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Development of Strain-softening Model for Geosynthetic-involved Interface Using Disturbed State Concept

DSC를 이용한 토목섬유가 포함된 경계면의 변형율 연화 모델 개발

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요 지

본 연구에서는 DSC를 이용한 구성방정식을 이용하여 토목섬유 사이의 접촉전단 응력과 변위와의 관계를 모델링하였다. DSC 모델은 두 개의 기준 상태, 즉 상대적으로 손상되지 않은 RI 상태와 완전히 파괴된 FA 상태와 한가지의 교란 함수로 구성된다. 본 모델은 통합된 모델로서, RI 상태를 탄성-완전 소성 모델, 계층적 단일 항복곡면 (HiSS) 모델 등 다양한 모델을 이용하여 모사할 수 있다. 한편 본 모델은 탄성과 소성 변위를 동시에 고려할 수 있다는 장점을 가지고 있다. 4가지의 대형 직접전단 시험으로부터 측정된 자료와 측정자료로부터 도출된 모델 변수를 이용하여 재해석한 결과를 서로 비교하여, 둘 사이의 비교 결과가 상당히 일치함을 발견하였으며, 특히 표면이 매끄러운 지오멤브레인의 접촉면에서는 매우 상관관계를 보였다. 비록 표면이 거친 지오멤브레인이 포함된 접촉면에서는 예측 최대 전단강도가 실험결과와 약간의 차이를 보이기는 하였지만, 전체적으로 본 모델이 최대 전단응력이 나타나는 변위점과 대변형에서의 전단강도를 상당히 정확히 예측하였으며, 이를 통해 본 모델이 변형율 연화 현상을 보이는 접촉면 전단거동의 모델링에 유용함을 확인하였다.

Abstract

In this study, a constitutive model called the disturbed state concept (DSC) was modified to be applied to the interface shear stress-displacement relationship between geosynthetics. The DSC model is comprised of two reference states, namely the relative intact (RI) and the fully adjusted (FA) state, and one function, namely the disturbance function. This model is a unified approach and can allow for various models as an RI state such as elastic-perfectly plastic model, hierarchical model, and so on. In addition, by using this model, the elastic and plastic displacements can be considered simultaneously. Comparisons between the measured data and predicted results through the parameters determined from four sets of large direct shear tests showed good agreements with each other, especially for the smooth geomembrane-involved interface. Although there are slight differences at peak shear strength for textured geomembrane-involved interface, this model can still be useful to predict the position of displacement at peak strength and the large displacement (or residual) shear strength.

Keywords : Disturbed state concept(DSC), Geosynthetic, Interface, Modeling, Shear behavior, Strain-softening effect

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1. Introduction

It is required for municipal and hazardous waste containment facilities to be equipped with the liner and cover system containing combinations of low-permeability liners and liquid collection drainage layers (Stark et al., 1996). The functions of composite liners that are composed of various geosynthetics include the isolation of waste and leachate from the environment. As a result of these geosynthetic liners and covers, many weak interfaces between geosynthetics form within such environmental containment facilities. Another issue about these geosynthetic interfaces is the significant displacements during filling of waste or after the completion of the waste landfilling prior to the overall failure faced by the facility (Filz et al., 2001).

The subject of contacts or interfaces between two similar or dissimilar materials has been studied for a long time. Although a number of models (Herrmann, 1978; Desai et al., 1984; Day and Potts, 1994; Esterhuizen et al., 2001; Zeghal and Edil, 2002) have been proposed for the contact problem, there have not been sufficient researches conducted to fully understand or characterize the shear behavior of the contacts or interfaces. Especially, it should be considered that significant factors influence the interface shear behavior (Desai, 2001). In order to solve the various limitations in the foregoing available models, whereby such factors cannot be allowed for as continuous yielding or hardening, softening leading to post-peak degradation, stiffening, and viscous effect, Desai (2001) proposed the unified DSC (disturbed state concept) model.

The main aim of this paper is to identify the availability of DSC model for the geosynthetic-involved interfaces, especially for the interfaces including both smooth and textured (rough) HDPE (high-density polyethylene) geomembrane. By comparing the measured data and predicted results, it was confirmed that the DSC can be an effective model that is capable of both allowing for strain-softening effect and even covering significant large displacements.

