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System Identification and Damage Estimation via Substructural Approach

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ABSTRACT

For system identification of large structures, it is not practical to identify the entire structure due to the prohibitive computational time and difficulty in numerical convergence. This paper explores the possibility of performing system identification at substructure level, taking advantage of reduction in both the number of unknowns and the number of degrees of freedom involved. Another advantage is that different portions (substructures) of a structural system can be identified independently and even concurrently with parallel computing. Two substructural identification methods are formulated on the basis whether substructural approach is used to obtain first-order or second-order model. For substructural first-order model, identification at the substructure level will be performed by means of the Observer/Kalman filter Identification (OKID) and the Eigensystem Realization Algorithm (ERA) whereas identification at the global level will be performed to obtain second-order model in order to evaluate the system's stiffness and mass parameters. In the case of substructural second-order model, identification will be performed at the substructure level throughout the identification process. The efficiency of the proposed technique is shown by numerical examples for multi-storey shear buildings subjected to random forces, taking into consideration the effects of noisy measurement data. The results indicate that both the proposed methods are effective and efficient for damage identification of large structures.

Keywords: system identification, damage assessment, substructural

1. Introduction

The problem of structural identification becomes important, particularly in relation to increasing number of aging structures. As research interest intensifies, it is noted that many methods proposed are suitable to small systems only due to the ill-conditioned nature of inverse analysis. In real world, analysis of engineering structures often requires mathematical models with many degrees of freedom (DOFs) to simulate their behaviour. A bigger challenge for structural identification is to identify large systems, i.e. system with many DOFs and unknown parameters. In practice, measurement and identification for the entire structure in one go is a difficult task. Therefore, in this paper, the substructuring technique is employed to decompose the large structural system into some smaller substructures (hence with less DOFs and unknown parameters) for more effective identification. This can be described as a "divide and conquer" strategy. Nevertheless, substructures interact with from the remainder of the structure (or adjacent substructures), and it is therefore necessary to account for the interaction forces.

Koh *et al.* (1991) first proposed substructural system identification (SSI) and used the Extended Kalman Filter (EKF) as the numerical tool to identify unknown structural parameters. This substructuring formulation of system identification not only reduces the computation time considerably but also helps to improve the convergence of the structural parameters. Zhao *et al.* (1995) reported their work on the substructural identification in frequency domain for the identification of frequency dependent systems such as soil-structure interaction systems. Procedures for assembling substructure transfer function data, substructure state-space models, and substructure Markov parameters were presented by Su *et al.* (1994). Yun (1997) proposed a SSI using the sequential prediction error method and an auto-regressive and moving average with stochastic input (ARMAX) model. More recently, Koh and Shankar (2003) proposed a frequency-domain approach

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of SSI with a numerical example of 50-DOF systems. An attractive advantage of this approach is that identification can be performed without measuring input excitation to the whole structure.

Substructural system identification can follow either of the two procedures. Briefly speaking, one can either (a) first synthesize substructure data and then carry out system identification at the global level based on the assembled data, or (b) first perform system identification at the substructural level and then employ substructure synthesis to assemble substructure models. Although both approaches are theoretically feasible, it is preferred to perform system identification at the substructure level, mainly because substructures are easier to identify than the assembled and bigger structure.

In this paper, two SSI methods are proposed using the Observer/Kalman filter Identification (OKID) and the Eigensystem Realization Algorithm (ERA). In the first method, identification will be performed at the substructure level by means of the OKID and the ERA whereas identification at the global level will be performed to obtain second-order model in order to evaluate the system's stiffness and mass parameters. In the second method, identification will be performed at the substructure level throughout the identification process.

