Error Performance Analysis of DS-CDMA System in Wireless Channel

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Abstract—This paper discusses the spectral efficiency and performance of asynchronous direct sequency spread spectrum multiple access systems strict bandwidth limitation by Nyquist filtering. The signal to noise plus interference ratio(SNIR) at the output from the correlation receiver is derived analytically taking the cross correlation characteristics of spreading sequences into account, and also an approximated SNIR of a simple form is presented for the systems employing Gold sequences. Based on the analyzed result of SNIR, bit error rate performance and spectral efficiency are also estimated. and at last, we analyzed improvement rate using RS, convolution as a method for improving functions.

I. INTRODUCTION

Direct sequence spread spectrum multiple access (DS-SSMA) systems draw much attention because of the attractive features such as easiness of overcoming interference and multipath effect, and capability of asynchronous access.

Since spread spectrum signals should have wide bandwidth and low power spectral density, the effects of the system band limitation have been often neglected. In DS-CDMA systems, however, numerous signals are transmitted at the same time and the total out of band emission power may become very large, even though each signal has a little out of band emission. Thus the signals have to be strictly band limited to avoid interference to other systems in the adjacent frequency bands.

Several studies have been made on the performance of band limited DS-CDMA systems [1]-[2]. It does not appear to us, however, that there have been analytical results of asynchronous band limited DS-CDMA systems taking the cross correlation characteristics of actual spreading sequences into consideration. Almost all of previous works are made by simulation or the analysis in which the random spreading sequences are assumed.

The objective of this paper is to evaluate the performance of a practical band limited DS-CDMA system analytically taking account of the cross correlation characteristics of spreading sequences.

For this purpose, the signal to noise plus interference ratio(SNIR) of the be spread signal, as a measure of performance, is derived for the system, in which quaternary phase shift keyed (QPSK) signals are band limited by Nyquist filtering.

Then, Gold sequence, which is most commonly used for spectrum spreading, is applied and an approximation of the performance of a very simple form is derived.

As example of application, we also estimate the bit error rate performance and the spectral efficiency using the analyzed result of SNIR.

II. SYSTEM MODEL

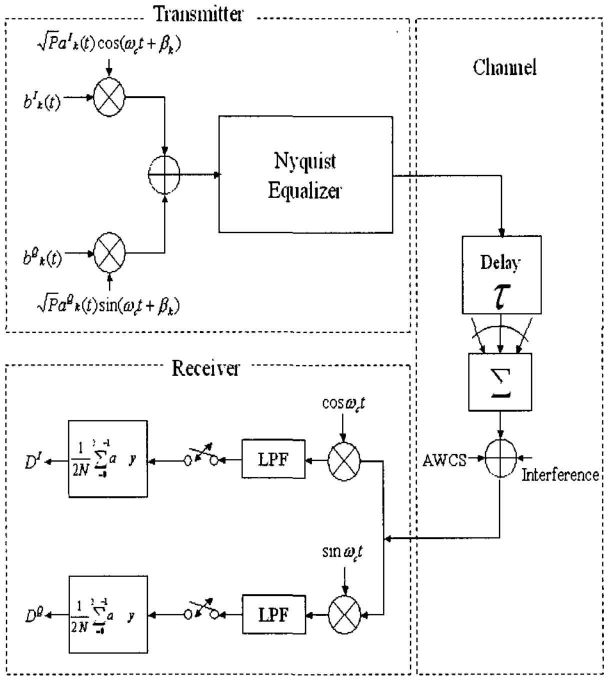

Fig. 1 System model

The models of band limited DS-CDMA systems with QPSK are shown in Fig. 1 At the QPSK transmitter for k-th user ($1 \le k \le K$), the information signal $b_k(t)$, which has a rectangular waveform of duration T and takes the value +1 or -1, is first multiplied by the spreading signal,

$$a_k(t) = \sum_{l=-\infty}^{\infty} a_{k,l} c(t - lT_c), \quad (1)$$

where $\{a_{k,l}\}$ is the spreading sequence assigned for the k-th user's signal which takes the values +1 or −1, and

$$c(t) = \begin{cases} 1; & |t| \le \frac{T_c}{2} \\ 0; & otherwise \end{cases} \quad (2)$$

Manuscript received January 1, 2003

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The period of the spreading sequence $\{a_{k,l}\}$ is set to be $N=T/T_c$, which is the number of chips in one bit duration of $b_k(t)$.

