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# Fuzzy Modeling and Control of Wheeled Mobile Robot

**Jin-Shik Kang**

**Dept. Telecommunication and Computer Eng. Cheju Nat'l Univ.**

**1 Ara 1, Cheju, 690-756, Korea**

**Abstract**

: In this paper, a new model, which is a Takagi-Sugeno fuzzy model, for mobile robot is presented. A controller, consisting of two loops the one of which is the inner state feedback loop designed for stability and the outer loop is a PI controller designed for tracking the reference input, is suggested. Because the robot dynamics is nonlinear, it requires the controller to be insensitive to the nonlinear term. To achieve this objective, the model is developed by well known T-S fuzzy model. The design algorithm of inner state-feedback loop is regional pole-placement. In this paper, regions, for which poles of the inner state feedback loop are lie in, are formulated by LMI's. By solving these LMI's, we can obtain the state feedback gains for T-S fuzzy system. And this paper shows that the PI controller is equivalent to the state feedback and the cost function for reference tracking is equivalent to the LQ( linear quadratic ) cost. By using these properties, it is also shown in this paper that the PI controller can be obtained by solving the LQ problem.

**Key Words** :. Takagi-Sugeno fuzzy model, wheeled mobile robot, PI, state feedback, LQ, LMI

## 1. Introduction

The wheeled mobile robot and its control schemes have been studied by many researchers with various degrees of application and success [1-6]. Most of these studies are concentrated on the development, control and planning the strategy of mobile robot. But, because of the wheeled mobile robot is modeled and controlled by a nonlinear system framework, its treatment is very complicated and conservative.

In this paper, a new model, which is a Takagi-Sugeno fuzzy model, for mobile robot is presented. A controller, consisting of two loops the one of which is the inner state feedback loop designed for stability and the outer loop is a PI controller designed for tracking the reference input, is suggested. Because the robot dynamics is nonlinear, it requires the controller to be insensitive to the nonlinear term. To achieve this objective, the model is developed by well known T-S fuzzy model. The design algorithm of inner state-feedback loop is regional pole-placement. In this paper, regions, for which poles of the inner state feedback loop are lie in, are formulated by LMI's. By solving these LMI's, we can obtain the state feedback gains for T-S fuzzy system. And this paper shows that the PI controller is equivalent to the state feedback and the cost function for reference tracking is equivalent to the LQ( linear quadratic ) cost. By using these properties, it is also shown in this paper that the PI controller can be obtained by solving the LQ problem

## 2. Fuzzy Modeling of Wheeled Mobile Robot

### 2.1. Dynamic Modeling of Wheeled Mobile Robot

The structure of the mobile robot, considered in this paper, is shown in Fig. 1. The relation between the forward velocity and the wheel angular velocity is described by

$$\begin{bmatrix} v \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ \frac{r}{2b} & -\frac{r}{2b} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \tag{1}$$

where, $v$ and $\dot{\phi}$ are forward and rotation velocities of the robot, respectively, and $r$ is the ratio of the wheel. And $b$ is the displacement from center robot to center of wheel. The kinetic equation is`

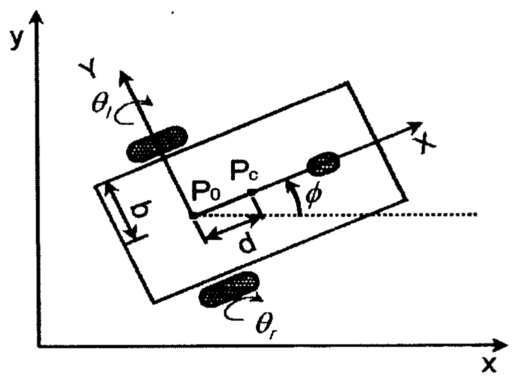


Fig. 1. The structure of robot

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \cos\phi & 0 \\ \sin\phi & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \dot{\phi} \end{bmatrix} \quad (2)$$

In order to derive the dynamic equations, we now define some variables.

$I_c$ : robot inertia except wheels and rotor
$I_w$ : motor rotor inertia for wheels and wheel axis
$I_m$ : motor rotor inertia for wheels and wheel diameter
$m$ : mass of robot except wheels and motor rotor
$m_c$ : mass of wheels and motor rotor