2. Basic Formulation

The dynamic response of a *N*-DOF linear structural system can be represented by

$$\mathbf{M}\ddot{\mathbf{q}}(t)+\mathbf{L}\dot{\mathbf{q}}(t)+\mathbf{K}\mathbf{q}(t)=\mathbf{f}(t) \tag{1}$$

where **M, L** and **K** are the mass, damping, and stiffness matrices of the structure, respectively, **q** is a displacement vector and the overdot denotes differentiation with respect to time *t*. The above model is referred to as the second-order model. The equations of motion and the measurement equations can be written in the first-order state space form as

$$\mathbf{x}(k+1)=\mathbf{A}\mathbf{x}(k)+\mathbf{B}\mathbf{u}(k) \tag{2}$$

$$\mathbf{y}(k)=\mathbf{C}\mathbf{x}(k)+\mathbf{D}\mathbf{u}(k) \tag{3}$$

where $\mathbf{x}(k)$ is a $n\times 1$ state vector, $\mathbf{y}(k)$ a $m\times 1$ observation vector, and $\mathbf{u}(k)$ a $r\times 1$ input vector. Matrices **A, B, C** and **D** represents the system matrix, the input influence matrix, the output influence matrix and the direct force input term respectively.

2.1 Observer/Kalman Filter Identification (OKID)

For zero initial conditions, Eqs. (2) and (3) can also be written in matrix form for a sequence of '*l*' consecutive time steps as

$$\underset{m\times l}{y}=\underset{m\times[(r+m)p+r]}{\bar{Y}}\;\underset{[(r+m)p+r]\times l}{V} \tag{4}$$

where

$$y=\begin{bmatrix}y(0) & y(1) & y(2) & \dots y(p) & \dots y(l-1)\end{bmatrix}$$

$$\bar{Y}=\begin{bmatrix}D & C\bar{B} & C\overline{AB} & \dots C\bar{A}^{p-1}\bar{B}\end{bmatrix}$$

$$V=\begin{bmatrix}u(0) & u(1) & u(2) & \dots & u(p) & \dots & u(l-1)\\ & v(0) & v(1) & \dots & v(p-1) & \dots & v(l-2)\\ & & v(0) & \dots & v(p-2) & \dots & v(l-3)\\ & & & \dots & \dots & \dots & \dots\\ & & & & v(0) & \dots & v(l-p-1)\end{bmatrix}$$

Having identified the observer Markov parameters, the systems Markov parameters can be retrieved using the recursive formula. (Juang *et al.*, 1993)

2.2 Eigensystem Realization Algorithm (ERA)

ERA begins by forming the generalized Hankel matrix, composed of the Markov parameters. The ERA process starts with the factorization of the first Hankel matrix using singular value decomposition, $H(0)=R\Sigma S^T$. This is the basic formulation of realization for the ERA (Juang *et al.*, 1985). The triplet

$$A=\Sigma_n^{-1/2}R_n^T H(1)S_n\Sigma_n^{-1/2},\quad B=\Sigma_n^{1/2}S_n^T E_r,$$

$$C=E_m^T R_n\Sigma_n^{1/2} \tag{5}$$

is a minimum realization where $\mathbf{E}_r=[\mathbf{I}_{r\times r}\,\mathbf{0}_{r\times r}\,\mathbf{0}_{r\times r}\cdots\mathbf{0}_{r\times r}]^T$ with **I** denoting an identity matrix and 0 denoting a matrix whose elements are all zeros, and $\mathbf{E}_m$ is defined similarity.

2.3 Conversion from First-order Model to Second-order Model

By similarity transformation, the first-order model as represented by Eq. (2) can be related to the second-order model as represented by Eq. (1). Once the properly scaled eigenvectors **P** and **Γ** eigenvalues are evaluated, the mass, damping, and stiffness matrices of the structural model can be obtained (Lus *et al.*, 2001):

$$\mathbf{M}=(\mathbf{P\Gamma P}^T)^{-1},\ \mathbf{L}=-\mathbf{MP\Gamma}^2\mathbf{P}^T\mathbf{M},$$

$$\mathbf{K}=-(\mathbf{P\Gamma}^{-1}\mathbf{P}^T)^{-1},\quad \mathbf{PP}^T=\mathbf{0} \tag{6}$$

3. Identification with Substructural First-Order Model

A general systematic procedure for assembling multi-substructure first-order model is formulated. This procedure can be used to assemble the first-order model obtained for substructures, from the identification by OKID/ERA. Subsequently, the global first-order model is used to identify the second-order model. Assembling substructure Markov parameters is an alternative to assembling first-order models. However, identification with ERA and conversion from first-order model to second-order model will be in the global sense and this is impractical for large system. Thus, this approach is not presented in this paper.

