Guided Wave Mode Identification Using Wavelet Transform

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웨이블릿 변환을 이용한 유도초음파의 모드 확인

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Abstract

One of unique characteristics of guided waves is a dispersive behavior that guided wave velocity changes with an excitation frequency and mode. In practical applications of guided wave techniques, it is very important to identify propagating modes in a time-domain waveform for determination of defect location and size. Mode identification can be done by measurement of group velocity in a time-domain waveform. Thus, it is preferred to generate a single or less dispersive mode But, in many cases, it is difficult to distinguish a mode clearly in a time-domain waveform because of superposition of multi modes and mode conversion phenomena. Time-frequency analysis is used as efficient methods to identify modes by presenting wave energy distribution in a time-frequency. In this study, experimental guided wave mode identification is carried out in a steel plate using time-frequency analysis methods such as wavelet transform. The results are compared with theoretically calculated group velocity dispersion curves. The results are in good agreement with analytical predictions and show the effectiveness of using the wavelet transform method to identify and measure the amplitudes of individual guided wave modes.

Key Words : Guided Wave(유도초음파), Mode Identification(모드 확인), Wavelet Transform(웨이블릿변환), Group Velocity(군속도), Dispersion Diagram(분산선도)

1. Introduction

The guided wave techniques extensively and successfully applied methods for long-range nondestructive inspection. Dispersion is a very unique characteristic of guided waves; the velocity of guided waves changes

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with excitation mode and frequency. Wave penetration power, attenuation, and sensitivity to defects also highly depend on frequency and mode. Thus, in applications of guided wave techniques, it is very critical to identify propagating modes in a structure for determination of defect location and size. A common and simple way to identify guided wave modes is measuring group velocity under certain excitation conditions such as frequency, structure geometry, and wave incident angle. However, in many cases, it is not easy to generate a single mode or identify the modes in a time-domain waveform due to superposition of multi modes and mode conversion due to scattering from defects or a boundary of structures.

Recently, there has been considerable interest in the application of wavelet transform(1-3) to signal processing application. In contrast to the Fourier Transform, whereby a time domain signal is decomposed into its constituent frequency components without regard to the temporal order of the components, the wavelet analysis uses sets of scaled basis functions that can provide a decomposition in terms of both time and frequency analysis have been found for interpretation of acoustic, sonar, and radar signals and data compression. Significant number of studies has been carried out on the signal analysis for guided wave mode identification(1-3). Fourier transform is a well-known method and has been widely used to analyze frequency component in the entire signal at once. But, Fourier transform provides information only on a frequency spectrum which is not dependent on time. Thus, it is not suitable for analyzing signals varying with time. For guided waves, several modes can be generated in a frequency range and each mode travels at different velocities. Thus, it is impossible to extract frequency component of each mode using the Fourier transform. In order to overcome this problem, time-frequency analysis methods such as Wigner-Ville distribution, wavelet transform (WT), and Short Time Fourier Transform (STFT) have been developed and applied to time-varying signals analysis(4-7). In the time-frequency analysis methods, the wave energy distribution is represented in a time-frequency plane. A time-domain signal is split into a series of small pieces using a wavelet function and each piece is Fourier transforms. Consequently, the frequency spectrum of a small portion of the time-domain signal is displayed in time sequence. With a known wave travel distance, the time-frequency information can be transferred into the group velocity- frequency domain. By comparing the wave energy distribution obtained from WT with the theoretically calculated dispersion curves, modes in the waveform can be identified.

In this study, first, theoretical wavelet transform is applied to guided wave mode identification in structures such as a plate. The group velocity-frequency representations obtained by each method are then compared with theoretically calculated dispersion curves in the structures.

2. Mode Identification Analysis

2.1 Wavelet Transform(WT)

Classical Fourier analysis provides a spectral representation that is independent of time. However, many vibration processes exhibit nonstationary behavior, which cannot be effectively described using this analysis. A number of different time-variant methods exist for The continuous wavelet transform(CWT) or continuous-time wavelet transform of an arbitrary function f(t) as given by Strang and Nguyen(11) is defined nonstationary processes, including: adaptive techniques(short-time Fourier transform), time-frequency techniques (Wigner-Ville distribution), and time-scale procedures (wavelet transform). A fundamental difference between wavelet analysis and other methods is that instead of seeking to decompose a signal into its harmonics, which are global functions, the signal is broken down into local functions called wavelets. The concept of wavelet analysis has many different origins from mathematics to signal processing. For the sake of completeness, a brief introduction to the relevant wavelet theory is given in this section.