The dynamic equation of a of robot is described by[4,5]

$$M(q)\ddot{q} + V(q, \dot{q}) = E(q)\tau - \hat{A}^T(q)\lambda \quad (3)$$

where, $\lambda$ is Lagrangy multiplier, $\tau$ is the torque of each wheels, and $d$ is the displacement from the center of mass to the center of rotation, $q = \begin{bmatrix} x & y & \theta_1 & \theta_2 \end{bmatrix}^T$ and

$$M(q) = \begin{bmatrix} m & 0 & -m_c cd\sin\phi & m_c cd\sin\phi \\ 0 & m & m_c cd\cos\phi & -m_c cd\cos\phi \\ -m_c cd\sin\phi & m_c cd\cos\phi & I_c^2 + I_w & -I_c^2 \\ m_c cd\sin\phi & -m_c cd\cos\phi & -I_c^2 & I_c^2 + I_w \end{bmatrix}$$

$$V(q, \dot{q}) = \begin{bmatrix} 2m_c d\dot{\phi}^2\cos\phi \\ 2m_c d\dot{\phi}^2\sin\phi \\ 0 \\ 0 \end{bmatrix}, \quad \hat{A}(q) = \begin{bmatrix} -\sin\phi & \cos\phi & 0 & 0 \\ -\cos\phi & -\sin\phi & cb & cb \end{bmatrix}$$

$$E(q) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \tau = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}, \quad \lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}$$

In order to eliminate the Lagrange multiplier, we select the null space of $\hat{A}(q)$ as

$$S(q) = \begin{bmatrix} cb\cos\phi & cb\cos\phi \\ cb\sin\phi & cb\sin\phi \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (4)$$

then, equation (3) becomes

$$S^T(q)M(q)(S(q)\ddot{\theta} + \dot{S}(q)\dot{\theta}) + S^T(q)V(q, \dot{q}) = \tau \quad (5)$$

Equation (5) is a type of nonholonomic equation. This type of system cannot be linearized by using the state feedback.

We now present a LPD system model for the mobile robot. Equation (5) becomes

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \quad (6)$$

where,

$$M_{11} = M_{22} = mc^2b^2 + I_c^2 + I_w$$
$$M_{12} = M_{21} = mc^2b^2 - I_c^2$$
$$N_{11} = m_c cd(cb + r)\dot{\phi}, \quad N_{12} = m_c cd(cb - r)\dot{\phi},$$
$$N_{21} = -m_c cd(cb - r)\dot{\phi}, \quad N_{22} = -m_c cd(cb + r)\dot{\phi},$$

In equation (6), the variable $\dot{\phi}$ must be selected as a parameter. Because of the term $\dot{\phi}^2$, the dynamic equation is not linear with respect to the parameter value $\dot{\phi}$. After simple algebraic manipulation, we can obtain the LPD system representation of mobile robot system[7]. Define the state variables, input and the output as

$$x_1 \triangleq \theta_1, x_2 \triangleq \theta_2, x_3 \triangleq \dot{\theta}_1, x_4 \triangleq \dot{\theta}_2$$

$$u = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}, \quad y = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

then, the state space representation of mobile robot is

$$\dot{x}(t) = A_0 x(t) + A_1(\dot{\phi}(t))x(t) + B_0 u(t)$$
$$y(t) = Cx(t) \quad (7)$$

where,

$$A_0 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & a_{11} & a_{12} \\ 0 & 0 & a_{21} & a_{22} \end{bmatrix},$$

$$B_0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$a_{11} = \frac{\left[(2mc^2b^2+I_w)cb+(2I_c^2+I_w)r\right](m_c cd\dot{\phi})}{4mc^2b^2I_c^2+2mc^2b^2I_w+2I_c^2I_w+I_w^2}$$

$$a_{12} = \frac{\left[(2mc^2b^2+I_w)cb-(2I_c^2+I_w)r\right](m_c cd\dot{\phi})}{4mc^2b^2I_c^2+2mc^2b^2I_w+2I_c^2I_w+I_w^2}$$

$$a_{21} = -\frac{\left[(2mc^2b^2+I_w)cb-(2I_c^2+I_w)r\right](m_c cd\dot{\phi})}{4mc^2b^2I_c^2+2mc^2b^2I_w+2I_c^2I_w+I_w^2}$$

$$a_{22} = -\frac{\left[(2mc^2b^2+I_w)cb+(2I_c^2+I_w)r\right](m_c cd\dot{\phi})}{4mc^2b^2I_c^2+2mc^2b^2I_w+2I_c^2I_w+I_w^2}$$

$$b_{11}(=b_{22}) = \frac{mc^2b^2 + I_c^2 + I_w}{4mc^2b^2I_c^2 + 2mc^2b^2I_w + 2I_c^2I_w + I_w^2}$$

$$b_{12}(=b_{21}) = \frac{I_c^2 - mc^2b^2}{4mc^2b^2I_c^2 + 2mc^2b^2I_w + 2I_c^2I_w + I_w^2}$$

In the equation (7), controllability matrix $[A_0, B_0]$ is controllable and $[A_1, B_0]$ is controllable except when the variable $\dot{\phi}(t) = 0$.

### 2.2. Takagi-Sugeno Fuzzy Model of wheeled Mobile Robot

The fuzzy model proposed by Tagaki and Sugeno is described by IF-THEN rules which represent local linear input-output relations of a nonlinear system.[8] The main feature of a T-S fuzzy model is to express the local dynamics of each fuzzy rule by a linear system model.