As an example to illustrate the substructure first-order model for identification approach (without loss of generality), a 12-DOF lump mass system is considered as shown in Fig. 1. Noting that the substructures have overlap (or common) members, the substructural identification will be referred to as the SSI with overlap. This structure comprises three substructures 1, 2 and 3. It is required that each substructure is allocated an actuator/sensor pair at every interface DOF. At the interface, measurement can be displacement, velocity, or acceleration. The actuators and sensors located at the interior points do not have to be collocated.

The first-order state space models of substructures are represented by

$$\dot{\mathbf{x}}_s = \mathbf{A}_s\mathbf{x}_s + [\mathbf{B}_s^{I}\ \mathbf{B}_s^{J}]\begin{Bmatrix}\mathbf{u}_s^{I}\\ \mathbf{u}_s^{J}\end{Bmatrix}$$

$$\begin{Bmatrix}\mathbf{y}_s^{I}\\ \mathbf{y}_s^{J}\end{Bmatrix} = \begin{bmatrix}\mathbf{C}_s^{I}\\ \mathbf{C}_s^{J}\end{bmatrix}\mathbf{x}_s + \begin{bmatrix}\mathbf{D}_s^{II} & \mathbf{D}_s^{IJ}\\ \mathbf{D}_s^{JI} & \mathbf{D}_s^{JJ}\end{bmatrix}\begin{Bmatrix}\mathbf{u}_s^{I}\\ \mathbf{u}_s^{J}\end{Bmatrix} \tag{7}$$

for s = 1, 2 and 3. Superscript 'I' denotes internal DOFs of the substructure concerned and superscript 'J' denoted all interface DOFs of the substructure concerned. The preceding substructure first-order models are the results of system identification performed at the substructure level in OKID/ERA. Matrices $\mathbf{A}_s$, $\mathbf{B}_s$, $\mathbf{C}_s$ and $\mathbf{D}_s$ represent the system matrix, the input influence matrix, the output influence matrix, and the direct force input term of the sth substructure, respectively. Vector $\mathbf{x}_s$ is a state vector of the sth substructure.

The interface inputs and outputs of the substructures and the interface inputs and outputs of the global structure are related by a coupling matrix T as

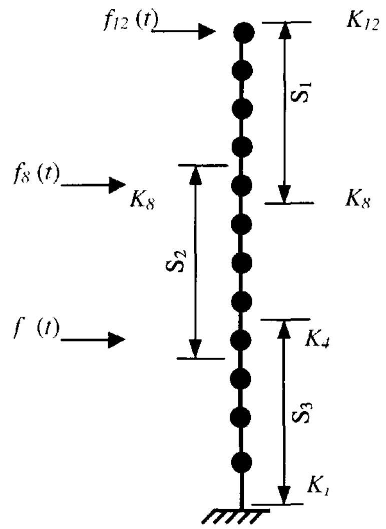

Fig. 1. A 12-DOF lump mass structure with 3 substructures (SSI with overlap)

$$\mathbf{y}_s^{J} = \mathbf{T}^{T}\mathbf{y}^{J} \qquad \mathbf{u}^{J} = \mathbf{T}\mathbf{u}_s^{J} \tag{8}$$

