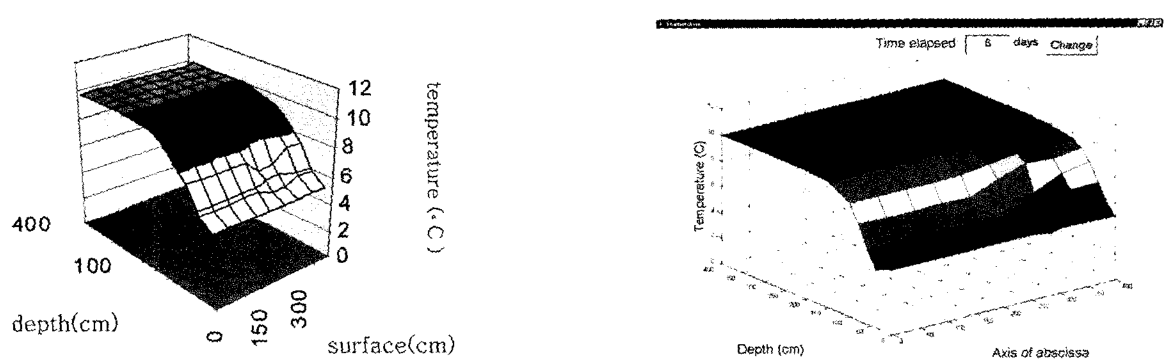
TEMP/W　　New model

(b) Temperature change on 4th day



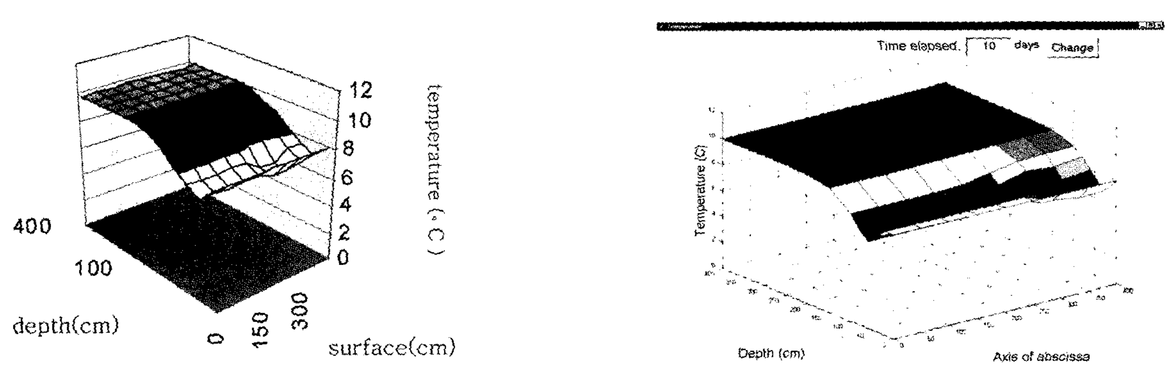
TEMP/W　　New model

(c) Temperature change on 6th day



TEMP/W　　New model

(d) Temperature change on 8th day



TEMP/W　　New model

(e) Temperature change on 10th day

Fig. 4. (Continue) Comparison of temperature changes by two numerical models

4. Sensitivity Analysis

A sensitivity analysis was performed to identify how model inputs affect the modeling analysis. The ASTM method is used for the sensitivity analysis of the model. The first step of a sensitivity analysis is to identify the model inputs that are likely to affect the model's conclusions. Then the model is run with the standard input values and is rerun with the new input values where one input value (or one group of input values) is varied while the rest are constant. Some input values should be varied geometrically while others should be varied arithmetically. The types of sensitivity of the model to model inputs are classified with four types of sensitivity depending on whether the changes to the residuals and model's conclusions are significant or insignificant. A residual is the difference between the output computed using the standard input values and the output computed using the new input values. The four types of sensitivity are described in the following and summarized in Figure 5.

Type I Sensitivity

When variation of an input causes insignificant changes in the residuals as well as the model's conclusions, then that model has a Type I Sensitivity to the input. Type I Sensitivity is of no concern because regardless of the value of the input, the conclusion will remain the same.

Type II Sensitivity

When variation of an input causes significant changes in the residuals but insignificant changes in the model's conclusions, then that model has a Type II Sensitivity to the input. Type II Sensitivity is of no concern because regardless of the value of the input, the conclusion will remain the same.

Type III Sensitivity

When variation of an input causes significant changes to both the residuals and the model's conclusions, then that model has a Type III Sensitivity to the input. Type III Sensitivity is of no concern because, even though the model's conclusions change as a result of variation of the input, the parameters used in those simulations cause the model to become uncalibrated.

Type IV Sensitivity

When variation of an input causes insignificant changes in the residuals but significant changes in the model's conclusions, then that model has a Type IV Sensitivity to the input. Type IV Sensitivity can invalidate model results because over the range of that parameter in which the model can be considered calibrated, the conclusions of the model change.

Type IV Sensitivity generally requires additional data collection to decrease the range of possible values of the parameter.

