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Structure-Control Combined Design with Structure Intensity

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ABSTRACT: *This paper proposes an optimum design method of structural and control systems, using a 2-D truss structure as an example. The structure is subjected to initial static loads and disturbances. For the structure, a FEM model is formed. Using modal transformation, the equation of motion is transformed into modal coordinates, in order to decrease D.O.F. of the FEM model. To suppress the effect of the disturbances, the structure is controlled by an output feedback* H_∞ *controller. The design variables of the combined optimal design of the control-structure systems are the cross sectional areas of truss members. The structural objective function is the structural weight. The control objective function is the* H_∞ *norm, the performance index of control. The second structural objective function is the energy of the response related to the initial state, which is derived from the time integration of the quadratic form of the state in the closed-loop system. In a numerical example, simulations have been perform. Through the consideration of structural weight and* H_∞ *norm, an advantage of the combined optimum design of structural and control systems is shown. Moreover, since the performance index of control is almost nearly optimiz, we can acquire better design of structural strength.*

1. Introduction

The combined design of structure and control systems is one of the most powerful and dynamic design techniques employed to design a highly developed machine structure. It is considered to be an especially an appropriate technique in the design of flexible space structures, such satellite and space stations, which are flexible, large-scale, and difficult to control. There are severe specications of the control system, vibration control, and the structure system that are necessary in the lightness of the design of such a large-scale structure. However, if the structure is light, its stiffness is decreased, and a big vibration will result from even a small external disturbance. And because the damping is small, the vibration caused once also has the defect that it is not easy to install. To secure the damping of vibration property for such a vibration, and the cost to the control system design grows. It is necessary to increase the stiffness of the structure in order to reduce the cost to the control system design, inversely which result in an increase to structure weigh. Thus, it is necessary to consider the structure system and the control system one system, when there is a close relationship between the structural design and the control system design. Several studies dealing with the combined consideration of structure and control system design have been performed. Salama et al., (1988) minimized the quadratic evaluation function of structure weight and linear regulator for 3 D.O.F beam models under the restriction of the natural frequency of a closed-loop system. Onoda and haftka (1987) minimized the linear sum of structure weight and the control energy under the restriction of the vibration energy. Rao et al., (1990) combined the design of structure weight and linear regulator by using game theory for the truss structure. Khot (1998) minimized the structure weight under the restriction of the natural frequency of closed-loop system or damping factor for the truss structure. Grandhi (1989) minimized the sturcture weight or frobenius norm, under the restriction of the natural frequency of closed-loop system or damping factor for the truss structure.

In structure design, carrying out an optimization considering the structural strength such as stress, transformation etc.. has been thought important, but research including strength design with an objective function in combined optimal design, of structural and control systems field, has been given very little attention.

It is particularly difficult to predict any fluctuation amount in the time response of maximum stress value by transient response, or any change occurring in a certain

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position. In this paper, evaluation on the structural strength, such as transform energy and kinetic energy etc was reflected by time integrating a quadratic function of closed loop system state within observation time. We conducted the combined optimal design of control-structure systems regarding the design object as 2-D truss structure of the state that another disturbance is imposed after vibration began with deformed state by receiving some static loading (initial external force).

First of all, the control is modeled design object by a finite element method, and is then dimensionized, lower by mode conversion again. Then, we design the H_∞ control system by output feedback not to misunderstand remainder mode, which is obtained by ignoring the process at the lower dimensionization as a modeling error or to repress the vibration of the control mode. The objective function in the optimization problem selects the structural weight for structure system and H_∞ norm of transfer function from disturbance to control output for the control system. Besides, the valuation on the structure strength, by having initial value of response energy, which is the expression including the term of no time integral initial state by changing the amount of quadratic time integral in the closed-loop system state, as an objective function of the control system. The optimization problem of this research formalizes the combined optimum designs, using the weighted factor method and ε-restriction method, as a multi-purpose plan problem. For the optimization technique, we use the complex method.

From the inquiry into the relationship between gross weight of structure which is an objective function regarding smallest weight design and H_∞ norm, which is an index function that expresses the measure of controlling the vibration effect, we can show the combined optimum design's effectiveness in reducing structural weight and H_∞ norm, compared with control system designs, alone. Also, from the inquiry into the relationship between H_∞ norm and initial value of response energy, the excellent design in structure strength with repressing the effect of disturbances to the system, is displayed.