Normally, the elements of the **T** matrix are ones and zeros. Finally, we obtain a first-order state space model for the global structural system with coupling matrix (Su *et al.*, 1994). The results are

$$\begin{aligned}
\mathbf{A} &= \tilde{\mathbf{A}} + \tilde{\mathbf{B}}^{J}\mathbf{Q}\tilde{\mathbf{C}}^{J}\\
\mathbf{B} &= [\tilde{\mathbf{B}} + \tilde{\mathbf{B}}^{J}\mathbf{Q}\tilde{\mathbf{D}}^{JI}\ \ \tilde{\mathbf{B}}^{J}(\tilde{\mathbf{D}}^{JJ})^{-1}\mathbf{T}^{T}\mathbf{S}^{-1}]\\
\mathbf{C} &= \begin{bmatrix}\tilde{\mathbf{C}} + \tilde{\mathbf{D}}^{IJ}\mathbf{Q}\tilde{\mathbf{C}}^{J}\\ \mathbf{S}^{-1}\mathbf{T}(\tilde{\mathbf{D}}^{JJ})^{-1}\tilde{\mathbf{C}}^{J}\end{bmatrix}\\
\mathbf{D} &= \begin{bmatrix}\tilde{\mathbf{D}}^{II} + \tilde{\mathbf{D}}^{IJ}\mathbf{Q}\tilde{\mathbf{D}}^{J} & \tilde{\mathbf{D}}^{IJ}(\tilde{\mathbf{D}}^{JI})^{-1}\mathbf{T}^{T}\mathbf{S}^{-1}\\ \mathbf{S}^{-1}\mathbf{T}(\tilde{\mathbf{D}}^{JJ})^{-1}\mathbf{D}^{JI} & \mathbf{S}^{-1}\end{bmatrix}
\end{aligned} \tag{9}$$

where

$$\mathbf{S} = \mathbf{T}(\tilde{\mathbf{D}}^{JJ})^{-1}\mathbf{T}^{T} \qquad \mathbf{Q} = (\tilde{\mathbf{D}}^{JJ})^{-1}\mathbf{T}^{T}\mathbf{S}^{-1}\mathbf{T}(\tilde{\mathbf{D}}^{JJ})^{-1} - (\tilde{\mathbf{D}}^{JJ})^{-1}$$

$$\tilde{\mathbf{A}} = \begin{bmatrix}\mathbf{A}_1 & & \\ & \mathbf{A}_2 & \\ & & \mathbf{A}_3\end{bmatrix},\quad \tilde{\mathbf{B}}^{I} = \begin{bmatrix}\mathbf{B}_1^{I} & & \\ & \mathbf{B}_2^{I} & \\ & & \mathbf{B}_3^{I}\end{bmatrix},\ \ldots \text{ etc.}$$

The first-order state space model with matrices defined in Eq. (9) will be referred to as the global structural first-order state space model. This model describes the dynamics of the substructures when their interface compatibility and equilibrium conditions are enforced. However, this first-order model is not a minimal-order model for the global structure. Elimination of these extra state variables can be accomplished by using a minimal realization algorithm. Fortunately, the global first-order model can be used directly in the conversion from first-order model to second-order model without using any minimal realization algorithm. The proposed substructure synthesis first-order models are valid only for assembling continuous-time models. The sampling time must be set small enough to obtain accurate results. It can be easily seen that only accelerations are required to compute the interface forces as opposed to displacements and velocities which are required in the following approach.

4. Identification with Substructural Second-Order Model

In order to write the equations of motion for the substructure, the equations of motion for the entire structure can be written as follows

$$\begin{bmatrix} \mathbf{M}^{II} & \mathbf{M}^{IJ} \\ \mathbf{M}^{JI} & \mathbf{M}^{JJ} \end{bmatrix} \begin{Bmatrix} \mathbf{q}^{I}(t) \\ \mathbf{q}^{J}(t) \end{Bmatrix} + \begin{bmatrix} \mathbf{L}^{II} & \mathbf{L}^{IJ} \\ \mathbf{L}^{JI} & \mathbf{L}^{JJ} \end{bmatrix} \begin{Bmatrix} \mathbf{q}^{I}(t) \\ \mathbf{q}^{J}(t) \end{Bmatrix} + \begin{bmatrix} \mathbf{K}^{II} & \mathbf{K}^{IJ} \\ \mathbf{K}^{JI} & \mathbf{K}^{JJ} \end{bmatrix} \begin{Bmatrix} \mathbf{q}^{I}(t) \\ \mathbf{q}^{J}(t) \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}^{I}(t) \\ \mathbf{f}^{J}(t) \end{Bmatrix} \tag{10}$$

where superscript J denotes the DOFs at the two interface ends of the substructure. These DOFs are referred to as interface DOFs. The remaining DOFs are denoted by superscript I and referred to as internal DOFs.

