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Intuitionistic Fuzzy Subgroupoids

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Abstract

In this paper, we introduce the concepts of intuitionistic fuzzy products and intuitionistic fuzzy subgroupoids. We investigate some properties of products and subgroupoids

Key words : intuitionitic fuzzy product, intuitionistic fuzzy subgroupoid, f-invariant, have the sup property.

1. Introduction

After the introduction of the concept of fuzzy sets by Zadeh [12], several researchers[1,7,10,11] have applied the notion of fuzzy sets to group theory.

As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov[2]. Recently, Çoker and his colleagues [5,6,8], and S.J.Lee and E.P.Lee[9] introduced the concept of intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. In 1989, R.Biswas[4] introduced the concept of intuitionistic fuzzy subgroups and stidied some of it's properties.

In 2003, Baldev Banerjee and Dhiren Kr. Basnet[3] investigated intuitionistic fuzzy subrings and ideals using intuitionistic fuzzy sets.

In this paper, we introduce the concepts of intuitionistic fuzzy products and intuitionistic fuzzy subgroupoids. And we study some properties of products and subgroupoids.

2. Preliminaries

We will list some concepts and results needed in the later sections.

Definition 1.1[2]. Let X be a nonempty set. An *intuitionistic fuzzy set*(in short, *IFS*) on X is an object having the form

$$A = \{(x,\ \mu_A(x),\ \nu_A(x)) : x \in X\}$$

where the functions $\mu_A : X \to I$ and $\nu_A \ X \to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\nu_A(x)$) of each $x \in X$ to A, respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each $x \in X$.

For the sake of simplicity, we shall use the symbol $A = (x,\ \mu_{A,}\ \nu_A)$ or $A = (\mu_{A,}\ \nu_A)$ for the IFS $A = \{(x,\ \mu_A(x),\ \nu_A(x)) : x \in X\}$(See, [2]).

We will denote the set of all he IFSs in X as IFS(X).

Definitions 1.2[2]. Let X be a nonempty set and let $A = (\mu_{A,}\ \nu_A)$ and $B = (\mu_{B,}\ \nu_B)$ be IFSs on X. Then:

(1) $A \subset B$ iff $\mu_A \le \mu_B$ and $\nu_A \ge \nu_{B.}$
(2) $A = B$ iff $A \subset B$ and $B \subset A$.
(3) $A^c = (\nu_{A,}\ \mu_A)$.
(4) $A \cap B = (\mu_A \wedge \mu_B,\ \nu_A \vee \nu_B)$.
(5) $A \cup B = (\mu_A \vee \mu_B,\ \nu_A \wedge \nu_B)$.
(6) $[\ ]A = (\mu_A, 1-\mu_A), \langle\rangle\ A = (1-\nu_A,\ \nu_A)$.

Definition 1.3[5]. Let $\{A_i\}_{i \in J}$ be an arbitrary family of IFSs in X, where $A_i = (\mu_{A_i}, \nu_{A_i})$ for each $i \in J$. Then:

(a) $\cap A_i = (\wedge \mu_{A_i},\ \vee \nu_{A_i})$.
(b) $\cap A_i = (\vee \mu_{A_i},\ \wedge \nu_{A_i})$.

Definition 1.4[5]. $0_\sim = (0,\ 1)$ and $1_\sim = (1,\ 0)$.

Result 1.A[5, Corollary 2.8]. Let $A,\ B,\ C,\ D$ be IFSs in X. Then:

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(1) $A \subset B$ and $C \subset D \Rightarrow A \cup C \subset B \cup D$ and $A \cap C \subset B \cap D$
(2) $A \subset B$ and $A \subset C \Rightarrow A \subset B \cap C$.
(3) $A \subset B$ and $B \subset C \Rightarrow A \cup B \subset C$.
(4) $A \subset B$ and $B \subset C \Rightarrow A \subset C$.
(5) $(A \cap B)^c = A^c \cap B^c$, $(A \cap B)^c = A^c \cup B^c$
(6) $A \subset B \Rightarrow B^c \subset A^c$
(7) $(A^c)^c = A$.
(8) $1_{\sim}^c = 0$, $0^c = 1_{\sim}$.