After the spreading, the signal $b_k(t)\ a_k(t)$ is QPSK modulated and gives

$$g_k(t)=\sqrt{P}b_k^I(t)a_k^I(t)\cos(\omega_c t+\beta_k) \quad \sqrt{P}b_k^Q(t)a_k^Q(t)\sin(\omega_c t+\beta_k) \tag{3}$$

where P is the signal power, ω_c is the carrier frequency in common among K signals, and β_k is an initial phase of the k-th user's signal and also where $b_k^I(t)$, $a_k^I(t)$ and $b_k^Q(t)$, $a_k^Q(t)$ are the information signals and the spreading signals for the in-phase and quadrature components, respectively. The spreading sequences assigned for $a_k^I(t)$ and $a_k^Q(t)$ are represented by $\{a_{k,l}^I\}$ and $\{a_{k,l}^Q\}$.

For the purpose, the symbol duration of QPSK becomes 2T. Accordingly, the period of are $\{a_{k,l}^I\}$ and $\{a_{k,l}^Q\}$, which is the same as the number of the chips included in one bit duration of the information signals $b_k^I(t)$ and $b_k^Q(t)$, becomes 2N.

III. PERFORMANCE ANALYSIS OF QPSK

At the receiver, K signals arrive with white Gaussian noise. In order to simplify the analysis, the spreading sequences with the length of 2N chips are assigned to the in-phase and quadrature channels, respectively. For simplicity of the analysis, information signals $b_k^I(t)$ and $b_k^Q(t)$ are assumed to be fixed at +1.

The k-th signal $h_k(t)$ is of the following form at the output from the receive filter:

$$\begin{aligned} h_k(t) = & \sqrt{P}\left\{\sum_{m=-\infty}^{+\infty} a^I{}_{k,m}\cdot x(t-mT_c-\tau_k)\right\} \\ & \cdot\cos(\omega_c+\phi_k) \\ & +\sqrt{P}\left\{\sum_{m=-\infty}^{+\infty} a^Q{}_{k,m}\cdot x(t-mT_c-\tau_k)\right\} \\ & \cdot\sin(\omega_c+\phi_k) \end{aligned} \tag{4}$$

where the time delay τ_k is a random variable uniformly distributed over [0, $2NT_c$]. In this paper, the analysis will be made only for the correlated output of in-phase channel, since the same result can be obtained for the quadrature channel.

The desired signal component of the output from the correlate of the i-th signal receiver is represented as

$$D_i^I=\frac{\sqrt{P}}{2} \tag{5}$$

The interfering component from the k-th user (k≠i) D_k^I is expressed as

$$\begin{aligned} D_k^I = & \frac{\sqrt{P}}{4N}\left[\cos\phi_k\sum_{q=-\infty}^{+\infty}\theta^{II}{}_{k,i}(q)\cdot x(qT_c-\tau_k)\right. \\ & \left. +\sin\phi_k\sum_{q=-\infty}^{+\infty}\theta^{QI}{}_{k,i}(q)\cdot x(qT_c-\tau_k)\right] \end{aligned} \tag{6}$$

where

$$\begin{aligned} \theta^{II}{}_{k,i}(q) &= \sum_{l=0}^{2N-1} a^I{}_{i,l}\cdot a^I{}_{k,2N-q+l} \\ &= \theta^{II}{}_{k,i}(q+2N) \end{aligned} \tag{7}$$

$$\begin{aligned} \theta^{QI}{}_{k,i}(q) &= \sum_{l=0}^{2N-1} a^I{}_{i,l}\cdot a^I{}_{k,2N-q+l} \\ &= \theta^{QI}{}_{k,i}(q+2N) \end{aligned} \tag{8}$$

we can obtain the variance of $D^I{}_k$.

$$\begin{aligned} Var[D^I{}_k] = & \frac{P}{64N^3T_c}\left\{\sum_{r=-\infty}^{+\infty}\Phi(rT_c)\rho^{II}{}_{k,i}(r)\right. \\ & \left. +\sum_{r=-\infty}^{+\infty}\Phi(rT_c)\rho^{QI}{}_{k,i}(r)\right\} \end{aligned} \tag{9}$$

where $\Phi(\tau)$ is the autocorrelation function of $x(t)$ and $\rho_{k,i}(r)$ is the autocorrelation function of cross correlation between the k-th and the i-th signals, and expressed as

$$\Phi(\tau)=\int_{-\infty}^{+\infty}x(t)\cdot x(t+\tau)dt \tag{10}$$

$$\rho^{II}{}_{k,i}(r)=\sum_{s=0}^{2N-1}\theta^{II}{}_{k,i}(s)\theta^{II}{}_{k,i}(s+r) \tag{11}$$

$$\rho^{QI}{}_{k,i}(r)=\sum_{s=0}^{2N-1}\theta^{QI}{}_{k,i}(s)\theta^{QI}{}_{k,i}(s+r) \tag{12}$$