The CWT or continuous-time wavelet transform of an arbitrary function f(t) as given by Strang and Nguyen(11) is defined

$$W_f(a,b)=\int_{-\infty}^{\infty} f(t)\Psi^{*}_{a,b}(t)dt \qquad (1)$$

including the analyzing function

$$\Psi_{a,b}(t)=\frac{1}{\sqrt{a}}\Psi\left(\frac{t-b}{a}\right) \qquad (2)$$

as wavelet functions, With the position variable b and the scale variable a, where $a>0$ and * denotes complex conjugation. The function $\Psi(t)$ is the mother wavelet (analyzing wavelet).

It satisfies the admissibility condition on $\Psi(t)$

$$\int_{-\infty}^{\infty}\frac{|\hat{\varphi}(\omega)|^2}{|\omega|}d\omega<\infty \qquad (3)$$

where $\hat{\Psi}(\omega)$ denote the Fourier transform $\Psi(\omega)$ of defined by

$$\hat{f}(\omega)=\int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}dt \qquad (4)$$

Although there are many choices for the analyzing wavelet, we adopt the Gabor wavelet, since it provides the best time frequency resolution as confirmed by the uncertainty principle.

The Gabor wavelet is expressed as[8]

$$\psi_g(t)=\frac{1}{\sqrt[4]{\pi}}\sqrt{\frac{\omega_0}{\gamma}}\cdot\exp\left[-\frac{1}{2}\left(\frac{\omega_0 t}{\gamma}\right)^2\right]\exp(i\omega_0 t) \qquad (5)$$

and its Fourier transform is expressed as

$$\psi_g(\omega)=\frac{\sqrt{2\pi}}{\sqrt[4]{\pi}}\sqrt{\frac{\gamma}{\omega_0}}\cdot\exp\left[-\frac{\left(\frac{\omega_0 t}{\gamma}\right)^2}{2}(\omega-\omega_0)^2\right] \qquad (6)$$

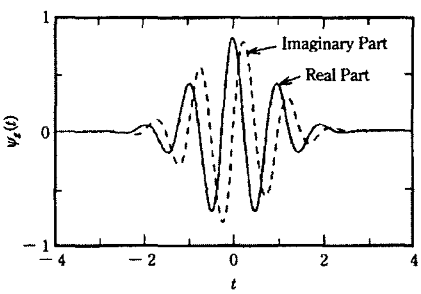
where ω_0 and γ are positive constants. Although the Gabor wavelet does not satisfy the admissibility condition(3) in the strict sense, it approximately satisfies the condition if γ is sufficiently large.

In this study, we set $\gamma=\pi\sqrt{2/\ln 2}$=5.336 according to Morlet. If eq (5) is substituted into eq (1) it is understood that the WT using the Gabor wavelet has a similar form to the Fourier transform with Gaussian windowing. Hence we set $\omega_0=2\pi$ such that $1/a$ takes the same value as the frequency $\omega/(2\pi)$.

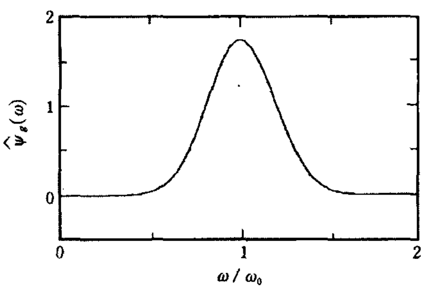
Fig. 1 shows the Gabor wavelet and its Fourier transform. the Gabor wavelet $\Psi_g^{(\omega)}$ is localized around the time (t)=0, and its Fourier transform $\Psi_g^{(\omega)}$ is localized around the angular frequency $\omega=\omega_0$. It is easily understood that the function $\Psi_g^{((t-b)/a)}$ is localized around t=b and that its Fourier transform [aexp(-ibω) $\Psi_g^{(a\omega)}$] is localized around $\omega=\omega_0/a$. Hence the magnitude of WT, $|(W_f)(a,b)|$ represents the "intensity" of the signal f(t) around the time t=b and the angular frequency $\omega=\omega_0/a$.

2.2 Wavelet Transform and Dispersion

To illustrate the use of the wavelet transform in the



(a) Gabor function



(b) Fourier transform of gabor function

Fig. 1 The Gabor wavelet and the Fourier Transform of Gabor wavelet $\gamma=\pi\sqrt{2/\ln 2}$ and $\omega_0=2\pi$

analysis of dispersion, let us consider two harmonic wave of unit amplitude and different angular frequency ω_1 and ω_2 propagating in the x-direction, given by

$$u(x,t) = e^{-i(k_1 x - \omega t)} + e^{-i(k_2 x - \omega_2 t)}$$
$$= 2\cos(\triangle kx - \triangle \omega t) e^{-i(k_c x - \omega_c t)} \qquad (7)$$

where k_1 and k_2 are wave numbers.