2. Disturbed State Concept

In the DSC, it is presumed that applied force is the cause for the disturbance in the micro-structure of the material (Park and Desai, 2000). And, as a result, an initially relative intact (RI) material transforms continuously and a part of which approaches the fully adjusted (FA) state. By using the disturbance function (D), which provides coupling between the responses of the RI and FA parts, the observed or actual response is expressed in terms of the response of the materials in the RI and FA states, which are referred to as reference states. Fig. 1 shows a schematic of the DSC model and the actual response ("a" in Fig. 1) with respect to RI ("i" in Fig. 1) and FA ("c" in Fig. 1) state.

2.1 Relative Intact (RI) State

The term of "relative" in the RI state is used to mean that it can be defined as a linear relative to nonlinear response, elasto-plastic associative relative to elasto-plastic nonassociative response, and so on. Thus, the RI state may be characterized as elastic, elastic-perfectly plastic, or by using any suitable constitutive model like the HiSS (Hierarchical Single Yield Surface). Fig. 2 shows that various model can be used to describe the RI state in relation to the real response, where the dotted lines indicate the model used for the RI state.

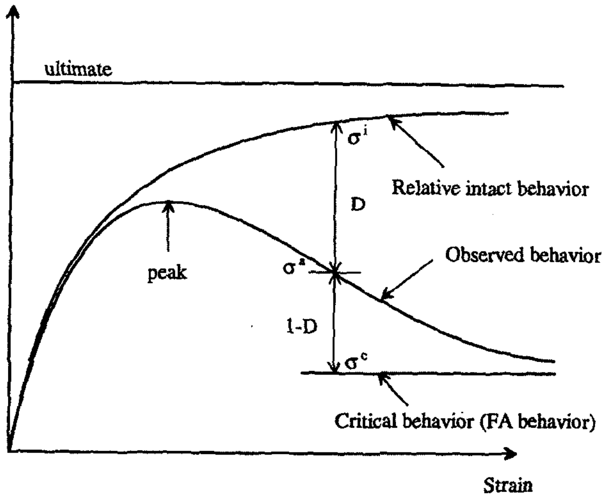


Fig. 1. Schematic of stress-strain behavior and disturbance (Desai et al., 1998)

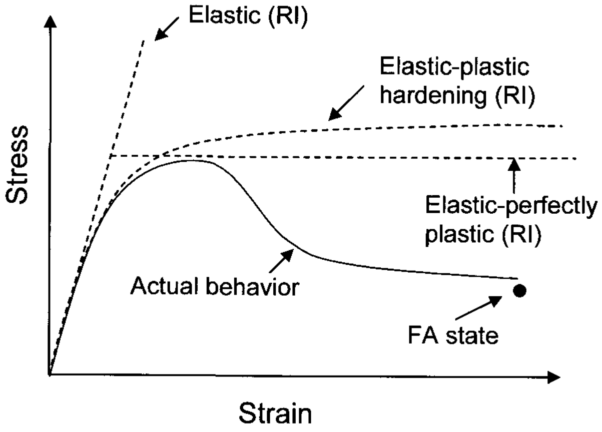


Fig. 2. Stress-strain relations for RI states (Desai, 2001)

As mentioned previously, the RI behavior can be simulated using an elastic or elasto-plastic model. In this paper, elastic-perfectly plastic model for the smooth geomembrane-involved interface and δ_0-version HiSS elasto-plastic model for the texured geomembrane-involved interface are considered, respectively, according to the types of geomembrane surface. The δ_0 version is based on the associative plasticity and isotropic hardening (potential function, Q = yield function, F) rule (Park and Jeon, 1998). In the HiSS model, the yield function, F, is given as :

$$F = \tau^2 + \alpha\sigma_n^n - \gamma\sigma_n^2 \tag{1}$$

where, τ is the interface shear stress, σ_n is the normal stress, and n and γ are material constants. α is the hardening function and it is expressed as :

$$\alpha = \frac{a}{\xi^b} \tag{2}$$

where, a and b are material parameters and ξ implies the trajectory of plastic shear strain or plastic displacement.

The yield function, F, means a continuous set of convex surface during plastic shear deformation. The ultimate shear strength, τ_{ult}, which represents the asymptotic failure stress, is found by setting α equal to zero in Equation (1).

$$\tau_{ult} = \sqrt{\gamma}\ \sigma_n \tag{3}$$

In case of softening, the intact ultimate shear stress is usually set between twentyfive and fifty percent greater than the peak shear stress (Ma and Desai, 1990)

2.2 Fully Adjusted (FA) State

The FA reference state refers to the asymptotic or equilibrium state which best characterizes the material at failure, at its ultimate stages of deformation (Rigby, 1996). For the case of soils, the critical state concept in which unlimited deformations can occur without changes made to the volume can be employed and the residual stress is also taken into account.