The $i$-th T-S fuzzy model is [8]

If $z_1(t) = M_{i1}$ and ..... and $z_p(t) = M_{ip}$

$$\text{THEN} \begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) \\ y(t) = C_i x(t), \end{cases} \quad i = 1, 2, \cdots, r \tag{8}$$

The final outputs of the fuzzy systems are inferred as follows:

$$\begin{aligned} \dot{x}(t) &= \frac{\sum_{i=1}^{r} w_i(z(t))\left\{A_i x(t) + B_i u(t)\right\}}{\sum_{i=1}^{r} w_i(z(t))} \\ &= \sum_{i=1}^{r} h_i(z(t))\left\{A_i x(t) + B_i u(t)\right\} \end{aligned} \tag{9.a}$$

$$\begin{aligned} y(t) &= \frac{\sum_{i=1}^{r} w_i(z(t))C_i x(t)}{\sum_{i=1}^{r} w_i(z(t))} \\ &= \sum_{i=1}^{r} h_i(z(t))C_i x(t) \end{aligned} \tag{9.b}$$

Because of the elements of matrices $A_0, B_0$ and $C$ are constant, fuzzy model for wheeled mobile robot described by the equation (8) becomes

If $\dot{\phi}(t) = M_i$

$$\text{THEN} \begin{cases} \dot{x}(t) = \left[A_0 + A_{1i}\right]x(t) + B_0 u(t) \\ y(t) = Cx(t), \end{cases} \quad i = 1, 2, \cdots, r \tag{10}$$

The final outputs of the fuzzy systems are inferred as follows:

$$\begin{aligned} \dot{x}(t) &= \frac{\sum_{i=1}^{r} w_i(z(t))\left\{\left[A_0 + A_{1i}\right]x(t) + B_0 u(t)\right\}}{\sum_{i=1}^{r} w_i(z(t))} \\ &= \sum_{i=1}^{r} h_i(z(t))\left\{\left[A_0 + A_{1i}\right]x(t) + B_0 u(t)\right\} \\ &= A_0 x(t) + B_0 u(t) + \sum_{i=1}^{r} h_i(z(t))A_{1i}x(t) \\ y(t) &= Cx(t) \end{aligned} \tag{11}$$

The equation (11) is a state-space representation of wheeled mobile robot in which terms $A_0, B_0$ and $C$ are constant, and in which the only term $A_i$ is dependent on the term $\dot{\phi}(t)$ which equal to the deference of velocities from the right wheel to left wheel.

## 3. Control of Mobile Robot

We are now state a controller structure presented in this paper, and a new control design algorithm for mobile robot.

### 3.1 Controller Structure

The controller presented in this paper consists of two loops, one of which is the inner state feedback loop designed to reduce the nonlinearity of the plant and the outer loop is PI(proportional-integral) control loop designed to satisfy the performance requirements, i.e., tracking error, overshoot, etc. The controller schematic is shown in Fig. 2.

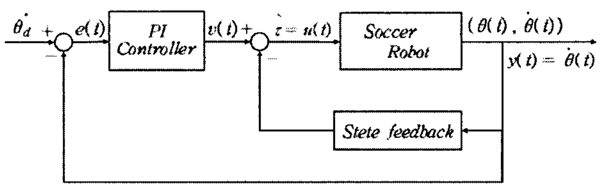


Fig. 2. Controller structure for the robot

### 3.2 State Feedback Design by Regional Pole-placement

The control input for inner state feedback loop considered in this paper is

$$\begin{aligned} u(t) &= \sum_{j=1}^{r} h_j(z(t))F_j(\mu(t))x(t) + v(t) \\ &\triangleq \left[F_0 + \sum_{j=1}^{r} h_j(z(t))F_{1j}(\mu(t))\right]x(t) + v(t) \end{aligned} \tag{12}$$

the inner closed loop dynamic equation is

$$\dot{x}(t) \triangleq A_{cl}x(t) + B_0 v(t) \tag{13}$$

where,

$$A_{cl} = \left[ [A_0 + B_0 F_0] + \sum_{i=1}^{r} h_i(z(t)) \sum_{j=1}^{r} h_j(z(t))[A_{1i} + B_0 F_{1j}(z(t))] \right]$$

The inner state-feedback loop is designed by regional pole-placement. The LMI region is defined following definition[9].

Definition 1. LMI regions are convex subset $D$ of the complex plan characterized by

$$D = \left\{ z \in C : L + Mz + M^T z^* \right\} \tag{14}$$

where $M$ and $L$ are fixed real matrices, and $z$ and $z^*$ are complex valued scalar and its complex conjugate pair.