Definition 1.5[5]. Let X and Y be nonempty sets and let $f : X \rightarrow Y$ a mapping. Let $A = (\mu_A, \nu_A)$ be an IFS in X and $B = (\mu_B, \nu_B)$ be IFS on Y.

Then
(a) the *preimage* of B under f, denoted by $f^{-1}(B)$, is the IFS in X defined by :

$$f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\nu_B)),$$

where $f^{-1}(\mu_B) = \mu_B \circ f$.
(b) the *image* of A under f, denoted by $f(A)$, is the IFS in Y defined by :

$$f(A) = (f(\mu_A), f(\nu_A)),$$

where for each $y \in Y$.

$$f(\mu_A)(y) = \begin{cases} \bigvee_{x \in f^{-1}(y)} \mu_A(x) & \text{if } f^{-1}(y) \neq \emptyset, \\ 0 & \textit{otherwise}, \end{cases}$$

and

$$f(\nu_A)(y) = \begin{cases} \bigwedge_{x \in f^{-1}(y)} \nu_A(x) & \text{if } f^{-1}(y) \neq \emptyset, \\ 1 & \textit{otherwise}. \end{cases}$$

Result 1.B[5, Corollary 2.10]. Let A, $A_i (i \in J)$ be IFSs in X, let B, $B_j (j \in K)$ IFSs in Y and let $f : X \rightarrow Y$ a mapping. Then

(1) $A_1 \subset A_2 \Rightarrow f(A_1) \subset f(A_2)$.
(2) $B_1 \subset B_2 \Rightarrow f^{-1}(B_1)(f^{-1}(B_2))$.
(3) $A \subset f^{-1}(f(A))$.
If f is injective, then $A = f^{-1}(f(A))$.
(4) $f(f^{-1}(B)) \subset B$.
If f is surjective, then $f(f^{-1}(f(B)) = B$.
(5) $f^{-1}(\cup B_j)) = \cap f^{-1}(B_j)$.
(6) $f^{-1}(\cap B_j)) = \cup f^{-1}(B_j)$.
(7) $f(\cup A_i)) = \cup f(A_i)$.
(8) $f(\cap A_i)) \subset \cap f(A_i)$.
If f is injective, then $f(\cap A_i)) = \cap f(A_i)$.
(9) $f(1_{\sim}) = 1_{\sim}$, if f is surjective and $f(0_{\sim}) = 0_{\sim}$.
(10) $f^{-1}(1_{\sim}) = 1_{\sim}$ and $f^{-1}(0_{\sim}) = 0_{\sim}$.
(11) $[f(A)]^c \subset f(A^c)$, if f is surjective.
(12) $f^{-1}(B^c) = [f^{-1}(B)]^c$.

Definition 1.6[9]. Let $\lambda, \mu \in (0, 1]$ and $\lambda + \mu \leq 1$. An *intuitionistic fuzzy point*(in short, *IFP*) $x_{(\lambda, \mu)}$ of X is the IFS in X defined by

$$x_{(\lambda, \mu)}(y) = \begin{cases} (\lambda, \mu) & \text{if } y = x \\ (0, 1) & \text{if } y \neq x \end{cases} \text{ for each } y \in Y.$$

In this case, x is called the *support* of $x_{(\lambda, \mu)}$ and λ and μ are called the *value* and *nonvalue* of $x_{(\lambda, \mu)}$, respectively.

An IFP $x_{(\lambda, \mu)}$ is said to *belong to* an IFS $A = (\mu_A, \nu_A)$ in X, denoted by $x_{(\lambda, \mu)} \in A$, IF $\lambda \leq \mu_A(x)$ and $\mu \geq \nu_A(x)$.

It is clear that an intuitionistic fuzzy point $x_{(\lambda, \mu)}$ can be represented by an ordered pair of fuzzy points as follows :

$$x_{(\lambda, \mu)} = (x_\lambda, 1 - x_{1-\mu}).$$

We will denote the set of all IFPs in X as $IF_p(X)$.

Result 1.C[9, Theorem 2.2]. Let $A = (\mu_A, \nu_A)$ be an IFS in X and let $x_{(\lambda, \mu)} \in IF_p(X)$. Then

$x_{(\lambda, \mu)} \in A$ if and only if $x_\lambda \in \mu_A$ and $x_{1-\mu} \in 1 - \nu_A$.