At the interface that reaches the critical or residual state, all possible damageable or disturbance-prone asperities are destroyed and the remaining non-damageable asperities provide a constant (residual) shear resistance which can be represented by the following empirical friction law :

$$\tau^c = c_0\ \sigma_n^m \tag{4}$$

where, c_0 and m are material constants. In addition, other similar empirical equations like a Mohr-Coulomb failure envelope can be also used to describe the FA state.

2.3 Disturbance Function

The disturbance function, D, is dependent on plastic displacements. Initially with the absence of disturbance, the material is assumed to be entirely in the RI state. In this case, D is zero (Park and Desai, 2000). The disturbance function, D, used in the present research is expressed by using following function (Equation (5)), commonly used for growth and decay process :

$$D = 1 - \exp(-A\xi^Z) \tag{5}$$

and the disturbance function, D, can be defined based on the interface shear stress as follows :

$$D = \frac{\tau^i - \tau^a}{\tau^i - \tau^c} \tag{6}$$

where i, a, and c denote RI, observed, and FA states, respectively. Both Equations (5) and (6) are required for determining the parameters in the disturbance functions, A and Z.

2.4 Incremental Formulation for the DSC

Based on the equilibrium of forces on the material elements which are composed of the clusters of particles in the RI and FA parts, actual response can be generalized in terms of shear stress :

$$\tau^a = (1-D)\tau^i + D\tau^c \tag{7}$$

where D means the disturbance function mentioned above, and other shear stresses are also defined in Equation (6). Equation (7) can also be expressed in the following manner :

$$d\tau^a \ = \ (1-D)d\tau^i \ + \ Dd\tau^c \ + \ (\tau^c - \tau^i)dD \tag{8}$$

The incremental constitutive equation, Equation (8), is derived to predict the observed response. If there is no change in stresses at FA state, $d\tau^c$ is assumed to be zero.

3. Experimental Data for Geosynthetic Interfaces

The strain-softening DSC constitutive model was developed for geomembrane(GM)/geosynthetic interfaces and verified, using the results of large-direct shear tests by Seo et al. (2002), Jones and Dixon (1998), and Triplett and Fox (2001). Two types of interfaces were considered in the analysis, where one interface involved smooth HDPE geomembrane (S-GM), while the other interface included textured or rough HDPE geomembrane (T-GM). The details about the characteristics of the testing and materials are explained in Seo et al. (2003).

Fig. 3 shows the typical behavior of interface shear stress and displacement for each interface, S-GM and T-GM, respectively. As shown in Fig. 3, strain-softening behavior is generally found in the interface shear stress and displacement relationship. Especially in this research, the entire region is modeled as one unified DSC constitutive equation without dividing entire region into two parts (pre-peak and post-peak).

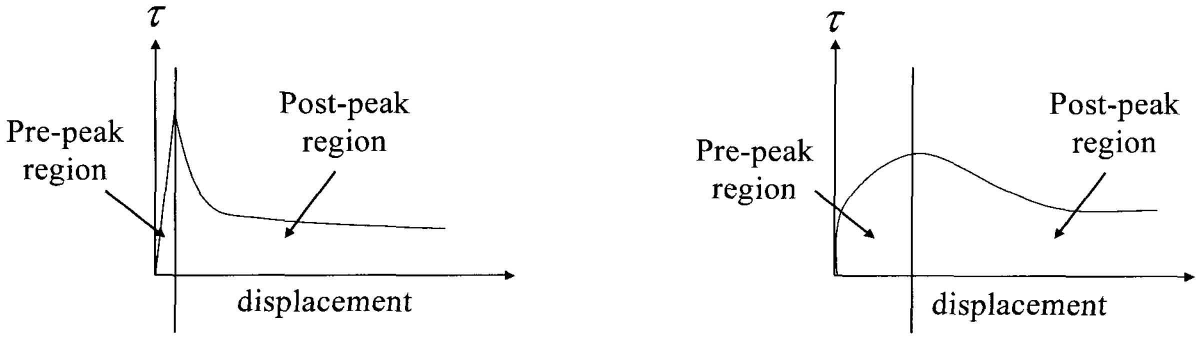
4. Determination of Constants

The proposed DSC model involves a number of material constants which can be determined through a series of large direct shear tests on geosynthetic interfaces. The material constants can be divided into four groups : the elastic constants, the constants for the RI state, the constants describing the critical (residual) state, and the constants of disturbance function (Ma and Desai, 1990).