2. Truss Structure and Control Problem

In this paper, a n D.O.F system, modeled by the finite element method, is expressed using the following equation of motion:

$$M_s\ddot{q} + D_s\dot{q} + K_s q = L_1 w + L_2 u \tag{1}$$

Where $M_s, D_s, K_s \in R^{n\times n}$ is a mass, the damping, and the stiffness matrices. $q\in R^n$, $w\in R^p$ and $u\in R^d$ are displacement, the disturbance input and control input and $L_1\in R^{n\times p}$ and $L_2\in R^{n\times d}$ are the disturbance and control input matrices. It is difficult to control n the number of the entire elasticity mode in a practical question, such that we make the model dimensionization lower, by the mode conversion as follows in this research.

$$q = \Phi\zeta \tag{2}$$

where $\Phi\in R^{n\times r}$ is a characteristic mode matrix of system and r is an adopted mode number. The equation of motion of Eq. (1) is transformed into the state equation as follows.

$$\dot{x}_s = A_s x_s + B_{1s} w + B_{2s} u \tag{3}$$

$$A_s = \begin{bmatrix} 0 & I_r \\ -\Omega^2 & -\Lambda \end{bmatrix}, B_{1s} = \begin{bmatrix} 0 \\ \Phi^T L_1 \end{bmatrix}, B_{2s} = \begin{bmatrix} 0 \\ \Phi^T L_2 \end{bmatrix}$$

$$\Omega^2 = \Phi^T K_s \Phi = diag(\omega_1^2, \omega_2^2, \cdots, \omega_r^2), \Lambda = \Phi^T D_s \Phi$$

Ω^2 is a eigenvalues matrix, ω_i expresses a vibration number of characteristic angle of i degree. Φ, characteristic mode matrix, is normalized such as $\Phi^T M_S \Phi = I_r$ I_r is a unit matrix of r degree. An observation output of system is expressed, as follows by using the state $\bar{x} = [\, q^T \quad \dot{q}^T \,]^T$. (Where superscript T means transposed matrix)

$$y = \bar{C}_{2s}\bar{x} + D_{21s} w \tag{4}$$

Suppose that tiny observation disturbance acts in the observation output, $D_{21s} = [0 \ \ \varepsilon I]$. An observation output in the case of the state x_s of mode coordinate system here is as follows:

$$y = C_{2s} x_s + D_{21s} w, \ C_{2s} = \bar{C}_{2s}\begin{bmatrix} \Phi & 0 \\ 0 & \Phi \end{bmatrix} \tag{5}$$

In this research the control value, z_s by using a matrix C_{1s}, D_{12s} is expressed as follows:

$$z_s = C_{1s} x_s + D_{12s} u \tag{6}$$

$$C_{1s}=\begin{bmatrix} diag(\omega_1,\cdots,\omega_r) & 0 \\ 0 & I_r \\ 0 & 0 \end{bmatrix},\ D_{12s}+\begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix} \tag{7}$$

A state space equation of control object is as follows:

$$\begin{cases} \dot{x}_s = A_s x_s + B_{1s} w + B_{2s} u \\ z_s = C_{1s} x_s + D_{12s} u \\ y = C_{2s} x_s + D_{21s} w \end{cases} \tag{8}$$

General control object of H_∞ control problem can be express as follows (Park et al., 2000; Kajiwara et al., 1993):

$$G(s)\equiv\begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix} \tag{9}$$

H_∞ control problem is to obtain the controller that H_∞ norm of closed-roof transfer function $G_{sw}(s)$ from disturbance w to control output z should satisfy the condition, Eq. (10) as follows:

$$J_n = \|G_{zw}(s)\|_\infty < \gamma \tag{10}$$

with internal stabilization of the closed-roof system. At this point, the smaller J_n or γ, the better performance of suppressing the disturbance.

In this research, for the efficient conduct of control performance and to achieve a more tenacious control system, to exclude modeling error, we established the frequency weighting factor $W_1(s)$, which takes an enlarged gain value in low frequency, the transfer function T_{yw} from disturbance w to observation output y. We also established the frequency weighting factor $W_2(s)$, which takes an enlarged gain value in high frequency, to the transfer function T_{uw} from disturbances to control input u. Regarding the remainder mode as modelling error, in this research, makes it possible to suppress the spillover by remainder mode. Additionally, from Eq. (1), it is evident that the disturbance that is imposed on the control object is unreliable, due to the impossible prediction of external force. We can expect that the disturbances usually have some effects on the system in the low frequency band, mainly day time. Therefore, we make it have frequency characteristics with frequency weighting factor W_F, which takes an enlarged gain value in low frequency for the disturbance w imposed to the system as filter.

