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COMMON FIXED POINT THEOREMS FOR WEAKLY COMPATIBLE MAPPINGS IN MENGER SPACES

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ABSTRACT. In this paper we prove common fixed point theorems for four mappings, under the condition of weakly compatible mappings in Menger spaces, without taking any function continuous. We improve results of [A common fixed point theorem for three mappings on Menger spaces. *Math. Japon.* **34** (1989), no. 6, 919–923], [On common fixed point theorems of compatible mappings in Menger spaces. *Demonstratio Math.* **31** (1998), no. 3, 537–546].

1. INTRODUCTION

Jungck [6] proved a common fixed point theorem for commuting maps generalizing the Banach's fixed point theorem. Banach fixed point theorem has many applications but it has one draw back, that the definition requires continuity of the function. There then follows a flood of papers involving contractive definition that do not require the continuity of the function. This result was further generalized and extended in various ways by many authors.

Sessa [19] defined weak commutativity and proved common fixed point theorem for weakly commuting mappings. Further, Jungck [7] introduced more generalized commutativity, so called compatibility, which is more general than that of weak commutativity. Since then various fixed point theorems for compatible mappings satisfying contractive type conditions and assuming continuity of at least one of the mappings, have been obtained by many authors.

It has been known from the paper of Kannan [9] that there exist maps that have a discontinuity in the domain but which has a fixed point. Moreover the maps involved in every case were continuous at the fixed point. In 1998, Jungck & Rhoades [8]

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introduced the notion of weakly compatible maps and showed that compatible maps are weakly compatible but converse need not to be true.

Menger [10] introduced the notion of probabilistic metric spaces, which is the generalization of metric space, and the study of these spaces was expanded rapidly with the pioneering works of Schweizer & Sklar [16, 17]. The theory of probabilistic metric spaces is of fundamental importance in probabilistic function analysis.

Recently fixed point theorems in Menger spaces have been proved by many authors including Bylka [1], Pathak, Kang & Baek [12], Stojakovic [20, 21, 22], Hadzic [4, 5], Dedeic & Sarapa [3], Rashwan & Hedar [15], Mishra [11], Radu [13, 14], Sehgal & Bhaucha-Reid [18], and Cho, Murthy & Stojakovic [2].

In this paper we prove some common fixed point theorems for weakly compatible mappings on complete metric spaces without using the condition of continuity. We improve results of Dedeic & Sarapa [3], Rashwan & Hedar [15] and Mishra [11].

2. Preliminaries

Let R denote the set of reals and R^+ the non-negative reals. A mapping $F : R \to R^+$ is called a distribution function if it is non-decreasing and left continuous with $\inf F = 0$ and $\sup F = 1$. We will denote by L the set of all distribution functions. A probabilistic metric space is a pair (X, F), where X is a non empty set and F is a mapping from $X \times X$ to L.

For $(u, v) \in X \times X$, the distribution function $F(u, v)$ is denoted by $F_{u,v}$. The functions $F_{u,v}$ are assumed to satisfy the following conditions:

(P_1) $F_{u,v}(x) = 1$ for every $x > 0$ if and only if $u = v$,

(P_2) $F_{u,v}(0) = 0$ for every $u, v \in X$,

(P_3) $F_{u,v}(x) = F_{v,u}(x)$ for every $u, v \in X$,

(P_4) if $F_{u,v}(x) = 1$ and $F_{v,w}(y) = 1$, then $F_{u,w}(x + y) = 1$ for all $u, v, w \in X$ and $x, y > 0$.

In a metric space (X, d), the metric d induces a mapping $F : X \times X \to L$ such that

$$F(u, v)(x) = F_{u,v}(x) = H(x - d(u, v)),$$

for every $u, v \in X$ and $x \in R$, where H is a distribution function defined by

$$H(x) = \begin{cases} 0, & x \leq 0 \\ 1, & x > 0 \end{cases}$$

Definition 2.1. A function $t : [0,1] \times [0,1] \to [0,1]$ is called T-norm if it satisfies the following conditions:

(t_1) $t(a,1) = a$ for every $a \in [0,1]$ and $t(0,0) = 0$,
(t_2) $t(a,b) = t(b,a)$ for every $a, b \in [0,1]$,
(t_3) If $c \geq a$ and $d \geq b$, then $t(c,d) \geq t(a,b)$,
(t_4) $t(t(a,b),c) = t(a,t(b,c))$ for every $a,b,c \in [0,1]$.