The autocorrelation function $\Phi(rT_c)$ (r : integer) can be obtained from the power spectral density of $x(t)$ as

$$\begin{aligned} \Phi(rT_c) &= \left[\frac{1}{2\pi}\int_{-\infty}^{+\infty}X^2(\omega)e^{j\omega t}d\omega\right]_{\tau=rT_c} \\ &= T_c\cdot\delta_r-\frac{1}{4}\alpha\cdot T_c\cdot(-1)^r\cdot S_a(r\alpha\pi) \\ & -\frac{1}{8}T_c\cdot(-1)^r\left[S_a\{(r\alpha+1)\pi\}\right. \\ & \left. +S_a\{(r\alpha-1)\pi\}\right] \end{aligned} \tag{13}$$

where δ_r is Dirac delta function. Note that, since each signal component $h_k(t)$ is independent, each D_k is also independent and thus

$$Var\left[\sum_{k=1,k\neq i}^{K} D_k\right] = \sum_{k=1,k\neq i}^{K} Var[D_k] \tag{14}$$

Assuming that the thermal noise with single sided spectral density N_0 is added at the input to the receiver, we obtain the variance of the thermal noise component D_r as

$$Var\left[D'_n\right] = \frac{N_0}{8NT_c} \tag{15}$$

Then, the SNIR for QPSK is derived in the following:

$$SNIR_Q = \left\{\frac{1}{16N^3T_c}\sum_{k=1,k\neq 1}^{K}\sum_{r=-\infty}^{+\infty}\Phi(rT_c)(\rho^{II}{}_{k,i}(r) + \rho^{QI}{}_{k,i}(r)) + \frac{N_0}{2E_0}\right\}^{-1} \tag{16}$$

where E_b is the energy of the signal concerned to one-bit of the information signal $b^I{}_k(t)$.

IV. DERIVATION OF APPROXIMATED SNIR FOR GOLD SEQUENCES

In this section, Gold sequence is applied to the results of the preceding section and a simple equation representing the approximated SNIR performance is derived. For Gold sequences, the parameters $\rho_{k,i}(r)$, $\rho^{II}{}_{k,i}(r)$ and $\rho^{QI}{}_{k,i}(r)$ are predominant where r=0, and also the summation of them on r≠0 are very small compared to the values at r=0. It can be seen from (19) that the autocorrelation function $\Phi(rT_c)$ of $x(t)$ becomes maximum at r=0 irrespective of the value of roll off factor α. Taking these facts into account, we can make the following approximation,

$$\sum_{r=-\infty}^{+\infty}\Phi(rT_c)(\rho^{II}{}_{k,i}(r) + \rho^{QI}{}_{k,i}(r)) \cong \Phi(0)(\rho^{II}{}_{k,i}(0) + \rho^{QI}{}_{k,i}(0)) \tag{17}$$

Thus (16) can be modified as follows:

$$SNIR_Q \cong \left\{\frac{1}{16N^3T_c}\Phi(0)\sum_{k=1,k\neq i}^{K}\rho^{II}{}_{k,i}(0) + \rho^{QI}{}_{k,i}(0)) + \frac{N_0}{2E_b}\right\}^{-1} \tag{18}$$

where from(13),

$$\Phi(0) = \left(1 - \frac{\alpha}{4}\right)\cdot T_c \tag{19}$$

The parameter $\rho^{II}{}_{k,i}(0)$ $\rho^{QI}{}_{k,i}(0)$ is expressed from (11), (12) as

$$\rho^{II}{}_{k,i}(r) = \sum_{s=0}^{2N-1}\theta^{II}{}_{k,i}(s)\theta^{II}{}_{k,i}(s+r) \tag{20}$$

$$\rho^{QI}{}_{k,i}(r) = \sum_{s=0}^{2N-1}\theta^{QI}{}_{k,i}(s)\theta^{QI}{}_{k,i}(s+r) \tag{21}$$

The cross-correlation values $\theta^{II}{}_{k,i}(s)$, $\theta^{QI}{}_{k,i}(s)$ of expected values $\rho^{II}{}_{k,i}(s)$, $\rho^{QI}{}_{k,i}(s)$ is changed by Gold sequences.

$$E\left[\rho^{II}{}_{k,i}(0)\right] = E\left[\rho^{QI}{}_{k,i}(0)\right] \cong 4N(N+1) \tag{22}$$