Including the transfer function, $T_{z,}w$ from disturbance w to control output z_s, H_∞ the goal of the control system design problem, which utilizes a frequency weighting factor, is to obtain the controller that satisfies the following equation:

$$\left\| \begin{matrix} T_{z,w} \\ W_1 T_{yw} \\ W_2 T_{uw} \end{matrix} \right\|_\infty < \gamma \tag{11}$$

When you set the state of each weighting factor as x_1, x_2, x_f, the output as $z_1, z_2, w_{f,}$, frequency weighting factors $W_1(s)$, $W_2(s)$, $W_F(s)$ are expressed as follows:

$$W_1(s) = C_{W1}(sI - A_{w1})^{-1} B_{w1} + D_{w1} \tag{12}$$

$$W_2(s) = C_{W2}(sI - A_{w2})^{-1} B_{w2} + D_{w2} \tag{13}$$

$$W_F(s) = C_F(sI - A_F)^{-1} B_F + D_F \tag{14}$$

and the space expression of the control object is as follows:

$$G(s)\equiv\begin{bmatrix} A_s & 0 & 0 & B_{1s}C_F & B_{1s}D_F & B_{2s} \\ B_{w1}C_{2s} & A_{w1} & 0 & 0 & B_{w1}D_{21s} & 0 \\ 0 & 0 & A_{w2} & 0 & 0 & B_{w2} \\ 0 & 0 & 0 & A_F & B_F & 0 \\ C_{1s} & 0 & 0 & 0 & 0 & D_{12s} \\ D_{w1}C_{2s} & C_{w1} & 0 & 0 & D_{w1}D_{21s} & 0 \\ 0 & 0 & C_{w2} & 0 & 0 & D_{w2} \\ C_{2s} & 0 & 0 & 0 & D_{21s} & 0 \end{bmatrix} \tag{15}$$

In this paper, the following assumptions are made:

$$D_{w1}=0,\ D_F=0\ D_{12s}^T\begin{bmatrix} C_{1s} \\ D_{12s} \end{bmatrix}=\begin{bmatrix} 0 \\ I \end{bmatrix} \tag{16}$$

In the case that each element of Eq. (15) correspondeds with the elements of Eq. (9), H_∞ controller, there exist X and Y, which satisfy the following two Raccati equations:

$$X(A - B_2 D_{12}^{\zeta} C_1) + (A - B_2 D_{12}^{\zeta} C_1)^T X + X(\frac{1}{\gamma^2} B_1 B_1^T - B_2 E_{12}^{-1} B_2^T) X + (D_{12}^{\perp} C_1)^T D_{12}^{\perp} C_1 = 0$$

$$Y(A - B_1 D_{21}^{\zeta} C_2)^T + (A - B_1 D_{21}^{\zeta} C_2) Y + Y(\frac{1}{\gamma^2} C_1^T C_1 - C_2^T E_{21}^{-1} C_2) Y + (B_1 D_{21}^{\perp} (B_1 D_{21}^{\perp})^T = 0$$

Now, we have the following H_∞ controller,

$$\begin{cases} \dot{\hat{x}} = A_k \hat{x} + B_k y \\ u = C_k \hat{x} \end{cases}$$

where A_K, B_K, C_K, F, L and Z are

$$\begin{cases} A_k = A + \frac{1}{\gamma^2} B_1 B_1^T X - B_2 F - ZL(C_2 + D_{21} B_1^T X) \\ B_k = ZL \\ C_k = -F \\ F = D_{12}^{\zeta} C_1 + E_{12}^{-1} B_2^T X \\ L = B_1 D_{21}^{\zeta} + Y C_2^T E_{21}^{-1} \\ Z = (I - \frac{1}{\gamma^2} YX)^{-1} \end{cases} \tag{17}$$