Result 1.D[9, Theorem 2.3]. Let $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ be IFSs in X. Then $A \subset B$ if and only if for each $x_{(\lambda, \mu)} \in IF_p(X)$, $x_{(\lambda, \mu)} \in A$ implies $x_{(\lambda, \mu)} \in B$.

Result 1.E[9, Theorem 2.4]. Let $A = (\mu_A, \nu_A)$ be an IFS in X. Then

$$A = \cup \{x_{(\lambda, \mu)} \mid x_{(\lambda, \mu)} \in A\}.$$

Definition 1.7[5]. Let X be a set and let $\lambda, \mu \in I$ with $0 \leq \lambda + \mu \leq 1$. Then the IFS $C_{(\lambda, \mu)}$ in X is defined by: for each $x \in X$, $C_{(\lambda, \mu)}(x) = (\lambda, \mu)$, i.e., $\mu_{C_{(\lambda, \mu)}}(x) = \lambda$ and $\nu_{C_{(\lambda, \mu)}}(x) = \mu$.

2. Intuitionistic fuzzy products

Definition 2.1. Let $(X, .)$ be a groupoid and let $A, B \in IFS(X)$. Then the *intuitionistic fuzzy product* of A and B, $A \circ B$, is defined as follows : for any $x \in X$,

$$\mu_{A \circ B}(x) = \begin{cases} \bigvee_{yz = x} [\mu_A(y) \wedge \mu_B(z)] & \text{for each} (y, z) \in X \times X \text{ with } yz = x, \\ 0 & \textit{otherwise}, \end{cases}$$

and

$$\nu_{A \circ B}(x) = \begin{cases} \bigwedge_{yz = x} [\nu_A(y) \vee \nu_B(z)] & \text{for each} (y, z) \in X \times X \text{ with } yz = x, \\ 1 & \textit{otherwise}. \end{cases}$$

Proposition 2.2. Let " $\circ$ " be as above, let $x_{(\alpha, \beta)}, y_{(\alpha', \beta')} \in IF_p(X)$ and let $A, B \in IFS(X)$. Then :

(1) $x_{(\alpha,\beta)} \circ y_{(\alpha',\beta')} = (xy)_{(\alpha\wedge\alpha',\beta\vee\beta')}$.
(2) $A \circ B = \bigcup_{x_{(\alpha,\beta)}\in A,\, y_{(\alpha',\beta')}\in B} x_{(\alpha,\beta)} \circ y_{(\alpha',\beta')}$.

Proof. (1) Let $z \in X$. Then :

$$\mu_{x_{(\alpha,\beta)}\circ y_{(\alpha',\beta')}}(z) = \begin{cases} \bigvee_{x'y'=z} [\mu_{x_{(\alpha,\beta)}}(x') \wedge \mu_{y_{(\alpha',\beta')}}(y')] & \text{for each}(x',y') \in X\times X \text{ with } x'y' = z \\ 0 & otherwise \end{cases}$$
$$= \begin{cases} \alpha\wedge\alpha' & \text{if } xy = z, \\ 0 & \text{otherwis}, \end{cases}$$

and

$$\nu_{x_{(\alpha,\beta)}\circ y_{(\alpha',\beta')}}(z) = \begin{cases} \bigwedge_{x'y'=z} [\nu_{x_{(\alpha,\beta)}}(x') \vee \mu_{y_{(\alpha',\beta')}}(y')] & \text{for each}(x',y') \in X\times X \text{ with } x'y' = z \\ 0 & otherwise \end{cases}$$
$$= \begin{cases} \beta\vee\beta' & \text{if } xy = z, \\ 1 & \text{otherwis}. \end{cases}$$

Hence $x_{(\alpha,\beta)} \circ y_{(\alpha',\beta')} = (xy)_{(\alpha\wedge\alpha',\beta\vee\beta')}$.