$$k_c = (k_1 + k_2) \ , \ \omega_c = (\omega_1 + \omega_2)/2 \qquad (8)$$

and

$$\triangle k = (k_1 - k_2)/2 \ , \ \triangle \omega = (\omega_1 - \omega_2)/2 \qquad (9)$$

If $\triangle \omega$ is sufficiently small, the group c_g at the angular frequency ω_c can be defined as

$$c_g = \triangle \omega / \triangle k \qquad (10)$$

When the Gabor wavelet is adopted as the analyzing wavelet, the magnitude of WT of $u(x,t)$ is obtained as (8)

$$|(Wu)(x,a,b)| = \sqrt{a}\{[\hat{\varphi}_g(a\omega_1)]^2 + [\hat{\varphi}_g(a\omega_2)]^2\} + 2\hat{\varphi}_g(a\omega_1)\hat{\varphi}_g(a\omega_2) \cos(2\triangle kx - 2\triangle \omega b)^{1/2} \qquad (11)$$

If $\triangle \omega$ is sufficiently small such that

$$\hat{\varphi}_g(a\omega_1) \cong \hat{\varphi}_g(a\omega_2) \cong \hat{\varphi}_g(a_c)$$

we obatin

$$|(Wu)(x,a,b)| \cong \sqrt{2a|\hat{\varphi}(a\omega_c)| \cdot [1 + \cos(2\triangle kx - 2\triangle \omega b)]}^{1/2} \qquad (12)$$

This equation indicates that the magnitude of WT takes its maximum value at $a = \omega_0/\omega_c$ and $b = (\Delta k/\Delta\omega)x = x/c_g$. Therefore, for fixed x, a three dimen sional plot of $|(Wu)(x,a,b)|$ on the (a,b)-plane has a peak at $(a,b) = \omega_0/\omega_c, x/c_g$. In other words, the location of the peak on the (a, b)-plane indicates the arrival time $b = xc_g$ of the wave having angular frequency $\omega_c = \omega_0 / a$.

3. Experimental Setup

For the excitation of guided waves, a couple of 0.5 MHz commercial type contact longitudinal transducers were used with variable angle shoes. Guided waves were excited on sample steel plate with the thickness of 6.25mm. The center frequency of the excited signals was 0.5 MHz and the angle of incident angle was 60 degrees. Pitch-catch and pulse-echo method(transducer spacing:24 inches) were used for the analysis of the characteristics of excited guided waves. The excitation signals were made by a tone burst system(Ritec RAM 10000) that could control signal duration and frequency. Therefore, the center of excitation bandwidth of phase velocity was between A0 and S0 at the fd value of 0.5 MHz.

Once the experimental data are saved on a workstation with sampling rate 6.35 MHz, the signal processing procedure follows. It is important to window the signals before applying the Fourier transform. In this paper a Hanning window gives best results. However, the maximum of the Hanning window is at the half of the input sequence length of the signal, whereas the interesting part of the signal might not be at the same place as the maximum of the Hanning window. Therefore by using a common Hanning window, the interesting part of the signal may be reduced in amplitude whereas the less interesting part of the signal may be unchanged. To avoid this problem, zeros are added at the beginning or the end of the signal in order to shift the interesting part of the signal to the corresponding maximum of the Hanning window. The signals are then transformed into the frequency domain using the fast Fourier transform(FFT). The discrete form

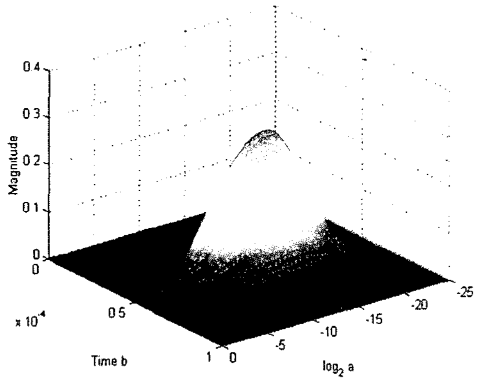


Fig. 2 3-D plot of the wavelet transform of Lamb wave signal

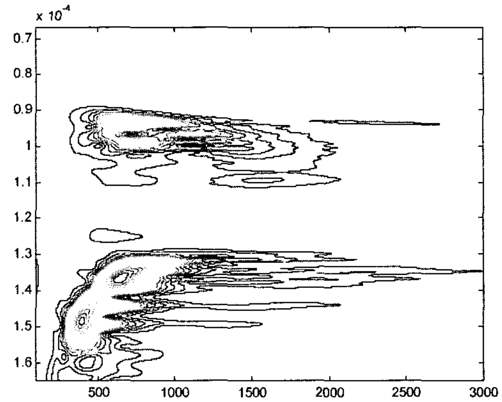


Fig. 3 Contour plot of the wavelet transform of Lamb wave signal

of Eq. (5) leads to the wavelet domain of the four signals. The parameters used in this transform are