4.1 Smooth Geomembrane-Involved Interface

4.1.1 Constants for Elastic and Relative Intact (RI) State

Two kinds of elastic-perfectly plastic model were used to describe the behavior of the RI state, respectively. The Mohr-Coulomb model was applied as an RI state model for the first one (Jones and Dixon, 1998), and nonlinear exponential model was used for the other one (Seo et al. 2002). Normally, the elastic perfectly-plastic models



(a) Interface involving S-GM

(b) Interface involving T-GM

Fig. 3. Generalized shear stress and displacement relationship for geosynthetic interface (Seo et al., 2003)

assume that the material remains elastic until the shear stress reaches a certain level, which is called the yield strength. Then, the material deforms plastically under the constant stress (Rigby, 1996). This theoretical yielding boundary is often called a yield surface and is assumed fixed in the stress space. Displacements are assumed to be composed of both a recoverable elastic part and an irreversible or so-called permanent plastic part. The values of elastic stiffness are listed in Table 1 for the two sets of testing results. In back-predicton, the averaging value was used as a representative value.

For the S-GM/geotextile (GT) interface, it is assumed that the interface shear strength envelope for the peak value has zero cohesion intercept. Two types of failure envelopes, linear and nonlinear failure envelope, were calculated for each interface. Among those two types of yield function, more accurate yield functions were chosen through the comparison of correlation coefficients. The yield functions defined for each interface are presented in Table 2.

4.1.2 Constants for Fully Adjusted (FA) State

In case of S-GM/GT interfaces, the large displacement stress or residual stress was regarded to be in a FA state. For the soils, the various empirical equations were used as an expression for critical shear and normal stress. Similarly to the RI state, the linear and nonlinear function were calculated to describe the FA state. The equations describing FA state are listed in Table 3.

4.1.3 Disturbance Function

The constants for the disturbed state are A and Z (Equation (5)). First, disturbance is calculated by using Equation (6) and then plastic displacements are obtained from the measured data using the elastic shear stiffness. Secondly, taking the logarithm twice for both sides of

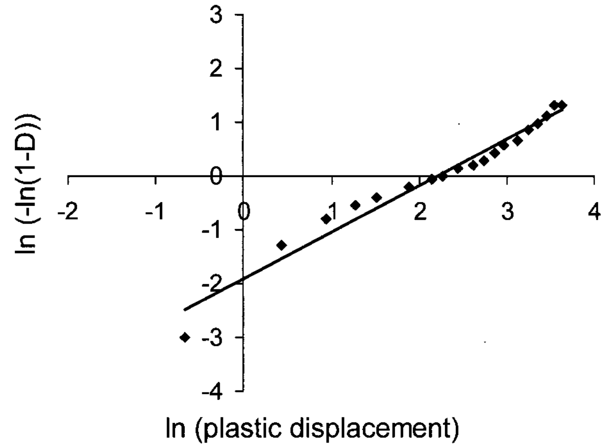


Fig. 4. Representative plot of relationship between ln(-ln(1-D)) and ln(ξ) for σ_n=100kPa (S-GM/GT; Jones and Dixon, 1998)

Table 1. Elastic shear stiffness

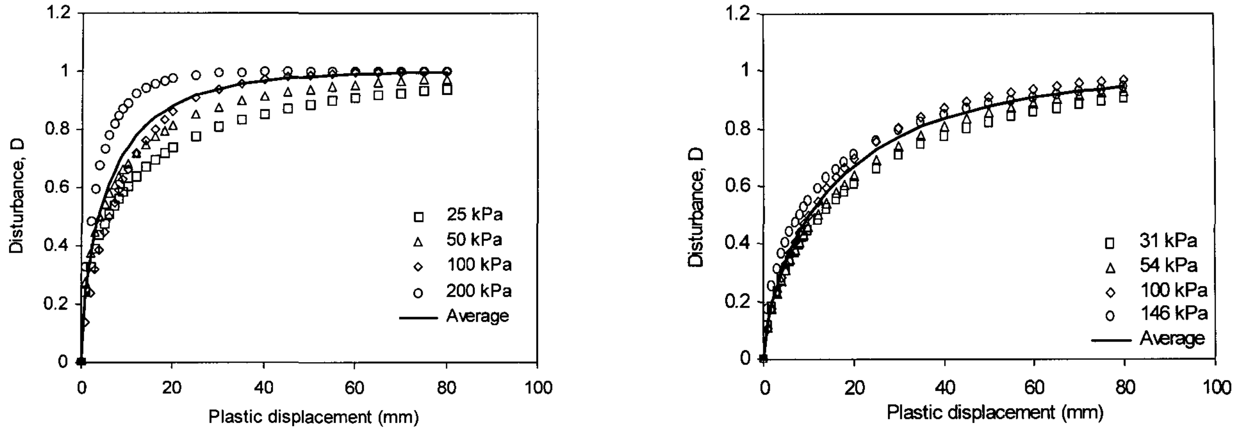
Normal stress (kPa)	Shear stiffness (kPa/mm)	Reference	Normal stress (kPa)	Shear stiffness (kPa/mm)	Reference
25	8.0	Jones and Dixon (1998)	31	7.1	Seo et al. (2002)
50	12.4		54	9.3	
100	28.0		100	6.2	
200	21.9		146	11.4	
average	17.6		average	7.8	