(2) Let $C = \bigcup_{x_{(\alpha,\beta)}\in A,\, y_{(\alpha',\beta')}\in B} x_{(\alpha,\beta)} \circ y_{(\alpha',\beta')}$ Then, by the proof of Proposition 1.1(ii) in [10], we have :

$$\mu_{A\circ B}(w) = \mu_C(w), \text{ ie., } \mu_{A\circ B} = \mu_C.$$

Thus it is sufficient to show that $\nu_{A\circ B}(w) = \nu_C(w)$.

Let $w \in X$ and we may assume that there exist $u, v \in X$ such that $uv = w$, $\nu_A(u) \neq 1$ and $\nu_B(u) \neq$without loss of generality. Then:

$$\nu_{A\circ B}(w) = \bigwedge_{uv=w} [\nu_A(u) \vee \nu_B(v)]$$
$$\leq \bigwedge_{uv=w} \bigwedge_{x_{(\alpha,\beta)}\in A,\, y_{(\alpha',\beta')}\in B} [\nu_{x_{(\alpha,\beta)}}(u) \vee \nu_{y_{(\alpha',\beta')}}(v)]$$
$$= \nu_C(w).$$

Since $u_{(\mu_A(u),\,\nu_A(u))} \in A$ and $v_{(\mu_B(v),\,\nu_B(v))} \in B$,

$$\nu_C(w) = \bigwedge_{x_{(\alpha,\beta)}\in A,\, y_{(\alpha',\beta')}\in B} \bigwedge_{uv=w} [\nu_{x_{(\alpha,\beta)}}(u) \vee \nu_{y_{(\alpha',\beta')}}(v)]$$
$$= \bigwedge_{uv=w} (\bigwedge_{x_{(\alpha,\beta)}\in A,\, y_{(\alpha',\beta')}\in B} [\nu_{x_{(\alpha,\beta)}}(u) \vee \nu_{y_{(\alpha',\beta')}}(v)])$$
$$\leq \bigwedge_{uv=w} [\nu_{u(\mu_A(u),\nu_A(u))}(u) \vee \nu_{v(\mu_A(v),\nu_A(v))}(v)]$$
$$= \bigwedge_{uv=w} [\nu_A(u) \vee \nu_B(u)]$$
$$= \nu_{A\circ B}(w).$$

Thus $\nu_{A\circ B} = \nu_C$. Hence

$$A \circ B = \bigcup_{x_{(\alpha,\beta)}\in A,\, y_{(\alpha',\beta')}\in B} x_{(\alpha,\beta)} \circ y_{(\alpha',\beta')}.$$

The following proposition holds from Definition 2.1.

Proposition 2.3. Let $(X, \cdot)$ be a groupoid and let " $\circ$ " be as above.

(1) If " $\cdot$ " is associative [resp. commutative] in X, then so is " $\circ$ " in IFS(X).

(2) If " $\cdot$ " has a unity $e \in X$, then $e_{(1,0)} \in IF_p(X)$ is a unity of " $\circ$ "in $IFS(X)$, i.e., $A \circ e_{(1,0)} = A = e_{(1,0)} \circ A$ for each $A \in IFS(X)$.

3. Intuitionistic fuzzy subgroupoids and ideals

Definition 3.1. Let $(G, \cdot)$ be a groupoid and let $0_\sim \neq A \in IFS(G)$. Then A is called an *intuitionistic fuzzy subgroupoid* in G(in short, $IFGP$ in G) if $A \circ A \subset A$.

The followings are the immediate results of Definition 2.1 and Definition 3.1.

Proposition 3.2. Let $(G, \cdot)$ be a groupoid and let $0_\sim \neq A \in$ IFS(G) Then the followings are equivalent :

(1) A is an IFGP in G.

(2) For any $x_{(\alpha,\beta)}, y_{(\alpha',\beta')} \in A$, $x_{(\alpha,\beta)} \circ y_{(\alpha',\beta')} \in A$, i.e., $(A, \circ)$ is a groupoid.

(3) For any $x, y \in X$, $\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y)$ and $\nu_A(xy) \leq \nu_A(x) \vee \nu_A(y)$.

Proposition 3.3. Let A be an IFGP in a groupoid $(G, \cdot)$.