Definition 2.2. A Menger space is a triple (X, F, t), where (X, F) is a probabilistic metric space and t is a T-norm with the following condition:

$$(\mathrm{P}_5) \qquad F_{u,v}(x+y) \geq t\big(F_{u,w}(x), F_{w,v}(y)\big)$$

for all $u, v, w \in X$ and $x, y \in R^+$.

The concept of neighbourhood in probabilistic metric space was introduced by Schweizer & Sklar [16]. If $u \in X, \varepsilon > 0$ and $\lambda \in (0,1)$, then an (ε, λ)-neighbourhood of u, denoted by $U_u(\varepsilon, \lambda)$, is defined by

$$U_u(\varepsilon, \lambda) = \{v \in X : F_{u,v}(\varepsilon) > 1 - \lambda\}.$$

If (X, F, t) is a Menger space with the continuous T-norm t, then the family

$$\{U_u(\varepsilon, \lambda) : u \in X,\ \varepsilon > 0,\ \lambda \in (0,1)\}$$

of neighbourhoods induces a Hausdorff topology on X and if $\sup_{a<1} t(a,a) = 1$, it is metrizable.

An important T-norm is the T-norm

$$t(a,b) = \min\{a,b\} \text{ for all } (a,b) \in [0,1]$$

and this is the unique T-norm such that $t(a,a) \geq a$ for every $a \in [0,1]$.

Indeed if it satisfies this condition, we have

$$\min\{a,b\} \leq t(\min\{a,b\}, \min\{a,b\}) \leq t(a,b) \leq t(\min\{a,b\}, 1) = \min\{a,b\}.$$

Therefore, $t = \min$.

In the sequel, we need the following definitions due to Radu [13].

Definition 2.3. A sequence $\{x_n\}$ in a Menger space X is said to be convergent to a point $x \in X$ if for every $\in > 0$

$$\lim_{n\to\infty} F_{x_n,x}(\varepsilon) = 1.$$

The sequence $\{x_n\}$ is called a Cauchy sequence if for each $\varepsilon > 0$ and $\lambda \in (0,1)$, there is an integer $N(\varepsilon, \lambda)$ such that $F_{x_n,x_m}(\varepsilon) > 1 - \lambda$ whenever $n, m \geq N$.

If every Cauchy sequence in X is convergent, then (X, F) is called a complete probabilistic metric space.

Theorem 2.1 (Schweizer & Sklar [16]). *Let t be a T-norm defined by $t(a,b) = \min\{a,b\}$. Then the induced Menger space (X,F,t) is complete if a metric space (X,d) is complete.*

Definition 2.4 (Jungck & Rhoades [8]). A pair of mappings A and S is called a *weakly compatible pair* if they commute at coincidence points.

Example 2.1. Define the pair $A, S : [0,3] \to [0,3]$ by

$$A(X) = \begin{cases} x & \text{if} \quad x \in [0,1) \\ 3 & \text{if} \quad x \in [1,3], \end{cases} \qquad S(X) = \begin{cases} 3-x & \text{if} \quad x \in [0,1) \\ 3 & \text{if} \quad x \in [1,3]. \end{cases}$$

Then for any $x \in [1,3], ASx = SAx$, showing that A, S are weakly compatible maps on $[0,3]$.

Example 2.2. Let $X = R$ and define $A, S : R \to R$ by $Ax = x/3, x \in R$ and $Sx = x^2, x \in R$. Hence 0 and 1/3 are two coincidence points for the maps A and S. Note that A and S commute at 0, *i. e.*, $AS(0) = SA(0) = 0$, but $AS(1/3) = A(1/9) = 1/27$ and $SA(1/3) = S(1/9) = 1/81$ and so A and S are not weakly compatible maps on R.