Table 2. Yield functions for S-GM/GT interface

Interface	Yield function (F)	Type of failure envelope	Reference
S-GM/GT	$F = \tan 7.6^\circ\ \sigma_n$	linear envelope	Jones and Dixon (1998)
S-GM/GT	$F = 0.51\ \sigma_n^{0.73}$	nonlinear envelope	Seo et al. (2002)

Table 3. Residual shear stress for the FA state (S-GM involved interface)

Interface	Residual shear stress (kPa)	Type of failure envelope	Reference
S-GM/GT	$F = \tan 5.9^\circ\ \sigma_n$	linear envelope	Jones and Dixon (1998)
S-GM/GT	$F = 0.16\ \sigma_n^{0.91}$	nonlinear envelope	Seo et al. (2002)



(a) The case of Jones and Dixon (1996) (b) The case of Seo et al. (2002)

Fig. 5. Disturbance curves for smooth geomembrane(S-GM)-involved interfaces

Table 4. Parameters for disturbance function, D (A and Z)

Normal stress (kPa)	A	Z	Reference	Normal stress (kPa)	A	Z	Reference
25	0.279	0.527	Jones and Dixon (1998)	31	0.128	0.668	Seo et al. (2002)
50	0.322	0.552		54	0.118	0.716	
100	0.148	0.863		100	0.113	0.785	
200	0.398	0.747		146	0.194	0.619	
average	0.286	0.672		average	0.138	0.697	

Equation (5), the following equation is obtained :

$$\ln(-\ln(1-D)) = \ln(A) + Z\ln\xi \qquad (9)$$

The constants, A and Z, can be determined from the ln(-ln(1-D)) versus ln(ξ) plot, which is shown in Fig. 4. The values of disturbance parameters, A and Z, are summarized and plotted in Table 4 and Fig. 5, respectively.

4.2 Textured Geomembrane-Involved Interface

4.2.1 Constants for Elastic and Relative Intact (RI) State

Plasticity theory assumes the existence of a yield function, F, separating the virgin loading process associated with the generation of plastic displacements and the unloading or reverse loading process. In contrast to the perfectly plastic concept, hardening plasticity allows for the growth or increase in yield strength as a result of the plastic work or plastic displacements (Rigby, 1996).

The elasto-plastic HiSS model was used to represent the RI state for interfaces including textured geomembrane (T-GM), which allows for the plasticity in the pre-peak range. First, the values of initial shear stiffness at each normal stress were calculated for the two sets of testing results (Table 5).

Table 5. Initial shear stiffness for the textured GM-involved interface

Normal stress (kPa)	Initial shear stiffness (kPa/mm)	Reference (Interface)	Normal stress (kPa)	Intial shear stiffness (kPa/mm)	Reference (Interface)
25	35	Jones and Dixon (1998) (T-GM/GT)	72	15	Triplett and Fox (2001) (T-GM/GCL*)
50	50		141	25	
100	60		279	30	
200	95		average	23.3	
average	60				

* GCL : Geosynthetic clay liner

The elasto-plastic model requires plastic parameters and some of the plastic parameters are included in the yield function, F. The ultimate yield strength and the growth function are related with the plasticity. The ultimate state is reached when the growth function, α, is equal to zero, and the ultimate yield strength can be made by using Equation (3), assuming the ultimate shear strength is 1.5 times of peak shear strength in this research, where the value of 1.5 was determined, as mentioned previously at section 2.1, based on the suggestion by Ma and Desai (1990).