The matrix valued function

$$f_D(z) \triangleq L + Mz + M^T z^* \tag{15}$$

is called the characteristic function of the region $D$.

In this paper, a complex region where the inner closed loop poles are lie in is described by figure 3.

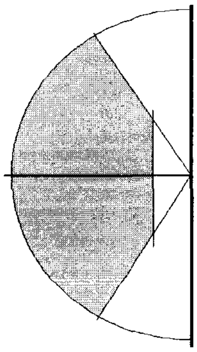

Figure 3. D Region

In the figure 3, the region $D$ is the common region of tree regions, i.e., inner parts of the conic sector, left part of the vertical line, and interior region of a circle. The characteristic function and its LMI representation of the region $D$ is consisted of tree parts. The characteristic function and its LMI representation of a conic sector is

$$f_{\sec}(z) = \begin{pmatrix} \sin\theta(z+z^*) & \cos\theta(z-z^*) \\ -\cos\theta(z-z^*) & \sin\theta(z+z^*) \end{pmatrix}$$

$$\begin{aligned} &\begin{pmatrix} \sin\theta(A_{cl}X + XA_{cl}^T) & \cos\theta(A_{cl}X - XA_{cl}^T) \\ -\cos\theta(A_{cl}X - XA_{cl}^T) & \sin\theta(A_{cl}X + XA_{cl}^T) \end{pmatrix} < 0 \\ &X > 0 \end{aligned} \tag{16}$$

The characteristic function and its LMI representation of a vertical line is

$$\begin{aligned} &f_{v-line}(z) = (z+z^*) - 2h \\ &A_{cl}X + XA_{cl}^T - 2h < 0 \\ &X > 0 \end{aligned} \tag{17}$$

The characteristic function and its LMI representation of a disk with center at (0, 0) and radios $r$ is

$$\begin{aligned} &f_D(z) = \begin{pmatrix} -r & z^* \\ z & -r \end{pmatrix} \\ &\begin{pmatrix} -r & XA_{cl}^T \\ A_{cl}X & -r \end{pmatrix} < 0 \\ &X > 0 \end{aligned} \tag{18}$$

Following tree theorems describe the regional pole-placement conditions and parts of the main results of this paper.

**Theorem 1**[9]: The closed loop poles lie in the LMI region $D$

$$D = \left\{ z \in C : L + Mz + M^T z^* \right\}$$

where,

$$L = L^T = \left[ \lambda_{jk} \right]_{1 \le j,\, k \le m}, \quad M = \left[ m_{jk} \right]_{1 \le j,\, k \le m}$$

if and only if there exists a symmetric matrix $X$ satisfying following four inequalities.

$$\begin{aligned} &\left[ \lambda_{jk} X + m_{jk} A_{cl} X + m_{kj} A_{cl}^T \right]_{1 \le j,\, k \le m} < 0 \\ &X > 0 \end{aligned} \tag{19}$$

proof) Proof of this theorem is omitted and refer Chilali and Gahinet's work [9].

We are now state a local pole placement of $i$-th fuzzy model. Define, $Y_0 \triangleq F_0 X$, $Y_i \triangleq F_i X$ then conditions of local pole placement is summarized by theorem 2.

**Theorem 2 :** The closed loop poles lie in the LMI region $D$ if and only if there exists a symmetric matrix $X$ satisfying following inequalities.

$$\begin{aligned} &[\lambda_{jk} X + m_{jk}(A_0 X + A_{1i} X + BY_0 + BY_i) \\ &\quad + m_{kj}(A_0 X + A_{1i} X + BY_0 + BY_i)^T ]_{1 \le j,\, k \le m} < 0 \\ &X > 0 \end{aligned} \tag{20}$$

then, the $i$-th state-feedback gain matrix is

$$F_0 = Y_0 X^{-1}, \quad F_i = Y_i X^{-1}$$

proof) The proof of this theorem is very simple extension of the results of Chilali and Gahinet's work [9].

The regional pole placement conditions of the global T-S fuzzy model are stated by following theorem.

Theorem 3: The closed loop poles lie in the LMI region $D$ if and only if there exists a symmetric matrix $X$ satisfying following inequalities.

$$X > 0$$

$$\begin{bmatrix} \lambda_{jk} X \\ +m_{jk}\left[A_0 X + B_0 Y_0 + \sum_{i=1}^{r} h_i(z(t)) \sum_{j=1}^{r} h_i(z(t))[A_{1_i} X + B_0 Y_{1j}]\right] \\ +m_{kj}\left[XA_0 + Y_0 B_0^T + \sum_{i=1}^{r} h_i(z(t)) \sum_{j=1}^{r} h_i(z(t))[XA_{1_i} + Y_{1j}^T B_0^T]\right]^T \end{bmatrix}_{1\le j,\, k\le m} < 0 \quad (21)$$

then, state-feedback gain matrices are

$$F_0 = Y_0 X^{-1}, \quad F_j = Y_j X^{-1}$$

Proof) Omitted. Because the proof of this theorem is simple extension of the results of Chilali and Gahinet's work [8].