The values of each elements is as follows:

$$\begin{cases} E_{12} = D_{12}^T D_{12} \\ E_{21} = D_{21} D_{21}^T \\ D_{12}^{\zeta} = (D_{12}^T D_{12})^{-1} D_{12}^T \\ (D_{12}^{\perp})^T D_{12}^{\perp} = I - D_{12} D_{12}^{\zeta} \\ D_{21}^{\zeta} = D_{21}^T (D_{21} D_{21}^T)^{-1} \\ D_{21}^{\perp} (D_{21}^{\perp})^T = I - D_{21}^{\zeta} D_{21} \end{cases}$$

From Eq. (15), (17), the state space equation of closed-loop system is as follows:

$$\begin{cases} \dot{\tilde{x}} = \mathcal{A} \tilde{x} + \mathcal{B} w \\ z = \mathcal{C} \tilde{x} \end{cases} \tag{18}$$

$$\tilde{x} = \begin{bmatrix} x \\ \hat{x} \end{bmatrix} \qquad \mathcal{A} = \begin{bmatrix} A & B_2 C_k \\ B_k C_2 & A_k \end{bmatrix}$$

$$\mathcal{B} = \begin{bmatrix} B_1 \\ B_k D_{21} \end{bmatrix} \qquad \mathcal{C} = [\ C_1 \quad D_{12} C_k\ ]$$

This time, the Riccati equation is formed as follows:

$$\mathcal{A}^T P + P \mathcal{A} + \gamma^{-2} P \mathcal{B} \mathcal{B}^T P + \mathcal{C}^T \mathcal{C} = 0 \tag{19}$$

$$P = \begin{bmatrix} \gamma^2 Y^{-1} & -(\gamma^2 Y^{-1} - X) \\ -(\gamma^2 Y^{-1} - X) & \gamma^2 Y^{-1} - X \end{bmatrix}$$

We shall examine the objective function, which not only occupies an important part in a combined optimum design, but also has an inseparable relationship with design specification. The design request specification of structure and control systems, in this research, is considered to achieve light-weight structure, to suppress the effect of the disturbance to system, and to increase the structure strength. To meet these design purposes, it is necessary to set the appropriate objective function. The objective function in this research is expressed as follows:

- $J_m = W$ (structural weight)
- $J_n = \|G_{zw}(s)\|_\infty$ (the transfer function, H_∞ norm from disturbance to control output)
- $J_{e1} = \int_0^\infty \tilde{x}^T Q \tilde{x}\, dt$ (the time integration of the quadratic form of the state $\tilde{x}$. Q is the matrix of weighting factor)

When $\mathcal{C}^T \mathcal{C}$ is set to be as weighting factor matrix Q of objective function J_{e1} from closed-loop system state equation of Eq. (18), it is possible to be transformed into the follows:

$$\begin{aligned} J_{e1} &= \int_0^\infty \tilde{x}^T Q \tilde{x}\, dt = \int_0^\infty \tilde{x}^T \mathcal{C}^T \mathcal{C} \tilde{x}\, dt \\ &= \int_0^\infty z_1^T z_1 dt + \int_0^\infty z_2^T z_2 dt + \int_0^\infty \{ q^T K_s q + \dot{q}^T M_s \dot{q} + u^T u \} dt \end{aligned} \tag{20}$$

The third term of Eq. (20) displays transform energy, and expense control. The objective function J_{e1} includes the term for the transform energy; it implies that this function includes structure strength's objective as well.

According as J_{e1} is supposed to be $Q = \mathcal{C}^T \mathcal{C}$, it is possible to be transformed into the following, using Eq. (19).

$$J_{e1} = \int_0^\infty \tilde{x}^T \mathcal{C}^T \mathcal{C} \tilde{x}\, dt \le \gamma^2 \int_0^\infty w^T w\, dt + \tilde{x}_0^T P \tilde{x}_0 \tag{21}$$