Remark 2.1. Weakly compatible maps need not be compatible. Let $X = [2,20]$ and d be the usual metric on X. Define mappings $A, S : X \to X$ by $Ax = x$ if $x = 2$ or $> 5, Ax = 6$ if $2 < x \le 5, Sx = x$ if $x = 2, Sx = 12$ if $2 < x \le 5, Sx = x - 3$ if $x > 5$. The mappings A and S are non-compatible since sequence $\{x_n\}$ defined by $x_n = 5 + \frac{1}{n}, n \ge 1$. Then

$$\lim_{n\to\infty} Sx_n = 2, \ \lim_{n\to\infty} SAx_n = 2 \quad \text{and} \quad \lim_{n\to\infty} = 6.$$

But they are weakly compatible since they commute at coincidence point at $x = 2$. In this paper, we assume that if $x_n \to x, y_n \to y$, than $F_{x_n,y_n} \to F_{x,y}$.

3. Common fixed point theorems

Theorem 3.1. *Let A, B, S and T be self mappings on a complete Menger space (X,F,t) where t is continuous and $t(x,x) \ge x$ for all $x \in [0,1]$, satisfying the following conditions:*

(3.1) $A(X), B(X)$ are closed sets of X and $A(X) \subset T(X), B(X) \subset S(X)$,
(3.2) The pairs $\{A, S\}$ and $\{B, T\}$ are weakly compatible,
(3.3) There exists $k \in (0,1)$ such that

$$F_{Au,Bv}(kx) \geq t\Big(F_{Au,Su(x)}, t\big(F_{Bv,Tv}(x), t\big(F_{Au,Tv}(\alpha x), F_{Bv,Su}(2x - \alpha x)\big)\big)\Big)$$

for all $u, v \in X, x > 0$ and $\alpha \in (0,2)$. Then A, B, S and T have a unique common fixed point in X. We need the following lemma proved by Mishra [11] for our first result.

Lemma 3.1. *Let A, B, S and T be self mappings of the Menger space (X, F, t), where t is continuous and $t(x, x) \geq x$ for all $x \in [0,1]$, satisfying the condition (3.1) and (3.3). Then the sequence $\{y_n\}$ defined by (3.4) is a Cauchy sequence in X.*

Proof of **Theorem 3.1**. Since $A(X) \subset T(X)$, for any $x_0 \in X$, there exists a point $x_1 \in X$ such that $Ax_0 = Tx_1$. Since $B(X) \subset S(X)$, for this point x_1, we can choose a point $x_2 \in X$, such that $Bx_1 = Sx_2$ and so on. Inductively, we can define a sequence $\{y_n\}$ in X such that

$$y_{2n-1} = Ax_{2n-2} = Tx_{2n-1} \text{ and } y_{2n} = Sx_{2n} = Bx_{2n-1}, \tag{3.4}$$

for $n = 1, 2, \cdots$.

Let $\{y_n\}$ be the sequence in X defined above. By using Lemma 3.1, $\{y_n\}$ is a Cauchy sequence in X and hence by completeness of X, the sequence converge to some point z in X. Also the subsequences $\{Ax_{2n}\}, \{Sx_{2n}\}, \{Bx_{2n-1}\}$ and $\{Tx_{2n-1}\}$ all converges to z.

Since $B(X) \subset S(X)$, there exists $p \in$X, such that $z = Sp$. Then using (3.3), we have

$$\begin{aligned} &F_{Ap,Bx_{2n-1}}(kx) \\ &\geq t\Big(F_{Ap,Sp},(x), t\big(F_{Bx_{2n-1}}, Tx_{2n-1}(x), t\big(F_{Ap,Tx_{2n-1}}(\alpha x), F_{Bx_{2n-1},Sp}(2x - \alpha x)\big)\big)\Big) \end{aligned}$$

Taking $n \to \infty$ and $\alpha \to 1$, we have

$$F_{Ap,z}(kx) \geq t\Big(F_{Ap,z}(x), t\big(F_{z,z}(x), t(F_{Ap,z}(x), F_{z,z}(x))\big)\Big) \geq F_{Ap,z}(x),$$

which means that $Ap = z$. Hence $Ap = Sp = z$.