$$a = 2^{m/4} \quad \text{and} \quad b = n\Delta t \tag{13}$$

where m and n are integers. Newland[12] suggested the use of the wavelet transform starting in the time domain, transforming into the frequency domain, and finally transforming into the wavelet domain. This is preferred against direct transformation of the time-domain signal into the wavelet domain as shown in Eq. (1). The advantage is a savings in computational time by about two orders of magnitude. Fig. 2 is an example of the 3-D plot of the magnitude of the wavelet transform of guided wave signal. The maximum of the plot can be easily seen, Fig. 3 shows the contour plot of the time-frequency analysis using the wavelet transform of guided wave signal. As mentioned before, each peak in Fig. 3 represents the arrival time of a guided wave traveling with the group velocity. According to such a plot, the traveling time of corresponding wave mode between two points can be obtained for each value f=1/a. Since the distance between two measuring points is known, the group velocity can be identified at each frequency by Eq. (13).

4. Results and Discussion

Guided wave modes and their dispersive characteristics can be obtained by solving wave equation with proper boundary conditions The result of frequency analysis using the wavelet transform of guided waves excited by an angle transducer is shown in Fig. 4 The guided wave signals in time domain indicate that the waves are dispersive and may be superposed by multi-modes. The wavelet transform analysis reveals the patterns of dispersion that is closely related with the group velocity dispersion curves obtained by theoretical calculations. The mode identification was performed by the comparison of the patterns of dispersion. It was revealed that two different mode groups were propagating and the mode group containing A0 and S0 modes were dominant. And the group velocity at 0.5 MHz was of A0 mode = 3.21 mm/µsec, S0 mode = 2.53 mm/µsec. From the comparison of the dispersive patterns obtained from both the wavelet transform and theoretical dispersion diagram, the compact packet mode and the widespread mode were identified as A0 and S0 modes, respectively. And, the wavelet transform can feasible to identifying not only modes but also the frequency bandwidth of each mode. The shaded areas in Fig. 4 (b) represent the guided wave modes identified in dispersion curves. These results agreed with the

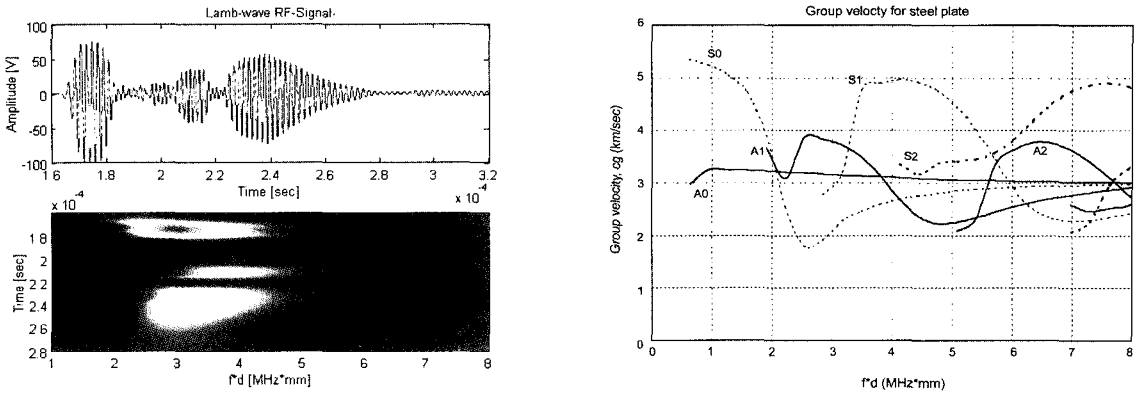


Fig. 4 Guided wave mode identification by wavelet transform

theoretical expectation for the given excitation conditions.

5. Conclusions

The application of the wavelet transform to the time-frequency analysis of guided waves propagating in a dispersive medium has been presented. It was found that the wavelet transform using the Gabor wavelet was an effective tool for the experimental analysis of dispersive waves in steel plate. The arrival times of each frequency component needed in the group velocity calculation could be determined from the peak of the magnitude of wavelet transform data on the time-frequency domain.

And, experimental guided wave mode identification is carried out in a steel plate using time-frequency analysis methods such as wavelet transform. The results are compared with theoretically calculated group velocity dispersion curves. The results are in good agreement with analytical predictions and show the effectiveness of using the wavelet transform method to identify and measure the amplitudes of individual guided wave modes.

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