The first term of the right side is decided by disturbance. The second term of the right side of Eq. (21) is used instead of the index function J_{e1}, which is derived from the time integration of the quadratic form of the state. That is, J_{e1} gets smaller by lessening the second term of right side as follows:

$$J_{e2} = \tilde{x}_0^T P \tilde{x}_0 \tag{22}$$

Even if it is thought that vibration is begun when the initial external force is imposed on the structure, in this research, it is considered to be impossible to have conclusive information about the initial external force, such that the initial external force p_0 is set as following equation:

$$P_0 = P'_0 + \Psi \tag{23}$$

$$E[P_0] = E[\ P'_0] + E[\Psi] \tag{24}$$

p'_0 is the supposed initial external force, Ψ is the random variable, expressing variance degree of supposed initial external force, and $E[\,]$ means the expectation operator. In the case of $E[p_0]=p'_0$, covariance matrix of the initial external force which is a random variable, is as follows:

$$Cov(P_0, P_0^T) = E\lfloor (p_0 - E[p_0])(p_0 - E[p_0])^T \rfloor = E\lfloor \Psi\Psi^T \rfloor \tag{25}$$

Due to the supposition that the initial external force is applied to the structure in this research, it is nessary to determine the initial state. Usually, an inverse matrix does not exist, because Φ a is not a square matrix, but if characteristic mode matrix Φ uses a fact that is normalized, the initial mode coordinate q'_0 is evaluated, as follows, from the initial displacement.

$$\xi'_0 = \Phi^{-1} q_0 = \Phi^T M_s q_0 = \Phi^T M_s K_s^{-1} p_0 \tag{26}$$

The initial state is expressed as follows:

$$x_0 = \begin{bmatrix} Sp_0 \\ 0 \end{bmatrix},\ S = \Phi^T M_s K_s^{-1} \tag{27}$$

The dimension that displays the degree of initial state can be evaluated by the sum of the state number of the control object and a frequency-weighting factor. By considering this stochastic initial external force, J_{e2} of Eq. (22) is stochastic, and the expectation value is as follows in Eq. (28) (the initial value of controller is derived from the supposed initial external force).

$$\begin{aligned} E[J_{e2}] &= E\lfloor \tilde{x}_0^T P \tilde{x}_0 \rfloor \\ &= \gamma^2 trace\left(\begin{bmatrix} SE[\Psi\Psi^T]S^T & 0 \\ 0 & 0 \end{bmatrix} Y^{-1} \right) \\ &\quad + [\,(Sp'_0)^T\ 0\,] X \begin{bmatrix} SP'_0 \\ 0 \end{bmatrix} \end{aligned} \tag{28}$$

Thus, in the case of the assumption that variance exists in the initial external force, objective function J_{e2} can be considered as the expected value, including the random variable, and this value is set as a new objective function J_e, which will be defined as the initial value response energy from this point on:

$$J_{e2} = E[\,J_{e2}\,] \tag{29}$$

3. Optimal Design Problem

In this paper, the optimization problem is formalized to be a multipurpose plan. H_∞ norm is the objective function of control system, which is established as single purpose control system, by utilizing the weighting factor for the initial value response energy. For the integration objective function of the structural system and the control system, structural weight is formalized as the equality restriction condition, by using the ε-restriction method. The design variable is set to be a cross-sectional area a of each truss member; the upper and inferior limit value of design variable are established as restriction condition. In the condition that the closed-loop system is stabilized, the controller, which is designed in lower dimensionization system, is applied into the whole mode system. Furthermore, for normalization of the objective function J_n, J_e, the objective function is established using the value divided by each objective function value J_{n0}, J_{e0}, when control system design is executed alone only by γ-iteration with the cross sectional area of truss member as initial structure. That is, it is formalized as follows:

$$\min_a\ J(a) = s\frac{J_n(a)}{J_{n0}} + t\frac{J_e(a)}{J_{e0}} \quad (s+t=1) \tag{30}$$

$$\text{subject to} \quad \left\{ \begin{array}{c} W(a) - W_c \le \varepsilon \\ a^{\min} < a < a^{\max} \\ \text{closed-loopsystem of all} \\ \text{mode system is stable} \end{array} \right\} \tag{31}$$

where $W(a)=\sum_1^n \rho_i l_i a_i$ is structural weight (ρ_i, l_i, a_i is density of truss member, length, cross sectional area) and W_c is the standard weight value, s and t are the weighting factors for the objective function. The first term of the restriction condition equation is meant with the formality of ε-restriction method. At optimization, the process of combined optimum design problem, J_n which is H_∞ norm, can be evaluated by γ-iteration, J_e is calculated by using the γ value at this point. Even in the case where the objective function is not a series for the design variable, by optimization technique, the complex method will be adopted; that is known as the one which is possible to search for the optimal solution (Kajiwara et al., 1993; Box et al., 1972).