From (18)-(22), and $SNIR_Q$ can be approximated as

$$SNIR_Q \cong \left\{\frac{N+1}{2N^2}\left(1-\frac{\alpha}{4}\right)(K-1) + \frac{N_0}{2E_b}\right\}^{-1} \tag{23}$$

V. PERFORMANCE ANNLYSIS

A. Bit Error Rate Performance

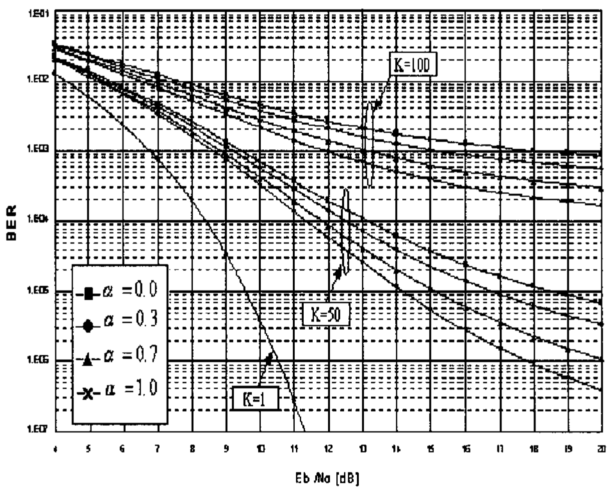

Fig. 2 Bit error rate performance

The correlator output consists of the desired signal, interference, and noise components. The noise component D_n has Gaussian distribution, as the noise input to the receiver is Gaussian. When the number of the signals K is large enough, the central limit theorem ensures that the interference component, sum of independent random values D_k, can be also assumed to be Gaussian.

In such a case, we can evaluate the bit error rate performance P_e of the system by SNIR obtained in the previous section as

$$P_e = \frac{1}{2} erfc(\sqrt{SNIR/2}) \tag{24}$$

where, erfc(*)is the complementary error function given by

$$erfc(x) = \frac{2}{\pi}\int_x^{\infty} e^{-y^2} dy \tag{25}$$

The curves drawn in Fig. 2 are the numerical examples given by (24) with (25), where the Gold sequences with the period 511 chips is employed. The analysis result by [5], in which the actual distribution of the cross correlation of the signals is considered, is also shown by the dots in the figure. Comparing the estimated bit error rate and the simulation result, we can confirm that the SNIR derived in the previous section is a good measure of the performance.

B. Spectral Efficiency

Equation (23) indicates that SNIR becomes maximum where the roll off of the cosine roll off filter is 1.0 for given T_c. In this subsection, we evaluate the effect of the roll-off factor from another point of view, the spectral efficiency. The spectral efficiency η is defined as follows: [5]

$$\eta = \frac{KR_b}{W_s} \qquad [bit/\sec/Hz] \tag{26}$$

where R_b is each user's bit rate, and W_s is the bandwidth assigned to the system.

Assuming the Nyquist spectrum shaping is adopted, we obtain R_b from (4) as

$$R_b = \frac{W_s}{N(1+\alpha)} \tag{27}$$

and substituting into (26) we obtain

$$\eta = \frac{K}{N(1+\alpha)} \tag{28}$$

When the number of user's K is much larger than one, SNIR in (23) is approximated as

$$SNIR \cong \left\{\frac{N+1}{2N^2}\left(1-\frac{\alpha}{4}\right)K + \frac{N_0}{2E_b}\right\}^{-1} \tag{29}$$

From (29) we obtain the maximum number of user's as

$$K = \frac{8N^2}{(4-a)(N+1)}\left(\frac{1}{SNIR} - \frac{N_0}{2E_b}\right) \tag{30}$$

substituting (30) into (28), we can write η as

$$\eta = \frac{8N}{(N+1)(4-\alpha)(1+\alpha)}\left(\frac{1}{SNIR} - \frac{N_0}{2E_b}\right) \tag{31}$$

Now, let us consider the condition that SNIR and E_b / N_0 are fixed. This is equivalent to saying that the bit error rate and the signal power, normalized by noise spectral density, are given. Under this condition, $(1/SNIR - N_0/2E_b)$ which corresponds to the amount of noise due to the interferences from other channels becomes constants and the spectral efficiency normalized by the value for the roll off factor $\alpha = 0$ is expressed as

$$\eta_n = \frac{4N}{(N+1)(4-\alpha)(1+\alpha)} \tag{32}$$

which is plotted in Fig. 3. As seen in this figure, the spectral efficiency decreases as the roll off factor increases.