The design procedure is summarized as formulate LMI and solve it. The LMI to be solved can be obtained by rewriting the equation (21) in to forms of the equation (16), equation (17) and equation (18). Solution can be obtained easily by using LMI TOOLBOX in MATLAB.

### 3.3 Design of PI Control

For the mobile robot, the reference input signal varies rapidly, and the design requires small tracking error, fast response, and small overshoot. These requirements are easily satisfied by using the well-known PI controller. The general description of the PI controller is

$$v(t) = K_P e(t) + K_I \int e(t)dt \quad (22)$$

Based on the definition of the state variables, and after simple algebraic manipulation and some modification, equation (22) becomes

$$\begin{aligned} v(t) &= \begin{bmatrix} K_I & K_P \end{bmatrix} \begin{bmatrix} \theta_d(t) \\ \dot{\theta}_d(t) \end{bmatrix} - \begin{bmatrix} K_I & K_P \end{bmatrix} \begin{bmatrix} \theta(t) \\ \dot{\theta}(t) \end{bmatrix} \\ &= -\begin{bmatrix} K_I & K_P \end{bmatrix} x(t) + \begin{bmatrix} K_I & K_P \end{bmatrix} x_d(t). \end{aligned} \quad (23)$$

We note that the equation (23) show that the PI controller described by equation (22) is equivalent to the state feedback. And hence the closed loop dynamic equation is obtained by

$$\dot{x}(t) = \begin{bmatrix} A_0 - BF_0 \\ +\sum_{i=1}^{r} h_i(z(t)) \sum_{j=1}^{r} h_i(z(t)) \left[A_i - BF_j - B[K_{I_j} \quad K_{P_j}]\right] \end{bmatrix} x(t) + B\begin{bmatrix} K_I & K_P \end{bmatrix} x_d(t) + d(t) \quad (24)$$

where,

$$d(t) \triangleq E\{(A_1 - BF_1)(\dot{\phi}(t))\} x(t) + \text{other noise terms.} \quad (25)$$

In the equation (25), the term $E\{(A_1 - BF_1)(\dot{\phi}(t))\}$ means the error between the actual system and model.

In order to translate the PI-control problem in to LQ optimal problem, some modification is needed on the closed loop dynamic equation. It is described as follows

$$\dot{x}(t) \triangleq A_F x(t) + B\hat{v}(t) + \hat{d}(t) \quad (26)$$

where,

$$A_F = \left[A_0 - BF_0 + \sum_{i=1}^{r} h_i(z(t)) \sum_{j=1}^{r} h_j(z(t))\left[A_i - BF_j\right]\right]$$

$$\bar{v}(t) = \sum_{j=1}^{r} h_j(z(t))[K_{I_j} \quad K_{P_j}]x(t)$$

$$\hat{d}(t) = B\begin{bmatrix} K_I & K_P \end{bmatrix} x_d(t) + d(t)$$

The PI controller must be designed to guarantee robust performance, i.e., well tracking the reference input. For satisfying this objective, let us consider the following cost function for i-th model

$$\min J = \int [e^T(t)Qe(t) + \bar{v}_i^T(t)R\bar{v}_i(t)]dt \quad (27)$$

by using the definition of state, error, input and after simple algebraic manipulation, equation (27) becomes

$$\begin{aligned} \min J &= \int [x^T Q_1 x + x_d^T Q_1 x_d - 2x^T Q_1 x_d + \bar{v}_i^T(t)R\bar{v}_i(t)]dt \\ &= \int [x^T Q_1 x + \bar{v}_i^T(t)R\bar{v}_i(t)]dt + \int [x_d^T Q_1 x_d - 2x^T Q_1 x_d]dt \end{aligned} \quad (28)$$

where,

$$Q_1 = C^T QC$$

The minimum cost is obtained by using the relationship

$$\min J \Rightarrow \min J = \int [x^T Q_1 x + \bar{v}_i^T(t)R\bar{v}_i(t)]dt \quad (29)$$

where, $K$ is the solution of following Riccati equation.

$$K_i A_{F_i} + A_{F_i}^T K_i - K_i B_0 R^{-1} B_0^T K_i + Q_1 = 0 \quad (30)$$

The cost function described by the equation (29) is equivalent to the general LQ cost. The PI gain is obtained by

$$[K^T{}_{I_i} \; K^T{}_{P_i}] = -R^{-1} B^T K_i \quad (31)$$

By using the cheap control properties, solutions of ARE described by equation (30) become unique and easily computed.