4. Numerical Simulation

The design object is 2-D truss structure of Fig. 1, and length of a long truss member is 2.0 m, short one is 0.2 m, density is 1.0 kg/m^3, vertical elastic modules is 2000 Pa, initial cross sectional area is 1.0 m^2, initial structure weight is 30.8 kg. x, y direction of structure is fixed at node 10, 11, 12, 13 . A sensor and actuator are arranged toward the x, y of node 7; the sensor can measure the displacement of node 7; and disturbance acts to node 7. It is supposed that the initial external force acts to node 6, 7, 8, and it has some fluctuations on each direction. In control system design, it adopts until by 2-D mode ($\omega_1 = 6.01$, $\omega_2 = 9.66$ in initial structure).

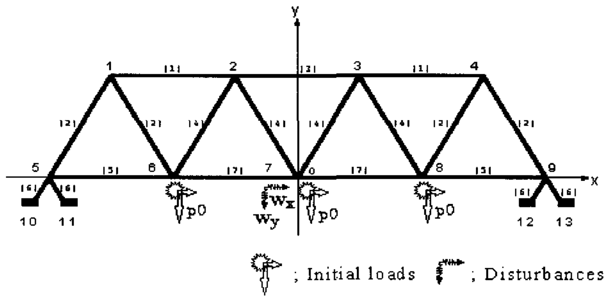
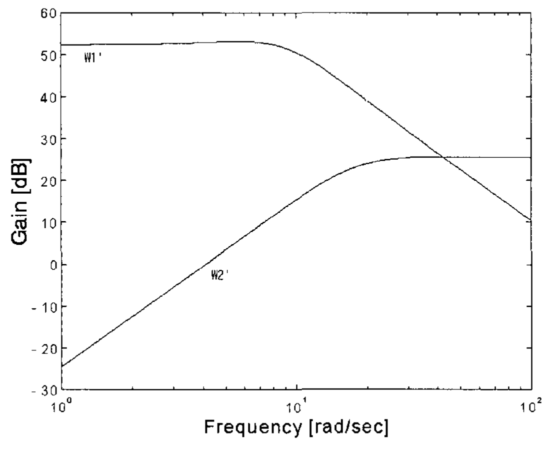


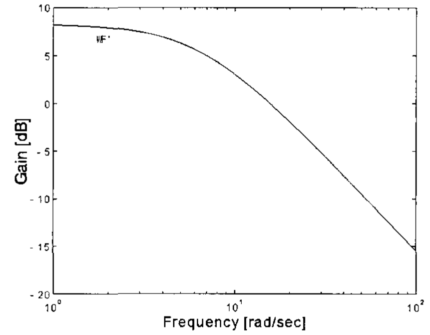
Fig 1. 2-D truss structure



$$W'_1 = \frac{3.351\times 10^4}{s^2+9.879s+81.63}$$

$$W'_2 = \frac{19.14\, s^2 + 0.03829s + 2.519\times 10^{-6}}{s^2+23.71s+322.7}$$

Fig 2. Frequency weighting factor W_1, W_2



$$W'_F = \frac{16.92}{s+6.524}$$

Fig 3. Frequency weighting factor W_F

In the actual design, it is not realistic to use the cross sectional area of all truss members as the design variable. Therefore, with the truss member in symmetry position, it is regarded as one truss member variable, the optimization executes by consideration([1] ~ [7]) that the design variable has seven kinds of the truss member variable. Frequency weighting factor W_1, W_2 is set in Fig. 2, W_F is in Fig. 3.

$$W_1 = diag[\,W'_1 \;\; W'_1\,],\quad W_2 = diag[\,W'_2 \;\; W'_2\,]$$
$$W_F = diag[\,W'_F \;\; W'_F\,] \qquad (32)$$

In this research, the supposed average initial external force, which interacts to node 6, 7, 8 is 1 on x direction, is -2 on y direction, and is independent on x, y directions, and has variance of 0.1 on each direction. With consideration of these facts, the covariance matrix is set as follows:

$$p'_0 = [0, \cdots, 0, [1, -2, 1, -2, 1, -2], 0, \cdots 0]^T$$
$$Cov(p_0, p_0^T) = diag[0, \cdots, 0[1, 1, 1, 1, 1, 1], 0, \cdots 0]\times 10^{-2}$$