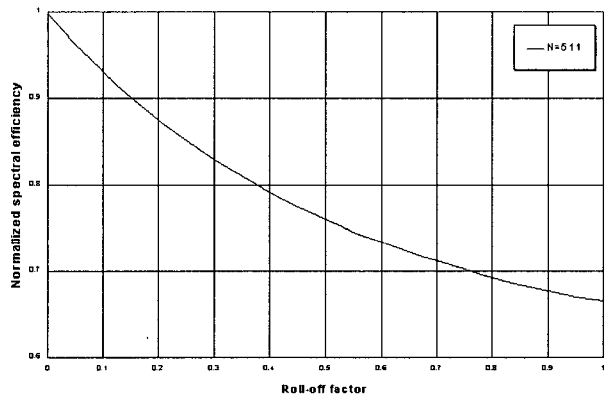

Fig. 3 Normalized spectral efficiency

On the other hand, it is appeared by variable SNIR as follow in Fig. 4

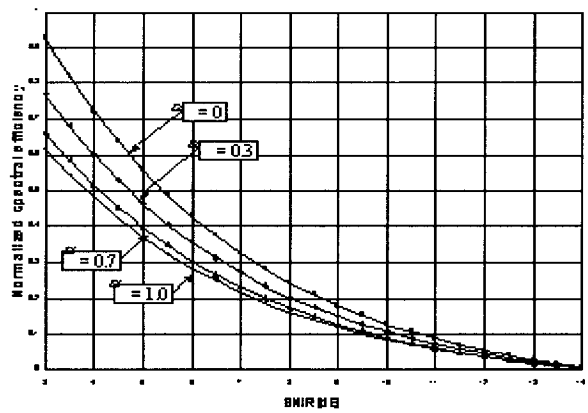

Fig. 4 Normalized spectral efficiency by SNIR

VI. CODED PERFORMANCE

A. RS Code

In this section, the performance of Reed-Solomon (RS) code is a nonbinary circulatory linear block code and it is a multiple error correction code. An (n,k) RS code which has the ability of error correction, t, is determined as followings.

$$n = 2^{m-1} \quad (33)$$

$$k = n - 2t \quad (34)$$

$$d_{\min} = 2t + 1 \quad (35)$$

BER is

$$p_{RS} = \sum_{i=t+1}^{n} \frac{i}{n} \binom{n}{i} p^{i}(1-p)^{n-i} \quad (36)$$

B. Convolution Code

(n,.k) convolution encoder is specially excellent in correcting errors and it can correct burst errors very easily with interleave/deinterleave even in channels burst errors occur.

For code rate, r = 1/2, BER of convolution code is as following,

$$p_c \le 0.5(7D^7 + 39D^8 + 104D^9 + 352D^{10} + \cdots) \quad (37)$$

where,

$$D = 2\sqrt{p_c}(1 - p_c) \quad (38)$$

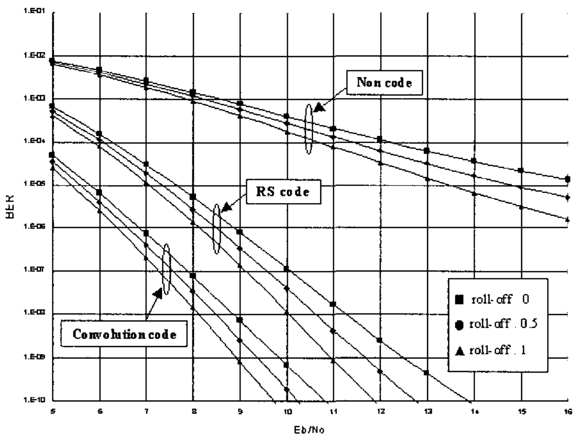

Fig. 5 BER Performance by coding Techniques

VII. CONCLUSION

In this paper, the signal to noise plus interference ratio(SNIR) of asynchronous band limited DS-CDMA systems has been derived analytically for QPSK systems. For band limitation, Nyquist filtering is employed, because this type of filtering is most commonly used for transmission of digital signals. The SNIR is evaluated under consideration of the cross correlation characteristics of spreading sequences. The simple approximation of SNIR is also given for the systems with Gold sequence.

The effects of band limitation is an important issue for practical DS-CDMA systems: the numerous signals transmitted at the same time could give the serious interference to the systems in adjacent frequency bands, even though the out of band emission of each signal is small. The analytical expressions of SNIR derived in this paper are simple and useful for the estimation of the performance of such practical DS-CDMA systems with the band-limitation. In addition to the derivation of SNIR, the bit error rate performance and spectral efficiency are also evaluated.

In Chapter VI, we compared improvement rate of the efficiency by RS with that of the efficiency by convolution in case of using coding techniques.

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