(1) If " $\cdot$ " is associative in G, then so is " $\circ$ " in A, i.e., for any $x_{(\alpha,\beta)}, y_{(\alpha',\beta')}, z_{(\alpha'',\beta'')} \in A$,
$$(x_{(\alpha,\beta)} \circ y_{(\alpha',\beta')}) \circ z_{(\alpha'',\beta'')} = x_{(\alpha,\beta)} \circ (y_{(\alpha',\beta')} \circ z_{(\alpha'',\beta'')}).$$

(2) If " $\cdot$ " is commutative in G, then so is " $\circ$ " in A, i.e for any $x_{(\alpha,\beta)}, y_{(\alpha',\beta')} \in A$, $x_{(\alpha,\beta)} \circ y_{(\alpha',\beta')} = y_{(\alpha',\beta')} \circ x_{(\alpha,\beta)}$.

(3) If " $\cdot$ " has a unity $e \in G$, then $e_{(1,0)} \circ x_{(\alpha,\beta)} = x_{(\alpha,\beta)} = x_{(\alpha,\beta)} \circ e_{(1,0)}$ for each $x_{(\alpha,\beta)} \in A$.

From Proposition 3.2, we can define an intuitionistic fuzzy subgroupoid in a groupoid G as follows.

Definition 3.1′. Let $(G, \cdot)$ be a groupoid and let $A \in IFS(X)$. Then A is called an *intuitionistic fuzzy subgroupoid*(in short, $IFGP$) of G if for any $x, y \in G$,

$$\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y) \text{ and } \nu_A(xy) \leq \nu_A(x) \vee \nu_A(y).$$

It is clear that $0_\sim$ and $1_\sim$ are both IFGPs of G.

Definition 3.4. Let G be a groupoid and let $A \in IFS(G)$. Then A is called an :

(1) *intuitionistic fuzzy left ideal*(in short, $IFLI$) of G if for any $x, y \in G$, $A(xy) \geq A(y)$, i.e., $\mu_A(xy) \geq \mu_A(y)$ and $\nu_A(xy) \leq \nu_A(y)$.

(2) *intuitionistic fuzzy right ideal*(in short, $IFRI$) of G if for any $x, y \in G$, $A(xy) \geq A(x)$, i.e., $\mu_A(xy) \geq \mu_A(x)$ and $\nu_A(xy) \leq \nu_A(y)$.

(3) *intuitionistic fuzzy ideal*(in short, IFI) of G if it is both an IFLI and an IFRI.

It is clear that A is an IFI of G if and only if for any $x, y \in G$, $\mu_A(xy) \geq \mu_A(x) \vee \mu_A(y)$ and $\nu_A(xy) \leq \nu_A(x) \wedge \nu_A(y)$. Moreover, an IFI(resp. IELI, IFRI) is an IFGP of G. Note that for any IFGP A of G we have $\mu_A(x^n) \geq \mu_A(x)$ and $\nu_A(x^n) \leq \nu_A(x)$ for each $x \in G$, where x^n is any composite of x's.

We will denote the set of all IFGPs of G as $IFGP(G)$.

Remark 3.5.

(1) If μ_A is a fuzzy subgroupoid of a groupoid G, then $A=(\mu_{A,}\ \mu_A^c)\in IFGP(G)$.

(2) If μ_A is a fuzzy (left, right) ideal of a groupoid$G$, then $A=(\mu_{A,}\ \mu_A^c)$ is an IFI(an IFLI, an IFRI) of G.

(3) If $A\in IFGP(G)$, then μ_A and ν_A^c are fuzzy subgroupoids of G.

(4) If A is an IFI(an IFLI, an IFRI) of G, then, μ_A and ν_A^c are fuzzy(left, right) ideals of G.

(5) If $A\in IFGP(G)$, then [ ]A, < > $A\in IFGP(G)$.

(6) If A is an IFI(an IFLI, an IFRI) of G, then [ ]A and < > A are IFIs(IFLIs, IFRIs) of G.

Definition 3.6. Let A be an IFS in a set X and let $\lambda,\ \mu\in I$ with $\lambda+\mu\le I$ Then the set $X_A^{(\lambda,\mu)}=\{x\in X: A(x)\ge C_{(\lambda,\mu)}(x)\}=\{x\in X: \mu_A(x)\ge\lambda$ and $\nu_A(x)\le\mu\}$ is called a *$(\lambda,\ \mu)$-level subset* of A.