### 3.4 Controller

The control input suggested in this paper is described by

$$\begin{aligned} u(t) = &\sum_{j=1}^{r} h_j(\dot{\phi}(t)) \left[F_j + [K_{I_j} \; K_{P_j}]\right] x(t) \\ &+ \sum_{j=1}^{r} h_j(\dot{\phi}(t))[K_{I_j} \; K_{P_j}] x_d(t) \end{aligned} \quad (31)$$

The design procedure is design state feedback and the next is PI controller.

The stability of the closed loop is guaranteed because the inner loop is designed by robust pole-placement algorithm and the outer loop PI controller is designed by LQR-algorithm which known as robust optimal controller.

## 4. Simulation

In simulation, the robot considered is MIROSOT soccer robot, and detailed specifications are summarized in the table 1.

Table 1. The specifications of MIROSOT robot

| | |
|---|---|
| Size | $70 \times 70 \times 70$ mm |
| Wheel diameter | 45 mm |
| Rpm | 8000 |
| Gear ratio | 8 : 1 |

The mass of the robot is 0.0612 Kg m/sec$^2$ and the mass of wheels is 0.0051 kg m/sec$^2$ . And other parameters used in this paper were

$$b = 35mm,\ \ c = r/2b, d = 10mm.$$

The robot inertia except wheels and rotor is 0.05 $Kg\,cm\sec^2$ and motor rotor inertia for wheels and wheel axis is 0.0176 $Kg\,cm\sec^2$ . These parameters were actually measured and computed for MIROSOT robot designed Yujin Robotics corp. In this paper, the maximum velocity of the wheel was the maximum velocity of the motor specification.

By using parameters described above, state space matrices for the mobile robot are

$$A_0 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},\ A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -40.7370\dot{\phi} & 40.5002\dot{\phi} \\ 0 & 0 & -40.5002\dot{\phi} & 40.7370\dot{\phi} \end{bmatrix}$$

$$B_0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.1055 & -22.0917 \\ -22.0917 & 0.1055 \end{bmatrix},\ C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Membership functions of this paper are shown in the figure 4. System matrices, state feedback gains for pole-placement, PI-gains are shown by the table 2 which is shown in the last page of this paper.

Simulations by using designed controller are shown in Fig. 5 to Fig. 11 for various possible input signals. The sinusoidal, pulse and saw-tooth signal was selected as a test signal because these signals are frequently used for the mobile robot test. Fig. 5 is a sample of a noise profile used in this paper. The maximum value of noise is selected as 15 percent of maximum value of the wheel velocity. This type of noise is added to control inputs in every simulation.

Fig. 6 and Fig. 7 are simulation results for the sinusoidal reference inputs. In the Fig. 6, the desired and actual velocities are shown. Tracking errors are shown in Fig. 7.

Fig. 8 and Fig. 9 are simulation results for the pulse command inputs. It is shown in the Fig. 8 that tracking errors jump at t=0 and t=5 because the signs of the command signals e change. Also, it is shown in the Fig. 8 that these abrupt changes in the reference signal also could be overcome by the controller presented.