The damping characteristic is supposed as follows:

$$D_s = 0.001M_s + 0.001K_s$$

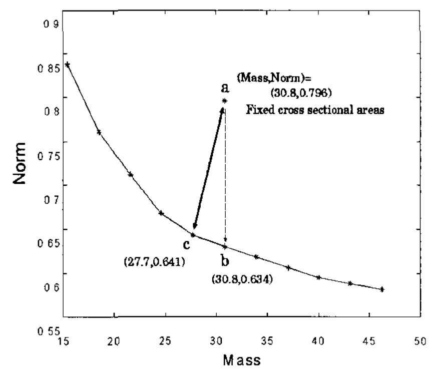


Fig 4. The structural weight and the optimal solution for H_∞ norm

4.1 Numerical Example 1

$$\min_{a} \frac{J_n(a)}{J_{n0}} \quad \text{with } W(a) - W_c \leq \varepsilon$$

From the result that executes combined optimum design by changing the standard structure weight value W_c of Eq. (31) to various kinds of numerical value with putting these weighting factors of Eq. (30) as $s=1.0$, $t=0$ into the index function, we arrived at the relationship such as in Fig. 4. In the case that 'a' point is a numerical value, when designed by only control system by γ-iteration fixed as initial cross sectional area, 'b' is a combined optimum design point when the standard structural weight value is the initial structure weight. In comparison between 'a' point and 'b' point, it is determined that H_∞ norm, which is impossible to achieve by only control system, can be fulfilled by doing a design that includes a structure system that has truss member cross sectional area that is designed variable changed. Also, when comparing 'a' point and 'c' point, 'c' point reduces the structural weight by 10% for 'a' point, shows that its control performance improves by from 0.641 to 0.796 in H_∞ norm. Therefore, confirmed the effectiveness of combined optimum design is comfirmed.

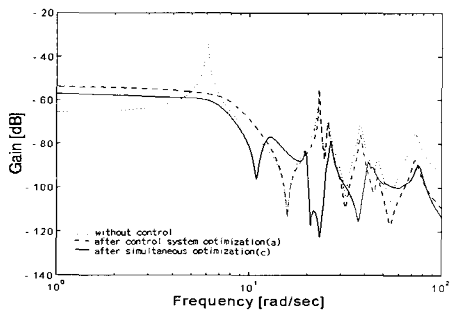


Fig 5. Frequency response

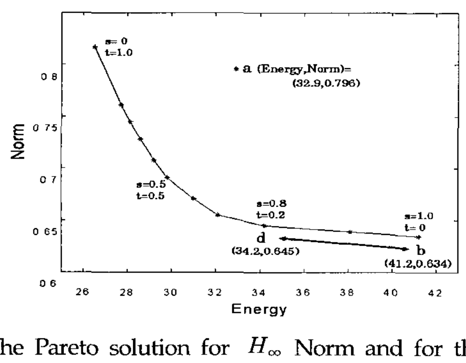


Fig 6. The Pareto solution for H_∞ Norm and for the initial response energy

Fig. 5 shows the frequency response on y directed displacement of node 7 in disturbance w_y. From this figure, the peak value of Gain is repressed in the case of combined optimum design, as opposed to when only the control system is designed. The first design vibration mode of design object is toward y direction, while the second vibration mode is of x direction; therefore, the second vibration mode does not appear in Fig. 5.

4.2 Numerical Example 2

$$\min_{a} \; s\frac{J_n(a)}{J_{n0}} + t\frac{J_e(a)}{J_{e0}} \quad \text{with fixed } W(a)$$

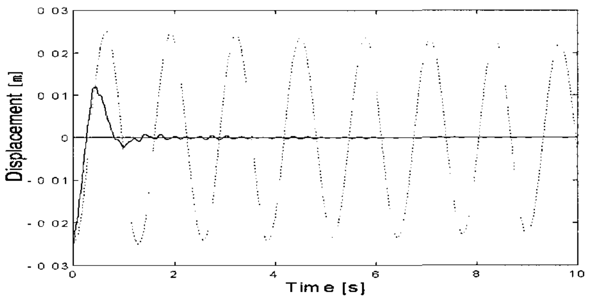
The values of each objective function, when combined optimum design is executed with changing s, t, which are the weighting factors of Eq. (30), after fixing the standard structural weight value as the initial one, are shown in Fig. 6. From Fig. 6, the relationship of competiton between two objective functions, it is impossible to minimize their values, simultaneously. At this point, we compare the design result between 'b' point ($t=0$), which is never considered as the initial value energy, and 'd' point ($t=0.2$), which is considered as the initial value energy.