Proposition 3.7. Let G be a groupoid and let $(\lambda,\ \mu)\in I\times I$ with $\lambda+\mu\le 1$. If $A\in IFGP(G)$ or A is an IFI(IFLJ, JFRI)of G, then $G_A^{(\lambda,\mu)}$ is a subgroupoid or a (left, right) ideal of G.

Proof. Suppose $A\in IFGP(G)$ and let $x,\ y\in G_A^{(\lambda,\mu)}$. Then $\mu_A(x)\ge\lambda,\ \nu_A(x)\le\mu$ and $\mu_A(y)\ge\lambda,\ \nu_A(y)\le\mu$. Since $A\in G\ IFGP(G)$, $\mu_A(xy)\ge\mu_A(x)\wedge\mu_A(y)$ and $\nu_{A(xy)}\le\nu_A(x)\vee\nu_A(y)$. Thus $\mu_A(xy)\ge\lambda$, and $\nu_A(xy)\le\mu$. So $xy\in G_A^{(\lambda,\mu)}$. Hence $G_A^{(\lambda,\mu)}$ is a subgroupoid of G.

Now suppose A is an IFLI of G, let $x\in G$ and let $y\in G_A^{(\lambda,\mu)}$. Then $\mu_A(y)\ge\lambda$ and $\nu_A(y)\le\mu$ Since A is an IFLI of G, $\mu_A(xy)\ge\mu_A(y)$ and $\nu_A(xy)\le\mu_A(y)$. Thus $\mu_A(xy)\ge\lambda$ and $\nu_A(xy)\le\mu$. So $xy\in G$. Hence $G_A^{(\lambda,\mu)}$ is a left ideal of G. By the similar argument, we can easily check that $G_A^{(\lambda,\mu)}$ is a (right) ideal of G. This complete the proof.

Proposition 3.8. Let G be a groupoid and let $T\in P(G)$. Then $A=(\chi_{T,}\ \chi_{T^c})$ is an IFGP or IFI(IFLI, IFRI) of G if and only if T is a subgroupoid or an ideal (left ideal, right ideal) of G, where χ_T is the charecteristic function of T.

Proof. A is an IFGP of G
iff for any $x,\ y\in G,\ \chi_T(xy)\ge\chi_T(x)\wedge\chi_T(y)$
and $\chi_{T^c}(xy)\le\chi_{T^c}(x)\vee\chi_{T^c}(y)$
iff for any $x,\ y\in G,\ \chi_T(x)=\chi_T(y)=1$
implies $\chi_T(xy)=1$
iff for any $x,\ y\in T,\ xy\in T$
iff T is a subgroupoid of G.
Similarly, A is an IFLI of G
iff for any $x,\ y\in G,\ \chi_T(xy)\ge\chi_T(y)$
and $\chi_{T^c}(xy)\le\ \chi_{T^c}(y)$
iff for each $x\in G$ and $y\in T,\ xy\in T$
iff T is a left ideal of G.
By the similar arguments, we can easily check that the remainders hold. This completes the proof.

From Definitions 1.2, 1.3, 1.4 and Result 1.A, for a groupoid G, IFS(G) is a complete lattice under the intuitionistic fuzzy set inclusion " $\subset$ " such that $0_\sim$ and $1_\sim$ are the least and greatest elements of IFS(G), respectively. It is clear that $0_\sim$ and $1_\sim$ are IFIs(and in particular, IFGPs)of G.

Proposition 3.9. Let $\{A_\alpha\}_{\alpha\in\Gamma}\subset IFGP(G)$. Then $\bigcap_{\alpha\in\Gamma}A_\alpha\in IFGP(G)$.