Similarly, since $A(X) \subset T(X)$, there exists $q \in X$, such that $z = Tq$. Then using (3.3) and taking $\alpha \to 1$, we have

$$F_{z,Bq}(kx) \geq t\Big(F_{z,z}(x), t\big(F_{Bq,z}(x), t(F_{z,z}(x), F_{Bq,z}(x))\big)\Big) \geq F_{Bq,z}(x)$$

which means that $Bq = z$. Hence $Bq = Tq = z$.

Therefore $z = Ap = Sp = Bq = Tq$. Since the maps A and S are weakly compatible, then $ASp = SAp$, *i. e.*, $Az = Sz$.

Now we show that z is a fixed point of A. By (3.3) and taking $\alpha \to 1$, we have

$$F_{Az,z}(kx) \geq t\Big(F_{Az,Sz}(x), t\big(F_{z,z}(x), t\big(F_{Az,z}(x), F_{z,Az}(x)\big)\big)\Big) \geq F_{Az,z}(x),$$

which means that $Az = z$. Hence $Az = Sz = z$. Similarly, a pair of maps B and T is weakly compatible By (3.3) and taking $\beta \to 1$, we have

$$F_{z,Bz}(kx) \geq t\Big(F_{z,z}(x), t\big(F_{Bz,Tz}(x), t\big(F_{z,Tz}(x), F_{Bz,z}(x)\big)\big)\Big) \geq F_{Bz,z}(x)$$

which means that $Bz = z$. Hence $Az = Sz = Bz = Tz = z$.

Thus z is a common fixed point of A, B, S and T. Finally in order to prove the uniqueness of z, suppose that z and w are common fixed points of A, B, S and T. Then by (3.3) and taking $\alpha \to 1$, we have

$$F_{z,w}(kx) \geq t\Big(F_{z,z}(x), t\big(F_{w,w}(x), t\big(F_{z,w}(x), F_{w,z}(x)\big)\big)\Big) \geq F_{z,w}(x)$$

which means that $z = w$. This completes the proof of the theorem. □

Remark 3.1. We note that Theorem 3.1 is still true if we replace the condition (3.3) by the following condition:

(3.5) there exists $k \in (0,1)$ such that

$$F_{Au,Bv}(kx) \geq \min\{F_{Au,Su}(x), F_{Bv,Tv}(x), F_{Au,Tv}(\alpha x), F_{Bv,Su}(2x - \alpha x)\}$$

for all $u, v \in X, x > 0$ and $\alpha \in (0,2)$.

Theorem 3.2. *Let A, B, S and T be self mappings on a complete Menger space (X, F, t), where t is continuous and $t(x,x) \geq x$ for all $x \in [0,1]$, satisfying conditions (3.1),(3.2) and*

(3.6) *there exists $k \in (0,1)$ such that*

$$F_{Au,Bv}(kx) \geq \min\{F_{Au,Su}(x), F_{Bv,Tv}(x), F_{Su,Tv}(x)\},$$

for all $u, v \in X, x > 0$. Then A, B, S and T have a unique common fixed point in X.

Proof. If the condition (3.6) is satisfied, then for any $\alpha \in (0,2)$, we have

$$\begin{aligned} F_{Au,Bv}(kx) &\geq \min\{F_{Au,Su}(x), F_{Bv,Tv}(x), F_{Su,Tv}(x)\}, \\ &\geq \min\{F_{Au,Su}(x), F_{Bv,Tv}(x), F_{Au,Tv}(\alpha x), F_{Bv,Su}(2x - \alpha x)\} \end{aligned}$$

Then using the Remark 3.1, the conclusion of Theorem 3.1 is still true. As a consequence of Theorems 2.1 and 3.1, we have the following: □

Theorem 3.3. *Let A, B, S and T be self mappings of a complete metric space (X, d), satisfying the following conditions:*

(3.7) *$A(X) \in T(X)$ and $B(X) \subset S(X)$,*

(3.8) *The pairs $\{A, S\}$ and $\{B, T\}$ are weakly compatible,*

(3.9) $d(Ax, By) \leq \max\{d(Ax, Sx), d(By, Ty), d(Sx, Ty), \frac{1}{2}[d(Ax, Ty) + d(Sy, By)]$

for all $x, y \in X$. Then A, B, S and T have a unique common fixed point in X.