To illustrate the formulation for the substructural second-order model identification, consider a smaller system that is a 7-DOF system due to space limitation. The building of 7-storey decomposed into three substructures 1, 2 and 3. DOFs are numbered upwards from bottom. The equation of motion for substructure can be written as:

$$\mathrm{M}_s^{II} q_s^{*I} + \mathrm{M}_s^{II} q_s^{*I} + \mathrm{K}_s^{II} q_s^{*I} = \mathrm{f}_s^{I} - \mathrm{M}_s^{II} 1 q_s^{J} + \mathrm{Z}^{J} \mathrm{K}_s^{IJ} q_s^{*J} + \mathrm{Z}^{J} \mathrm{L}_s^{IJ} q_s^{*J} \tag{11}$$

for s = 1, 2 and 3. $\mathbf{q}^*$ is the relative displacement with respect to $\mathbf{q}^J$ and $\mathbf{1}=[111\cdots\ 1]^T$. The elements of the $\mathbf{Z}^J$ are ones and zeros depending on whether particular DOF is considered to account for interaction force. If the particular interface DOF is used to account for interaction forces, the value of that location is equal to one.

4.1 Substructure 1 (S_1)

The first substructure S_1 comprises the 6th and 7th floors with the observed response at the 5th floor treated as an input motion. The equation of motion for S_1 can be formulated by assuming the substructure behaves as a structure subjected to support excitation as:

$$\begin{bmatrix} \mathrm{M}_6 & 0 \\ 0 & \mathrm{M}_7 \end{bmatrix} \begin{Bmatrix} \mathbf{q}_6^* \\ \mathbf{q}_7^* \end{Bmatrix} + \begin{bmatrix} \mathrm{L}_6+\mathrm{L}_7 & -\mathrm{L}_7 \\ -\mathrm{L}_7 & \mathrm{L}_7 \end{bmatrix} \begin{Bmatrix} \mathbf{q}_6^* \\ \mathbf{q}_7^* \end{Bmatrix} + \begin{bmatrix} \mathrm{K}_6+\mathrm{K}_7 & -\mathrm{K}_7 \\ -\mathrm{K}_7 & \mathrm{K}_7 \end{bmatrix} \begin{Bmatrix} \mathbf{q}_6^* \\ \mathbf{q}_7^* \end{Bmatrix} = \begin{Bmatrix} 0 \\ \mathrm{f}_7 \end{Bmatrix} - \begin{bmatrix} \mathrm{M}_6 & 0 \\ 0 & \mathrm{M}_7 \end{bmatrix} \begin{Bmatrix} \mathbf{q}_5 \\ \mathbf{q}_5 \end{Bmatrix} \tag{12}$$

In structural dynamics applications, however, accelerations are often the directly measured response by means of accelerometers. Though displacements and velocities can be obtained by numerical integration of the accelerations measured, numerical error would be introduced inevitably. To resolve this problem (for practical convenience), the substructural identification is formulated in such a way that only accelerations (as opposed to displacements or velocities) at interface DOFs are required to compute the interface forces at S_1.

4.2 Substructure 2 (S_2)

The second substructure S_2 comprises 3rd, 4th and 5th floors, with the 2nd and 6th floors as the interfaces. The equations of motion for S_2 can be written as:

$$\begin{bmatrix} \mathrm{M}_3 & 0 & 0 \\ 0 & \mathrm{M}_4 & 0 \\ 0 & 0 & \mathrm{M}_5 \end{bmatrix} \begin{Bmatrix} \mathbf{q}_3^* \\ \mathbf{q}_4^* \\ \mathbf{q}_5^* \end{Bmatrix} + \begin{bmatrix} \mathrm{L}_3+\mathrm{L}_4 & -\mathrm{L}_4 & 0 \\ -\mathrm{L}_4 & \mathrm{L}_4+\mathrm{L}_5 & -\mathrm{L}_5 \\ 0 & -\mathrm{L}_5 & \mathrm{L}_5+\mathrm{L}_6 \end{bmatrix} \begin{Bmatrix} \mathrm{q}_3^* \\ \mathrm{q}_4^* \\ \mathrm{q}_5^* \end{Bmatrix} + \mathrm{f}$$