Proof. Let $A=\bigcap_{\alpha\in\Gamma}A_\alpha$ and let $x,y\in G$. Then $A=(\bigwedge_{\alpha\in\Gamma}\mu_{A\alpha},\ \bigvee_{\alpha\in\Gamma}\nu_{A\alpha})$. By the proof of Proposition 3.1 in [12], $\mu_A(xy)\ge\mu_A(x)\vee\mu_{A(y)}$. Thus it is enough to show that $\nu_A(xy)\le=\nu_A(x)\vee\nu_{A(y)}$.

$$\begin{aligned}\nu_A(xy)&=\bigvee_{\alpha\in\Gamma}\nu_{A\alpha}(xy)\le\bigvee_{\alpha\in\Gamma}[\nu_{A\alpha}(x)\vee\nu_{A\alpha}(y)]\\&=\bigvee_{\alpha\in\Gamma}\nu_{A\alpha}(x)\vee\bigvee_{\alpha\in\Gamma}(\nu_{A\alpha}(y))\\&=\nu_A(x)\vee\nu_A(y).\end{aligned}$$

Hence $\bigcap_{\alpha\in\Gamma}A_\alpha$ is an IFGP of G.

Proposition 3.10. Let $\{A_\alpha\}_{\alpha\in\Gamma}$ be any family of IFIs(IFLIs, IFRIs). Then $\bigcap_{\alpha\in\Gamma}A_\alpha$ or $\bigcup_{\alpha\in\Gamma}A_\alpha$ is an IFI(IFLI, IFRI).

Proof. Let G be a groupoid and let $\{A_\alpha\}_{\alpha\in\Gamma}$ be any family of IFIs(IFLIs, IFRIs) of G. Let $A=\bigcap_{\alpha\in\Gamma}A_\alpha$ and let $x,\ y\in G$.

Suppose $\{A_\alpha\}_{\alpha\in\Gamma}$ is a family of IFLIs of G. Then

$$\mu_A(xy)=\bigvee_{\alpha\in\Gamma}\mu_{A\alpha}(xy)\ge\bigwedge_{\alpha\in\Gamma}\mu_{A\alpha}(y)$$

(Since A_α is an IFLI of G for each $\alpha\in\Gamma$)

and

$$\nu_A(xy)=\bigvee_{\alpha\in\Gamma}\nu_{A\alpha}(xy)\le\bigvee_{\alpha\in\Gamma}\nu_{A\alpha}(y)$$

(Since A_α is an IFLI of G for each $\alpha\in\Gamma$).

So $A=\bigcap_{\alpha\in\Gamma}A_\alpha$ is an IFLI of G. By the similar arguments, we can easily check that the remainders hold. Also we can see that $\bigcup_{\alpha\in\Gamma}A_\alpha$ is an IFI(IFLI, IFRI). This completes the proof.

It is clear that the collection of all the IFIs(IFLIs, IFRIs)is a complete sublattice of IFS(G).

4. Homomorphisms

Proposition 4.1. Let $f:G\to G'$ be a groupoid homomorphism and let $B\in IFS(G')$.

(1) If $B\in IFGP(G')$, then $f^{-1}(B)\in IFGP(G)$.

(2) If B is an IFI(IFLI, IFRI) of G' then $f^{-1}(B)$ is an IFI(IFLI, IFRI) of G.

Proof. (1) By Definition 1.5, $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\nu_B))$. Let $x, y \in G$. Then:

$$\begin{aligned} \mu_{f^{-1}(B)}(xy) &= f^{-1}(\mu_B)(xy) = \mu_B(f(xy)) \\ &= \mu_B(f(x)f(y)) \quad (f \text{ is a groupoid homomophism}) \\ &\geq \mu_B(f(x)) \wedge \mu_B(f(y)) \\ &= f^{-1}(\mu_B)(x) \wedge f^{-1}(\mu_B)(y) \end{aligned}$$

and

$$\begin{aligned} \nu_{f^{-1}(B)}(xy) &= f^{-1}(\nu_B)(xy) = \nu_B(f(xy)) \\ &= \nu_B(f(x)f(y)) \quad (f \text{ is a groupoid homomorphism}) \\ &\leq \nu_B(f(x)) \wedge \nu_B(f(y)) \\ &= f^{-1}(\nu_B)(x) \wedge f^{-1}(\nu_B)(y). \end{aligned}$$

Hence $f^{-1}(B) \in IFGP(G)$.

(2) By the similar arguments of the proof of (1), it is clear.