The sensitivity analyses were performed on the prediction of frost heave at surface and the predictions of water content and temperature at 2 m below the surface. Figure 6 shows a problem domain for the sensitivity analyses. The finite elements are the same as those of the previous example. The soil within the domain is homogeneous and isotropic. The model is run for the

Change in conclusions	Change in residuals: Insignificant	Change in residuals: Significant
Insignificant	Type I	Type II
Significant	Type IV	Type III

Fig. 5. Summary of sensitivity types

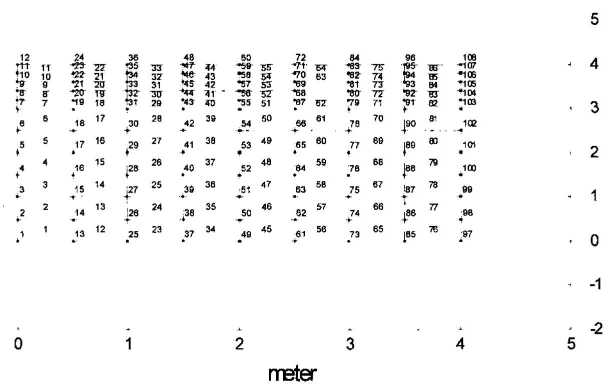


Fig. 6. Problem domain for sensitivity analysis

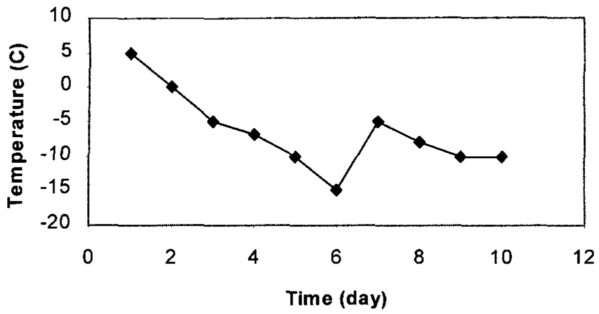


Fig. 7. Temperature changes at the upper boundary

period of 10 days. As a boundary condition there is 1 cm^3/hr infiltration on the 1^{st} day at the upper boundary. Figure 7 shows the temperature changes for 10 days at the upper boundary. The temperature at the bottom boundary was held constant for 10 days. The results of the sensitivity analyses are shown in Table 1. The sensitivity types for the input parameters were determined from the results of the sensitivity analysis.

Water content has Type III sensitivity for Kw, air-entry value, Cr, n, soil porosity, saturated permeability and initial water content. Water content is affected by initial water content and the soil parameters such as Kw, air-entry value, Cr, n, soil porosity and saturated permeability which are the parameters which affect the soil's ability to retain water and the soil's permeability. Temperature has Type III sensitivity for time increment, Kw, air-entry value, Cr, m, soil porosity, heat capacity, thermal conductivity, saturated permeability, bottom boundary temperature, initial water content. Temperature is affected by soil thermal parameters, initial and boundary temperature, and initial water content. Kw, air-entry value, Cr, m, soil porosity and saturated permeability are the parameters which affect the amount of moisture in the soil. They also affect the soil temperature due to the latent heat of the water.

5. Conclusions

A model to analyze two-dimensional coupled moisture and heat flow was presented. It is considered that two-dimensional analysis may be needed more than

Table 1. Sensitivity analysis results

Parameter	Standard input value	Range of variation	Water content sensitivity Type	Temperature sensitivity Type
Time increment (hour)	3	1 ~ 6	Type I	Type III
Parameters for SWCC				
- Gardner equation				
Aw	0.0045	0.0039 ~ 0.0051	Type I	Type I
a	0.61	0.59 ~ 0.63	Type I	Type I
Kw	1.00E-12	1E-13 ~ 1E-12	Type III	Type III
b	5.4	5.2 ~ 5.6	Type I	Type I
- Fredlund equation				
air-entry value	10	8 ~ 12	Type III	Type III
Cr	1500	500 ~ 2500	Type III	Type III
a	350	310 ~ 390	Type I	Type I
n	0.59	0.53 ~ 0.65	Type III	Type I
m	1.74	1.64 ~ 1.84	Type I	Type I
soil porosity	0.45	0.43 ~ 0.47	Type III	Type III
soil dry density (g/cm^3)	1.37	1.17 ~ 1.57	Type I	Type I
unfrozen water content	0.15	0.13 ~ 0.17	Type I	Type I
heat capacity (cal/cm^3-C)	0.2	0.18 ~ 0.22	Type I	Type III
thermal conductivity (cal/cm-hr-C)	5	4.8 ~ 5.2	Type I	Type III
saturated permeability (cm/hr)	0.03	0.003 ~ 0.3	Type III	Type III
boundary flux (cm^3/hr)	1	0.6 ~ 1.4	Type I	Type I
bottom boundary temperature (C)	10	8 ~ 12	Type I	Type III
field capacity	0.3	0.28 ~ 0.32	Type I	Type I
initial water content	0.43	0.41 ~ 0.45	Type III	Type III
initial temperature (C)	10	9 ~ 11	Type I	Type III

one-dimensional analysis in practice, because water content and temperature might change horizontally or vertically.

The verification for the analysis of water content and temperature variation within pavement has been tested. The results by the model presented here show good agreement with the results by GEO-SLOPE. The model can thus be used as an investigative tool or others for studying the effects of the various factors that affect the subsurface moisture and temperature.

A sensitivity analysis using ASTM method was performed to identify how model inputs affect the modeling analysis. The model has Type I, Type II and Type III Sensitivity to the inputs.

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