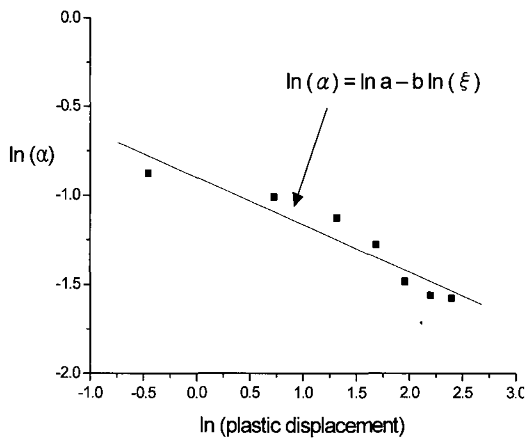


Fig. 6. Representative plot of ln(α) and ln(ξ) for T-GM/GCL (σ_n =279kPa)

After calculating the value of γ at the ultimate state and substituting Equation (2) into Equation (1), interface shear stress can be transformed as a function of plastic displacement, where the value of n is assumed as section 2.1 (Rigby, 1996).

Since α can be calculated with the stresses obtained from the shear tests and also be expressed as a function of ξ, the parameters of a and b were obtained from the relationship of ln(α) versus ln(ξ) plot (Fig. 6, Table 6).

4.2.2 Constants for Fully Aadjusted (FA) State

Similarly to the case of S-GM/GT interfaces, the large displacement shear stress (or residual stress) was chosen as a value at FA state. The equations for the FA state are listed in Table 7.

4.2.3 Disturbance Function

The constants for the disturbance function, A and Z,

Table 6. Lists of parameters, γ, a and b for T-GM involved interface

Normal stress (kPa)	γ	a	b	Reference (Interface)	Normal stress (kPa)	γ	a	b	Reference (Interface)
25	0.43	0.184	0.452	Jones and Dixon (1998) (T-GM/GT)	72	0.72	0.407	0.266	Triplett and Fox (2001) (T-GM/GCL*)
50		0.201	0.207		141		0.419	0.245	
100		0.210	0.187						
200		0.211	0.170		279		0.380	0.212	
average		0.202	0.254		average		0.402	0.241	

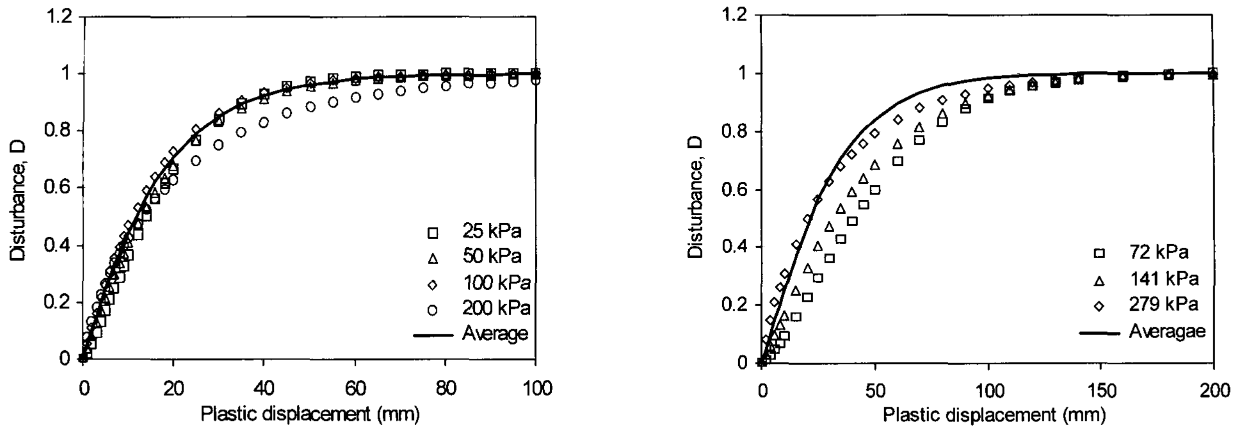
* GCL : Geosynthetic clay liner

Table 7. Residual shear stress for the FA state (T-GM involved interface)

Interface	Residual shear stress (kPa)	Type of failure envelope	Reference
T-GM/GT	$\tau_{FA} = 1.9 + \tan 11.3^{\circ}\ \sigma_n$	linear envelope	Jones and Dixon (1998)
T-GM/GCL	$\tau_{FA} = 0.90\ \sigma_n^{0.76}$	nonlinear envelope	Triplett and Fox (2001)