Remark 3.2. (i) Theorem 3.1 improves result of Rashwan & Hedar [15] by dropping the condition of continuity.

(ii) Theorem 3.2 improves the main result of Dedeic & Sarapa [3]. Following Bylka [1], we consider the family G of functions $g : [0, \infty) \to [0, \infty)$ such that g is non-decreasing in $[0, \infty)$,

$$\lim_{n \to \infty} g^n(x) = \infty \tag{3.10}$$

for every $x > 0$. Here g^n denotes the nth iteration of g.

Theorem 3.4. *Let A, B, S and T be self mappings on a complete Menger space (X, F, t), where t is continuous and $t(x, x) \geq x$ for all $x \in [0, 1]$, satisfying conditions (3.1) and (3.2), and suppose that there exists a function $g \in G$ such that*

$$F_{Au,Bv}(x) \geq F_{Su,Tv}(g(x)), \tag{3.11}$$

for all $x > 0$ and for every $u, v \in X$. Then A, B, S and T have a unique common fixed point in X.

In order to prove the theorem we need the following lemma due to Rashwan & Hedar [15].

Lemma 3.2. *Let $g \in G$, then*

(i) *$g(x) \geq x$ for all $x \geq 0$,*

(ii) *if $F_{u,v}(x) \geq F_{u,v}(g(x))$ for some $x > 0$, then $u = v$.*

Proof of **Theorem 3.4**. Let $\{y_n\}$ be the sequence in X defined by (3.4). Then for all $x > 0, n = 1, 2, \ldots$, we have

$$F_{y_{2n+1},y_{2n+2}}(x) = F_{Ax_{2n},Bx_{2n+1}}(x) \geq F_{Sx_{2n},Tx_{2n+1}}(g(x)) = F_{y_{2n},y_{2n+1}}(g(x))$$

Similarly, we have

$$\begin{aligned} F_{y_{2n},y_{2n+1}}(x) &= F_{Bx_{2n-1},Ax_{2n}}(x) \\ &\geq F_{Sx_{2n-1},Tx_{2n}}(g(x)) = F_{Tx_{2n-1},Sx_{2n}}(g(x)) = F_{y_{2n-1},y_{2n}}(g(x)) \end{aligned}$$

Therefore

$$F_{y_n,y_{n+1}}(x) \geq F_{y_{n-1},y_n}(g(x)) \geq \cdots \geq F_{y_0,y_1}(g^n(x)). \tag{3.12}$$

Now, we show that the sequence $\{y_n\}$ is a Cauchy sequence in X. Let ε, λ be positive reals. Then for $m > n$ and $l = m - n$, by using (3.12), we have

$$\begin{aligned} F_{y_n,y_m}(\varepsilon) \geq\ & t(F_{y_n,y_{n+1}}(\varepsilon/l), F_{y_{n+1},y_m}(\varepsilon(l-1)/l)) \\ \geq\ & t(F_{y_0,y_1}(g^n(\varepsilon/l)), F_{y_{n+1},y_m}(\varepsilon(l-1)/l)) \\ \geq\ & t\big(F_{y_0,y_1}(g^n(\varepsilon/l)), t(F_{y_{n+1},y_{n+2}}(\varepsilon/l), F_{y_{n+2},y_m}(\varepsilon(l-2)/l))\big) \\ \geq\ & t\big(F_{y_0,y_1}(g^n(\varepsilon/l)), t(F_{y_0,y_1}(g^{n+1}(\varepsilon/l)), F_{y_{n+2},y_m}(\varepsilon(l-2)/l))\big) \\ \geq\ & t\big(t(F_{y_0,y_1}(g^n(\varepsilon/l)), F_{y_0,y_1}(g^{n+1}(\varepsilon/l))\big), F_{y_{n+2},y_m}(\varepsilon(l-2)/l)). \end{aligned} \tag{3.13}$$