Although H_∞ norm, the objective of disturbance control at these two points, has very minimal changes; designing of 'd' point made the initial value response energy have a decrease by about 17%, compared with point 'b', by considering some initial value response energy with setting the weighting factor as $t=0.2$. It is shown by the initial state response of node 7 of y direction displacement in Fig. 7. Very few large differences appeared in the response waveform. It could be inferred that the strain energy is decreasing, from the certain fact that the initial value response energy, which is about the evaluation of structural intensity, has been decreased. Fig. 8 is showing the comparison of the initial state responses of stress in the left side, in the seventh elements from the object structure. (Fig. 1) It is thought that strain energy is also decreased, due to obvious inhibition of the stress response of design of 'd' point, compared with design of 'b' point from Fig. 8. As stated above, the combined optimum point 'd' not only achieves H_∞ norm with the same level of the only combined optimal solution, which is 'b' point of H_∞ norm, but also is an excellent design point, even in the structure strength which reduces strain energy.

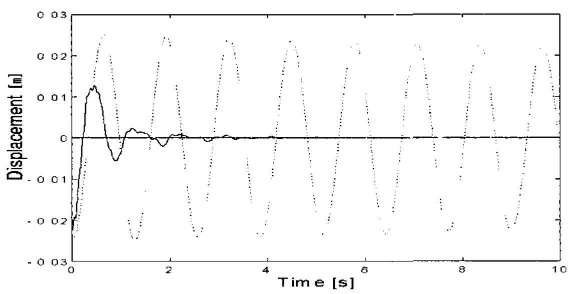
5. Conclusions

In this paper, the proposed combined optimal design of structure-control systems, regarding 2-D truss structure, is the design object. From the inquiry into the relationship between gross weight of structure and H_∞ norm, by showing the optimum point that reduces both structural weight and H_∞ norm, compared with control system design alone, we proved the effectiveness of combined optimum design. Also, from the inquiry into the relationship between H_∞ norm and initial value of response energy, the possibility of excellent design in structure strength, repressing the effect of disturbances to the system, is evident. In addition to the inquiry into both the objective function, H_∞ norm, and the initial value response energy, we recognized the relationship of competition between two objective functions.

We compared the design by the initial value response energy we presented in the paper to the single purpose design from H_∞ norm alone. This showed that the combined design method resulted in the reduction of the cost for structure system design and the improvement of the suppression for the effect of disturbances for the control system.



(b : $s=1.0$, $t=0$)

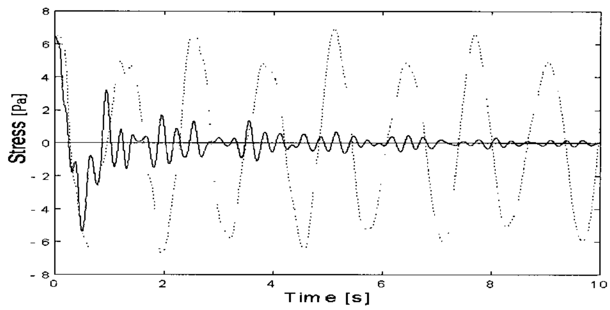


(d : $s=0.8$, $t=0.2$)

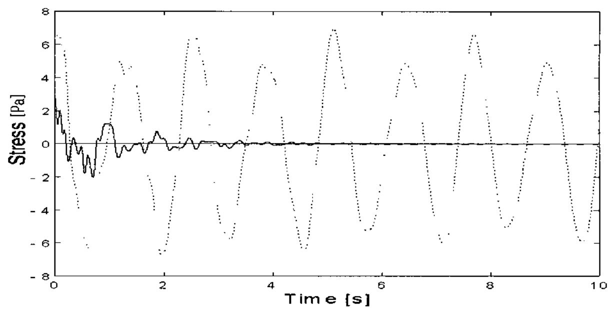
··· without control

— after combined optimization

Fig 7. Initial state response of displacement



(b : $s=1.0$, $t=0$)



(d : $s=0.8$, $t=0.2$)

··· without control

— after combined optimization

Fig 8. Initial state response of stress

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