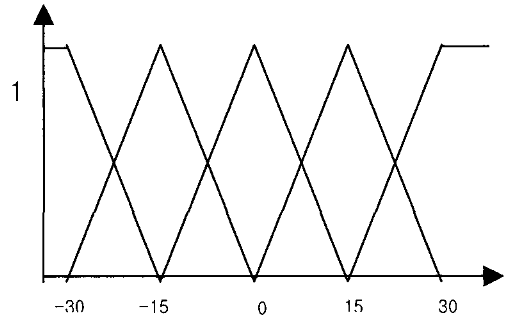


Figure. 4. Membership functions

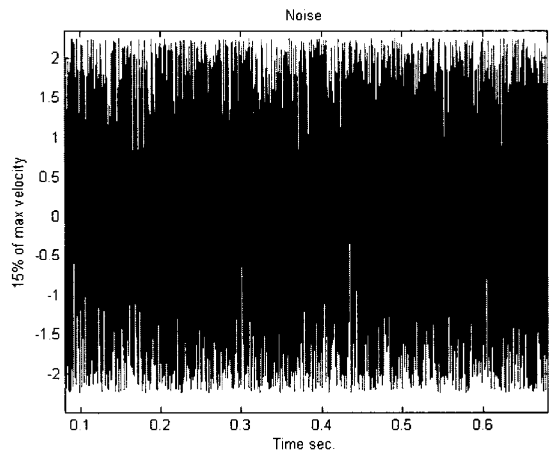


Figure. 5. Sample of noise profile

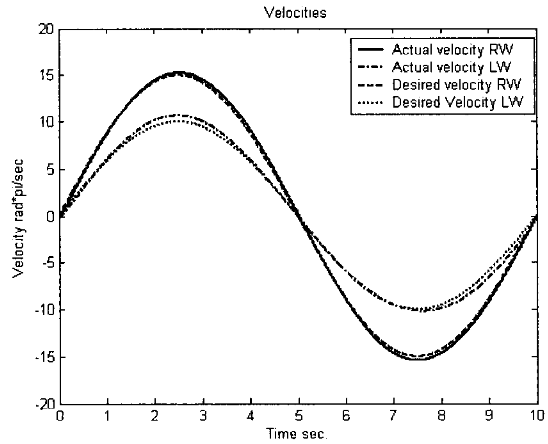


Figure. 6. Desired and actual velocities

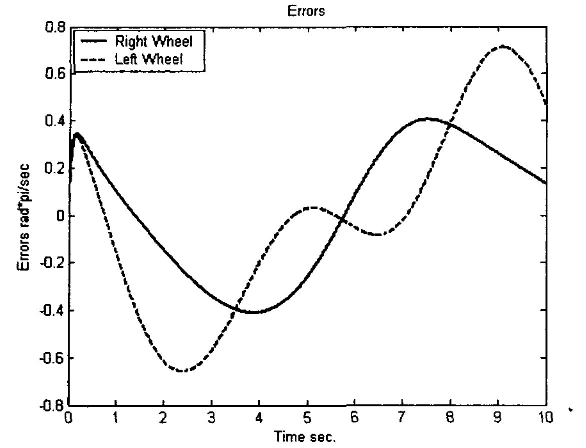


Figure. 7. Errors for sinusoidal signal

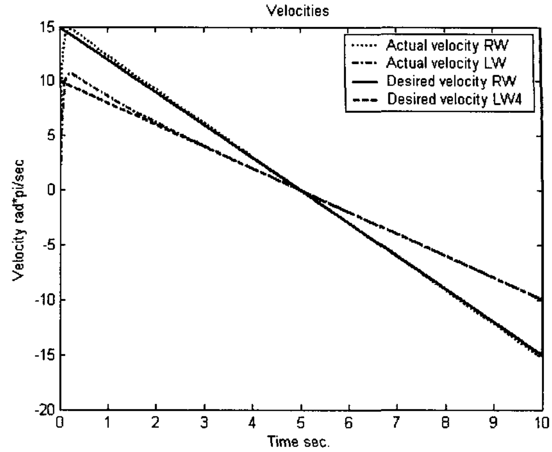


Figure 10 Desired and actual velocities of saw signal

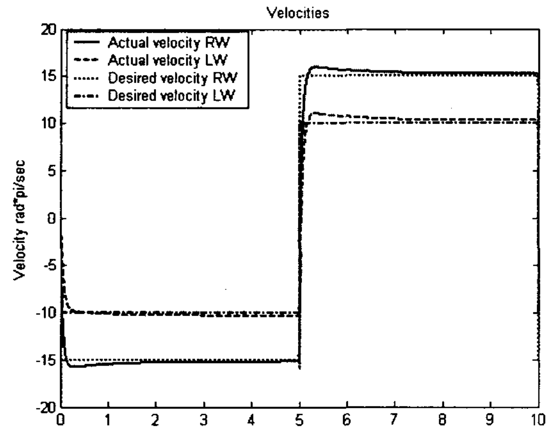


Figure. 8. Desired and actual velocities for pulse signal

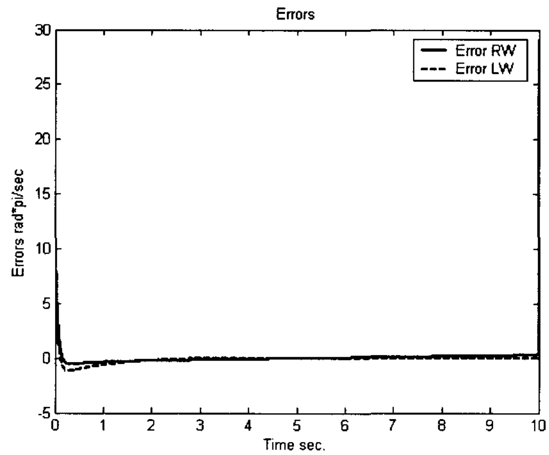


Figure. 11. Errors of saw signal

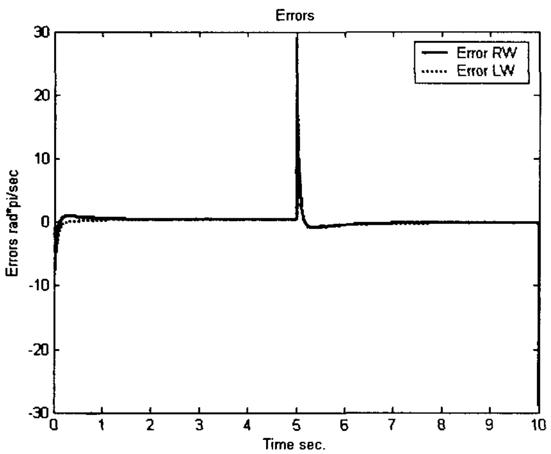


Figure. 9. Errors of left and right wheels for pulse signal

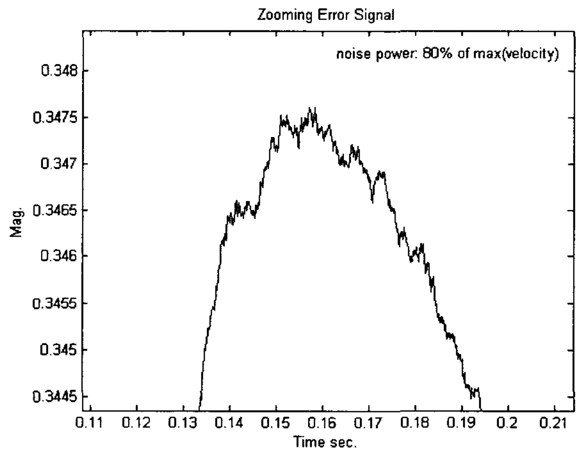


Figure. 12. Enlarging error signal

Figure 7 and figure 9 shows that noise signals added to the input channels cannot affect to the response. The figure 12 is the enlarging result of the error signal when reference inputs are sinusoidal signal and noise power is 30% of maximum velocity.

Fig. 10 and Fig. 11 are simulation results for saw reference inputs. In the Fig. 10, the desired and actual velocities are shown. Tracking errors are shown in Fig. 11

Table 2. Simulation parameters, state feedback gains, PIgains

| | *System matrix A1* | *State-feedback gain* | *PI gain Q=100 I, R=0.0001 I* |
|---|---|---|---|
| *-30* | *1.3036 -1.2960* | *0.0286 5.9854 54.7421 -54.2148* | *310.3 0.0280 314.9062 0.0063* |
| | *1.2960 -1.3036* | *5.9854 0.0286 -53.6897 -55.2573* | *0.0280 0.0280 0.0063 314.9062* |
| *-15* | *-0.6518 -0.6480* | *0.0286 5.9854 27.3743 -26.4229* | *310.3 0.0280 314.9062 0.0063* |
| | *0.6480 -0.6518* | *5.9854 0.0286 -26.1604 -27.6254* | *0.0280 0.0280 0.0063 314.9062* |
| *0* | *0 0* | *0.0286 5.9854 0.0065 1.3689* | *310.3 0.0280 314.9062 0.0063* |
| | *0 0* | *55.9854 0.0286 1.3689 0.0065* | *0.0280 0.0280 0.0063 314.9062* |
| *15* | *0.6518 0.6480* | *0.0286 5.9854 -27.3613 29.1608* | *310.3 0.0280 314.9062 0.0063* |
| | *-0.6480 0.6518* | *5.9854 0.0286 28.8983 27.6384* | *0.0280 0.0280 0.0063 314.9062* |