From Lemma 3.2, we deduce that $g^n(\varepsilon/l) \leq g^{n+1}(\varepsilon/l)$ and by the hypothesis $t(a,a) \geq a$. Then from (3.13) we obtain

$$F_{y_n,y_m}(\varepsilon) \geq t(F_{y_0,y_1}(g^n(\varepsilon/l), F_{y_{n+2},y_m}(\varepsilon(l-2)/l)). \tag{3.14}$$

Using the induction argument we obtain from (3.14) that

$$\begin{aligned} F_{y_n,y_m}(\varepsilon) \geq\ & t\big(F_{y_0,y_1}(g^n(\varepsilon/l), t(F_{y_{m-2},y_{m-1}}(\varepsilon/l), F_{y_{m-1},y_m}(\varepsilon/l))\big) \\ \geq\ & t\big(F_{y_0,y_1}(g^n(\varepsilon/l), t(F_{y_0,y_1}(g^{m-2}(\varepsilon/l), F_{y_0,y_1}(g^{m-1}(\varepsilon/l))\big) \\ \geq\ & t\big(F_{y_0,y_1}(g^n(\varepsilon/l), F_{y_0,y_1}(g^{m-2}(\varepsilon/l))\big) \\ \geq\ & F_{y_0,y_1}(g^n(\varepsilon/l)). \end{aligned}$$

Hence, we can choose $N \leq n$ such that $F_{y_0,y_1}(g^N(\varepsilon/l)) > 1 - \lambda$, and then $F_{y_n,y_m}(\varepsilon) > 1 - \lambda$ for all $m > n \geq N$.

This means that $\{y_n\}$ is a Cauchy sequence in X and hence by completeness of X, this sequence converges to some point z in X. Also the subsequences $\{Ax_{2n}\}$, $\{Sx_{2n}\}$, $\{Bx_{2n-1}\}$ and $\{Tx_{2n-1}\}$ all converge to z. Since $B(X) \subset S(X)$, there exists a point $p \in X$ such that $z = Sp$. Then using (3.11), we have

$$F_{Ap,Bx_{2n-1}}(x) \geq F_{Sp,Tx_{2n-1}}(g(x)),$$

Taking the limit $n \to \infty$, we have

$$F_{Ap,z}(x) \geq F_{z,z}(g(x)) = 1,$$

which implies that $Ap = z$. Hence $Ap = Sp = z$.

Similarly, since $A(X) \subset T(X)$, there exists a point $q \in X$ such that $z = Tq$. Then using (3.11), we have

$$F_{z,Bq}(x) \geq F_{z,Tq}(g(x)) = F_{z,z}(g(x)) = 1,$$

which implies that $z = Bq$. Hence $Bq = Tq = z$. Therefore $z = Ap = Sp = Bq = Tq$. Since the maps A and S are weakly compatible, then $ASu = SAu$, *i. e.*, $Az = Sz$. Now we show that z is a fixed point of A. By (3.11), we have

$$F_{Az,z}(x) \geq F_{Sz,z}(g(x)) = F_{Az,z}(g(x)).$$

By Lemma 3.2 we have $Az = z$. Hence $Az = Sz = z$. Similarly, a pair of maps B and T are weakly compatible. By (3.11) we have

$$F_{z,Bz}(x) \geq F_{z,Tz}(g(x)) = F_{z,Bz}(g(x)).$$

By Lemma 3.2 we have $Bz = z$. Hence $Az = Sz = Bz = Tz = z$.

Thus z is a common fixed point of A, B, S and T. Finally in order to prove the uniqueness of z, suppose that z and w $(z \neq w)$ are common fixed points of A, B, S and T. Then by (3.11), we have

$$F_{z,w}(x) \geq F_{z,w}(g(x)).$$

By Lemma 3.2, we have $z = w$. This completes the proof of the theorem. □

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