$$\begin{bmatrix} \mathrm{K}_3+\mathrm{K}_4 & -\mathrm{K}_4 & 0 \\ -\mathrm{K}_4 & \mathrm{K}_4+\mathrm{K}_5 & -\mathrm{K}_5 \\ 0 & -\mathrm{K}_5 & \mathrm{K}_5+\mathrm{K}_6 \end{bmatrix} \begin{Bmatrix} \mathbf{q}_3^* \\ \mathbf{q}_4^* \\ \mathbf{q}_5^* \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \mathrm{f}_5 \end{Bmatrix} - \begin{bmatrix} \mathrm{M}_3 & 0 & 0 \\ 0 & \mathrm{M}_4 & 0 \\ 0 & 0 & \mathrm{M}_5 \end{bmatrix} \begin{Bmatrix} \mathbf{q}_2^* \\ \mathbf{q}_2^* \\ \mathbf{q}_2^* \end{Bmatrix}$$

$$+ \begin{bmatrix} 0 \\ 0 \\ \mathrm{L}_6 \mathrm{q}_6^* \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \mathrm{K}_6 \mathrm{q}_6^* \end{bmatrix} \tag{13}$$

K_6 and L_6 and the corresponding displacement and velocity are obtained from the identification results of S_1.

4.3 Substructure 3 (S_3)

The third substructure S_3 comprises the 1st and 2nd floors, with the 3rd floor as the interface. The equations of motion for S_3 can be written as:

$$\begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix}\begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{Bmatrix} + \begin{bmatrix} L_1+L_2 & -L_2 \\ -L_2 & L_2+L_3 \end{bmatrix}\begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{Bmatrix} +$$

$$\begin{bmatrix} K_1+K_2 & -K_2 \\ -K_2 & K_2+K_3 \end{bmatrix}\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ f_2 \end{Bmatrix} + \begin{bmatrix} 0 \\ L_3\dot{q}_3 \end{bmatrix} + \begin{bmatrix} 0 \\ K_3 q_3 \end{bmatrix} \quad (14)$$

Similar to S_2, K_3 and L_3 and the corresponding displacement and velocity are obtained from the identification results of S_2.

5. Damage Detection

It is possible to detect damage by identifying storey stiffness values and comparing them with the corresponding values of the original (presumably undamaged) state. For this purpose, a simple stiffness integrity index is defined as

$$S_i = \frac{K_d(i)}{K_u(i)} \quad (15)$$

where K_d (i) and K_u (i) are the storey stiffness value of the i^{th} storey for the damaged state and undamaged state, respectively. The stiffness integrity index is 1 for no loss in stiffness (no damage) and 0 for complete loss of stiffness (complete damage).

6. Numerical Examples

A 12-storey shear building model (Figure 1) is studied to test the performance of the two proposed substructural identification that is identification with substructural first-order model and second-order model. The signals of Gaussian white noise are used as input force and are assumed as known. The numerical prediction of the system response ("measurement") is performed for 17 s with iterative time step of 0.085 s by numerically simulation time response of linear time invariant models using MATLAB toolbox.

Damage is simulated by reducing the storey stiffness value. Two damage scenarios are studied: (1) with single damage and (2) with multiple damages. Scenario 1 contains 20% damage in the fourth floor (i.e. the remaining stiffness is 80% of the original value). Scenario 2 has two damage locations: 30% damage in the sixth floor and 20% in the ninth floor.