Definition 4.2. Let $A \in IFS(G)$. Then A is said to have *the sup property* if for any $T \in P(G)$, there exists a to $t_0 \in T$ such that $A(t_0) = \bigcup_{t \in T} A(t)$, i.e., $\mu_A(t_0) = \bigvee_{t \in T} \mu_A(t)$ and $\nu_A(t_0) = \bigwedge_{t \in T} \nu_A(t)$ where $P(G)$ denotes the power set of G.

Remark 4.3. Let $A \in IFS(G)$. If A can take an only finitely many values(in particular, if they are charecteristic function), then A has the sup property.

Proposition 4.4. Let $f: G \to G''$ be a groupoid homomorphism and let $A \in IFS(G)$ have the sup property.

(1) If $A \in IFGP(G)$, then $f(A) \in IFGP(G'')$.

(2) If A is an IFI(IFLI, IFRI) of G, then $f(A)$ is an IFI(IFLI, IERI) of G''.

Proof (1) Let $y, y' \in G''$. Then we can consider four cases :

(i) $f^{-1}(y) \neq \phi$, $f^{-1}(y') \neq \phi$;

(ii) $f^{-1}(y) \neq \phi$, $f^{-1}(y') = \phi$;

(iii) $f^{-1}(y) = \phi$, $f^{-1}(y') \neq \phi$;

(iv) $f^{-1}(y) = \phi$, $f^{-1}(y') = \phi$.

We prove only the case (i) and omit the remainders. Since A has the sup property, there exist $x_0 \in f^{-1}(y)$ and $x_0' \in f^{-1}(y')$ such that:

$$\mu_A(x_0) = \bigvee_{x \in f^{-1}(y)} (\mu_A(x), \nu_A(x_0)) = \bigwedge_{x \in f^{-1}(y)} \nu_A(x)$$

and

$$\mu_A(x_0') = \bigvee_{x' \in f^{-1}(y')} (\mu_A(x_0'), \nu_A(x_0')) = \bigwedge_{x' \in f^{-1}(y')} \nu_A(x').$$

Then:

$$\begin{aligned} \mu_{f(A)}(yy') &= f(\mu_A)(yy') \\ &= \bigvee_{x \in f^{-1}(yy')} \mu_A(z) \geq \mu_A(x_0 x_0') \geq \mu_A(x_0) \wedge \mu_A(x_0') \\ &= (\bigvee_{x \in f^{-1}(y)} \mu_A(x)) \wedge (\bigvee_{x' \in f^{-1}(y)} \mu_A(x')) \\ &= f(\mu_A)(y) \wedge f(\mu_A)(y') \end{aligned}$$

and

$$\begin{aligned} \nu_{f(A)}(yy') &= f(\nu_A)(yy') \\ &= \bigwedge_{x \in f^{-1}(yy')} \nu_A(z) \leq \nu_A(x_0 x_0') \leq \nu_A(x_0) \vee \nu_A(x_0') \\ &= (\bigwedge_{x \in f^{-1}(y)} \nu_A(x)) \vee (\bigwedge_{x' \in f^{-1}(y')} \nu_A(x')) \\ &= f(\nu_A)(y) \vee f(\nu_A)(y'). \end{aligned}$$

(2) By the similar arguments of the proof of (1), it is clear.

Definition 4.5[3]. Let $f: X \longrightarrow Y$ be a mapping and let $A \in IFS(X)$. Then A is said to be *f-invariant* if $f(x) = f(y)$ implies $A(x) = A(y)$, i.e., $\mu_A(x) = \mu_A(y)$ and $\nu_A(x) = \nu_A(y)$.

It is clear that if A is f-invariant, then $f^{-1}(f(A)) = A$ (See Proposition 6.6 in [3]).

The following is the immediate result of Definition 4.5.

Proposition 4.6. Let $f: X \longrightarrow Y$ be a mapping and let $\mathcal{A} = \{A \in IFS(X) : A \text{ is f-invariant}\}$. Then f is a one-to-one correspondence between $\mathcal{A}$ and $IFS(f(X))$.

Corollary 4.6. Let $f: G \longrightarrow G''$ be a mapping and let $\mathcal{A} = \{A \in IFGP(G): A \text{ is f-invariant and has the sup property}\}$. Then f is a one-to-one corre-spondence between $\mathcal{A}$ and IFGP(f(G)).

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