| *30* | *1.3036 1.2960* | *0.0286 5.9854 -54.7291 56.9527* | *310.3 0.0280 314.9062 0.0063* |
| | *-1.2960 1.3036* | *55.9854 0.0286 56.4276 55.2703* | *0.0280 0.0280 0.0063 314.9062* |

The figure 12 shows that the response of the robot is not affected by noises added to the input. By this result, it is noted that the proposed controller guarantee the robust stability with model errors and the robust performance with input noise.

## 5. Conclusion

In this paper, we studied the modeling and control of a wheeled mobile robot. This paper presents tree main results.

The one is T-S fuzzy model of wheeled mobile robot which has two driving wheels. The T-S fuzzy model of wheeled mobile robot presented in this paper is very simple in the form and can be easily treated in the design of controller.

The control structure presented in this paper consists of two loops. The first one is the state feedback loop and the other is PI control loop. The state-feedback loop is designed for the poles of the state feedback loop lie in the desired region. For achieving this objective, regions, in which poles of the state feedback loop are lie, are formulated by LMI equations and a algorithm of computing state feedback gains is presented which is the second main result of this paper.

The last one is PI controller. The cost function of command tracking of PI controller transformed in to the LQ cost and it is shown that the minimization of tracking cost is equivalent to the LQ optimal cost. By using these properties, the PI gain is obtained by solving LQ optimal control problem.

This paper shows that the presented controller have robust stability and performance with modeling error and noise added to the input channel.

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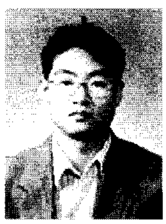

**Jin-Shik Kang**

He received BS degree(1986) in Telecommunication engineering from Cheju National University and received MS(1988) and Ph.D.(1992) degrees from Hanyang University. His research interests are control theory and Application, mobile robot, nonlinear system control and intelligent control.