Table 8. Parameters for disturbance function, D (T-GM involved interface)

Normal stress (kPa)	A	Z	Reference (Interface)	Normal stress (kPa)	A	Z	Reference (Interface)
25	0.024	1.277	Jones and Dixon (1998) (T-GM/GT)	72	0.004	1.389	Triplett and Fox (2001) T-GM/GCL
50	0.040	1.117		141	0.013	1.146	
100	0.056	1.051					
200	0.081	0.838		279	0.047	0.896	
average	0.050	1.071		average	0.021	1.143	



(a) T-GM/GT (Jones and Dixon, 1996)

(b) T-GM/GCL (Triplett and Fox, 2001)

Fig. 7. Disturbance curves for textured geomembrane-involved interfaces

were calculated by using Equation (9) similar to the methods suggested at section 4.1.3. The values of disturbance parameters, A and Z, are summarized and plotted in Table 8 and Fig. 7, respectively.

5. Verification of DSC Model

5.1 Procedures for Back-prediction

The verification of DSC model with respect to large direct shear testsa for geosynthetic-involved interface was made through back-prediction using numerical integration based on the DSC model. The parameters calculated and listed above were used for the back-prediction.

For the interface where S-GM is considered, $d\tau^i$ is assumed to be zero because the elastic and perfectly plastic model is regarded as a RI state. Therefore, the shear stress in the RI state is assumed to be constant for the ranges where the plastic displacements are observed.

At first, the displacements increased until the state reaches the yield function as suggested in Table 2, and then incremental DSC model was applied to allow for the strain-softening effect of the geosynthetic-involved interface shear behavior.

However, in case of T-GM involved interfaces, τ^i is not kept constant and, therefore, $d\tau^i$ is not assumed zero from the initial stage because plastic displacements are found from the initial stage of displacements, that is, even in the pre-peak area. Generally, when DSC model is used, the initial values of the hardening function, α, need to be found. To find the initial value of the hardening function, it is needed to solve the Equation (1), first, for obtaining the initial value of α, assuming the current stress state or initial shear and normal stresses is on the yield surface.

Finally, the residual shear strength is calculated using the suggested equations in Tables 3 and 7, and the constant value for a normal stress was included in the Equation (8) as a FA state on the each incremental step.

5.2 Comparisons between Testing Results and Measured Data

Fig. 8 and Fig. 9 show comparisons of predicted and observed interface shear stress between S-GM and geotextile for four different normal stresses. The values of averaging parameters given in the tables were used in

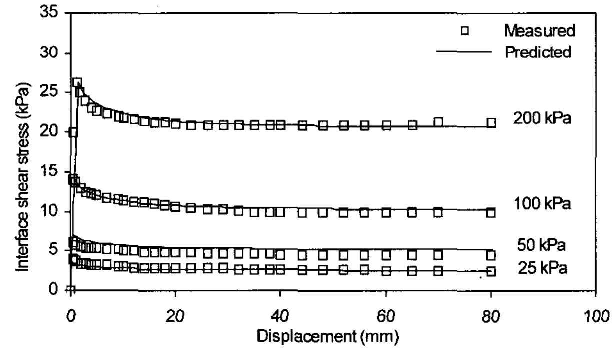


Fig. 8. Back-predicted interface shear stress with displacements for S-GM/GT (Jones and Dixon, 1998)

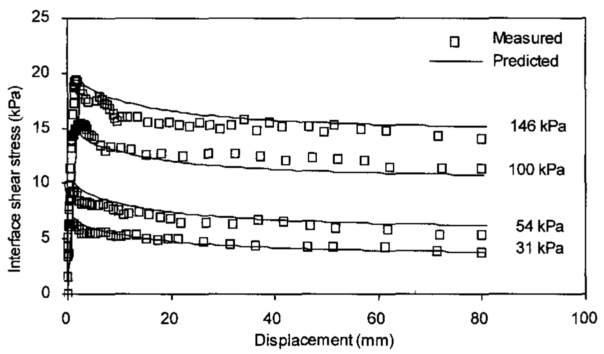


Fig. 9. Back-predicted interface shear stress with displacements for S-GM/GT (Seo et al., 2002)