6.1 Verification of Substructural First-order Model

The task is to identify stiffness, mass and damping matrices of the 12-DOF system. There are 36 unknown parameters to be identified. However, stiffness coefficients are required for damage identification therefore only identified stiffness is shown. Three excitation forces act on the 4^{th}, 8^{th} and 12^{th} nodes. Response "measurements" of accelerations are assumed to be available at all nodes. The structure is divided into three substructures: S_1 = [7~12], i.e. 7^{th} to 12^{nd} nodes inclusive, and S_2 = [3-8] and S_3 = [1-4]. Noting that the substructures have overlapping (or common) members, the substructural identification procedure will be referred to as the SSI with overlap.

6.2 Verification of Substructural Second-order Model

The structure is divided into the same three substructures as above. Three excitation forces act on the 4^{th}, 8^{th} and 12^{th} nodes. Response "measurements" of accelerations are assumed to be available at all nodes whereas displacements and velocities are available at the 2^{nd}, 5^{th} and 9^{th} nodes.

The first substructure is S_1 = [7-12], for which no damping forces and elastic forces are needed to input at the interface. Accordingly, only accelerations (as opposed to displacements or velocities) at interface DOFs are required to compute the interface forces at S_1. The second substructure is S_2 = [3-8], for which the displacements and velocities at 2^{nd} and 9^{th} nodes are treated as input. Identified K_9 from the S_1 will then be treated as known in the subsequent substructure, which is S_2. The third substructure is S_3 = [1-4], for which the displacement and velocity at 5^{th} node is treat as input. K_5 is identified in S_2 and taken as known in S_3.

6.3 Effects of I/O noise

For the noise free case, the identification with substructural first-order model and second-order model yields an exact model for the global system. Therefore, the results of noise-free system identification are not shown here. To simulate noise system, both the input and output signals are polluted with 10% root-mean-square Gaussian white noise. The three identified substructural first-order models are assembled by using Eq. (9). Then, the identified global first-order model can be used directly in the conversion from first-order model to second-order model without any minimal realization algorithm in order to evaluate the storey stiffness values. However, in the case of substructural second-order model, the identified storey stiffness values can be determined directly from the identified substructural second-order models without any synthesis process. This is because the distribution of the storey stiffness values in the stiffness matrix is known a priori. First, the undamaged case is considered. Table 1

Table 1. Identified storey stiffness for undamaged case with 10% I/O noise

Storey	Exact Stiffness (kN/m)	Identified Stiffness in kN/m (Error in Bracket)	
		Substructural First-order Model	Substructural Second-order Model
1	500	480 (-4.0%)	510 (2.0%)
2	500	530 (6.0%)	515 (3.0%)
3	500	523 (4.6%)	489 (-2.2%)
4	500	524 (4.8%)	520 (4.0%)
5	500	529 (5.8%)	528 (5.6%)
6	500	510 (2.0%)	511 (2.2%)
7	500	507 (1.4%)	492 (-1.6%)
8	500	492 (-1.6%)	465 (-7%)
9	500	496 (-0.8%)	520 (4.0%)
10	500	520 (4.0%)	509 (1.8%)
11	500	519 (3.8%)	472 (-5.6%)
12	500	489 (-2.2%)	498 (-0.4%)
Mean Error		3.42%	3.28%
Max. Error		6.0%	7.0%

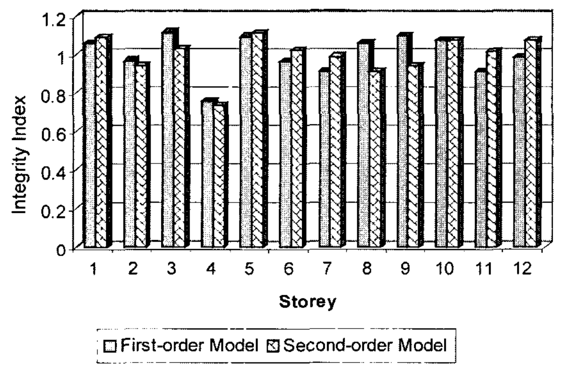

Fig. 2. Identified stiffness integrity index for Scenario 1 (single damage)

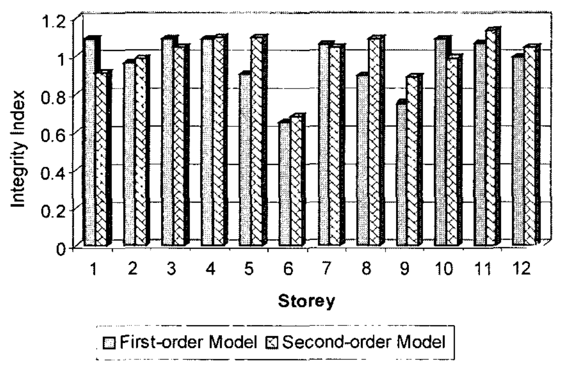

Fig. 3. Identified stiffness integrity index for Scenario 2 (multiple damage)

compares the identified stiffness with the exact values under the influence of I/O noise. Then, Fig. 2 and Fig. 3 present the identified stiffness integrity indices (as defined in Eq. (15)) for Scenarios 1 and 2, respectively.

First, the stiffness identification results are presented. The identification results of substructural first-order model and second-order model are almost the same with I/O noise level of 10%. The mean error and largest error is 3.42% and 6.0% respectively for the identification with substructural first-order model with overlap. The corresponding values are about 3.28% and 7.0% for the substructural second-order model with overlap. As expected, the identification errors increase with the noise level. Nevertheless, the identification results are reasonably good for such a high 10% noise as shown in Table 1. Normally, SSI without overlap is generally better than the SSI with overlap. This is due to the error propagation from one substructure to another in the SSI with overlap. However, there is not much error propagation problem from the proposed approaches as shown in Table 1 with reasonably good results for substructure with overlap.

Both the proposed approaches are effective in identifying the damage locations and extents. As shown in Fig. 2, the mean and maximum error in the identified stiffness integrity index is 6.72% and 11.5% for substructural first-order model approach and 5.88% and 9.3% for substructural second-order model approach in Scenario 1 under 10% I/O noise. The maximum error in Scenario 2 is larger -- 7.14% and 6.95% for the both approaches respectively, under 10% noise as shown in Fig. 3. Although full structure identification results are not shown in this paper, it can be realized that both the proposed substructure methods give much better results than the full structure identification method does. Identification of small structure like 12-DOF system with substructural first-order model is quite attractive. However, it may have numerical difficulties when one needs to determine the second-order model from the global first-order model of large system. The important point to note here is that the identification with substructural second-order model can avoid these numerical difficulties because the conversion from first-order model to second-order model is done in substructure level. In contrast, identification with substructural first-order model requires accelerations only whereas substructural second-order model needs to have velocities as well as displacements to compute the interface forces.

7. Concluding Remarks

The possibility of performing system identification at the substructure level in first-order model and second-order model has been investigated in this paper. These two approaches are developed for improving the identification results adopting the strategy of "divide and conquer". The motivation for the two proposed methods is to reduce the number of unknown by splitting a larger structure into several smaller structures. Hence, instead of full structure, identification is made easier in smaller substructures since the size of matrices involved reduces. It is found that coupling of substructure models can be accomplished by accounting for in some way the interaction forces at interfaces. Therefore, to produce exact substructure coupling in the case of substructural first-order model, the requirement is placing collocated actuators and sensors at all the interface DOFs whereas in the case of substructural second-order model, the requirement is velocities and displacements are needed at some of the interface DOFs. Therefore, only accelerations are required in the former approach whereas velocities and displacements in the latter approach. In the identification with the substructural first-order model, the proposed substructure synthesis first-order models are valid only for assembling continuous-time models. The sampling time must be set small enough to obtain accurate results. Identification with substructural second-order model is preferred when dealing with large structure in order to avoid numerical difficulties in conversion from first-order model to second-order model. From the studies in this paper, it is clearly shown that the substructural in first-order model and second-order model have better performance compared to the full structure identification method. The results are satisfactory considering the presence of I/O noise. The identified stiffness integrity index, in particular, is found to reveal the location and extent of damage in the numerical simulation study accounting for effects of I/O noise.

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