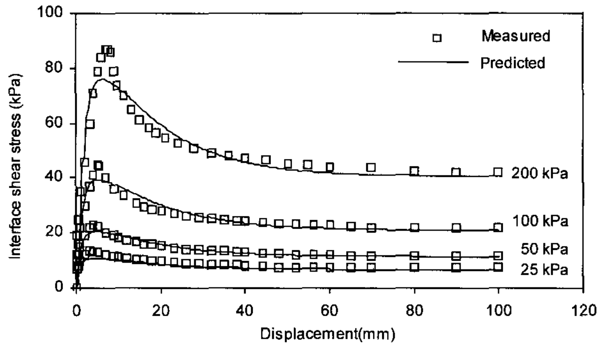


Fig. 10. Back-predicted interface shear stress with displacements for T-GM/GT (Jones and Dixon, 1998)

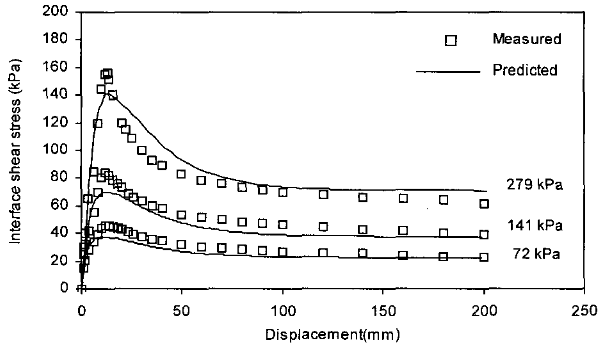


Fig. 11. Back-predicted interface shear stress with displacements for T-GM/GCL (Triplett and Fox, 2001)

the back-prediction.

From the comparison results, the predicted values appear to be well consistent with observed or measured data despite the fact that there exist some differences between observation and prediction for the case of Seo et al. (2002), which indicate that sophisticated or accurate laboratory tests are quite important for making more accurate back-prediction. Overall, the two sets of comparison results matched well for smooth geosynthetic-involved interface, especially for the values of peak and large displacement shear strength.

Fig. 10 and Fig. 11 show the comparison results between measured and predicted data for the interface of T-GM and geosynthetics. However, there are some differences with those of S-GM involved interface as shown in Fig. 8 and Fig. 9.

The back-predictions for T-GM/GT (Fig. 10) are, on the whole, satisfactory with respect to the position of displacements at peak strength and the large displacement shear stress (or residual stress) even though the degree of accuracy with respect to the prediction of peak values is shown to decrease with increasing normal stress. However, it can be confirmed from the back-prediction using the average DSC model parameters that the comparison results show, in general, good agreement between the two sets of data if the followings are considered in the present research; (1) the plasticity is taken into account from the initial stage, (2) the yield function grows continuously because plastic displacements are developed even from the initial stage of work, and (3) only one unified model is applied on the back-prediction for such a severe strain-softening behavior as textured geomembrane-involved interfaces. Similar comparison results are observed for the interface of T-GM/GCL (Fig. 11). Although the degree of differences between measured and predicted peak shear strengths was calculated to be about 15% of the measured value for all cases of normal stresses, the DSC model has an advantage in terms of the implementation of FEM (finite element method) since this model has been used in FEM process (Desai, 2001).

6. Conclusions

The verification of DSC model is conducted for the interface shear behavior between geosynthetics, which shows the characteristics of significant strain-softening.

(1) The DSC model can provide a unified model covering the large displacement area. In the DSC model, material constants can be categorized into four groups; the elastic constants, the constants for the RI state,

the constants describing the critical (residual) state, and the constants for the disturbance function. Most of the parameters have physical meaning and can be determined through the large direct shear tests.

(2) The DSC model can be conveniently implemented in finite-element procedure (FEM) and also predict the plastic displacements from the initial step of displacement space.

(3) For the two types of interfaces that involve smooth geomembrane (S-GM) and textured geomembrane (T-GM), respectively, back-prediction was made by using the averaging value of parameters. From the comparisons between measured data and predicted results, good agreements were observed, on the whole, especially on the smooth geomembrane-involved interface.

(4) The Applicability of DSC model for geosynthetic-including interfaces was confirmed despite some differences found in the textured geomembrane-involved interface with respect to peak interface shear strength. Especially, the position of displacements at peak shear strength and the large displacment (or residual) shear strengths were overall well predicted.

(5) Finally, the DSC model was verified to be a good alternative model for the interface shear behavior that shows the characteristics of high strain-softening, and also be able to provide good means to simulate large displacement (or residual) shear stress with elasticity and plasticity, simultaneously.

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