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Buckling Enhancement of Column Strips with Piezoelectric Layer

Quan Wang† **and Dajun Wang**[1]

†*Mechanical, Materials and Aerospace Engineering Department University of Central Florida, Orlando, FL 32816-2450, USA*
[1]*Department of Mechanics and Engineering Science, Peking University Beijing, P.R. China*

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ABSTRACT

This paper discusses the enhancement of the buckling capacity of column strips by use of piezoelectric layer. The analytical model for obtaining the buckling capacity of the piezoelectric coupled column with general boundary conditions modelled with different types of springs applied at the ends of the column is derived the first time. Based on this proposed model, the buckling capacity of the column strips can be accurately predicted by solving an eigenvalue problem. The computational results show the great potential of the piezoelectric materials in enhancing the buckling capacity of the column strips. The optimal locations of the piezoelectric layer for higher buckling capacity are also obtained for the columns with standard pinned-pinned, fixed-free, and fixed-pinned structures. In addition, the buckling capacity and the increase of buckling capacity are discussed for those columns with the general boundaries as well. This research may provide a benchmark for the buckling analysis of the piezoelectric coupled strips.

Keywords: column strips, buckling analysis of structures, piezoelectric coupled columns, piezoelectric layers, enhancement of buckling capacity of structures

1. Introduction

The analysis and application of coupled piezoelectric structures have recently been keenly researched because of the actuation and sensing functions of the piezoelectric materials. The analytical modelling and behaviour of a beam with surface-bonded or embedded piezoelectric sensors and actuators were presented by Bailey (1985), Lee and Moon (1989). The use of finite element method in the analysis of piezoelectric coupled structures has been studied by Tzou and Tseng (1990), Robinson and Reddy (1991), and Saravanos and Heyliger (1995). One of the successful applications by piezoelectric materials is in the vibration control. The use of piezoelectric materials in composite laminates and for vibration control was discussed by Ha and *et al.* (1992). Crawley and de Luis (1987) developed the analytical model for the static and dynamic response of a beam structure with segmented piezoelectric actuators either bonded or embedded in a laminated composite. As was pointed in this paper, the shear forces acting at the interfaces of the piezoelectric patches and the host structure can be modelled as concentrated forces at the ends of the patches. Owing to their good characteristics of lightweight and electro-mechanical coupling effects, the piezoelectric materials can be applied in other application fields, such as acoustic wave excitation, health monitoring of structures, and other applications (Milsom *et al.*, 1977; Monkhouse *et al.*, 2000; Morgan, 1998). Although the majority of the research work has been paid attention to the above mentioned application fields, the potential of the piezoelectric materials in the enhancement of buckling capacity of structures was received scant attention from the scientific community.

Chandrashekhara and Bhatia (1993) developed a finite element model for the active buckling control of laminated composite plates with surface bonded or embedded piezoelectric sensors that are either continuous or segmented. They also investigated the dynamic buckling behaviour of the laminated plate subjected to a linearly increasing compression load. In controlling the buckling of flexible columns, investigations were made by Meressi and Paden

† Corresponding author
Tel: +1-407-823-5828, Fax: +1-407-823-0208
E-mail address: qzwang@mail.ucf.edu

(1993). They showed that the buckling of a flexible beam could be postponed beyond the first critical load by means of feedback using piezoelectric actuators and strain gauges. The spillover problem was discussed as well in their work. Thompson and Loughlan (1995) reported their experimental results in verifying the potential to increase the load bearing strength of imperfection sensitive composite columns loaded in compression. The principle of their work is to apply a controlled voltage to the actuators to induce a reactive moment at the column centre. Such induced moment by piezoelectric actuator will remove the lateral deflections and enforce the column to behave in a perfectly straight manner. Chase and Bhashyam (1999) proposed optimal design equations to actively stabilize laminated plates loaded in excess of the critical buckling load using a large number of sensors and actuators. Berlin (1995) proposed several other designs, control and implementation issues for stabilising buckling using PZT actuators. The principle of most of the above work is to apply a controlled voltage to the actuators to induce a reactive moment at the column centre. Such induced moment by piezoelectric actuator will remove the lateral deflections and enforce the column to behave in a perfectly straight manner. Wang (2002) proposed a study on the enhancement of buckling capacity of a column strip with the piezoelectric effects fully modelled. The tensile effect by the piezoelectric layer was first employed for the buckling enhancement of structures. The finite difference method was employed to study the buckling capacity of the column strip with standard pinned-pinned, fixed-free boundaries. In view of all the above-cited researches on the enhancement of buckling capacities of the thin-walled structures, it is concluded that an accurate analytical model is necessary by using the tensile effect of the piezoelectric layer in the buckling enhancement of column strips with general boundaries.

This paper presents the computational results on the enhancement of buckling capacity of column strips with a pair of piezoelectric layers symmetrically bonded on the surfaces of the host column. These piezoelectric actuators bonded to the structural surfaces may be made to expand or contract depending on the direction of the applied electrical voltage and the poling direction of piezoelectric actuators. By ensuring that the piezoelectric layer expands, a tensile force may be introduced that will enhance the ability of a column structure against buckling, which is a different method from the previous researches in the enhancement of buckling capacity of structures. In order to obtain a general solution for this problem, we study the column strip with general boundary conditions. The general boundary conditions are modelled by applying two types of springs, translational springs and rotational springs, at the two ends of the column. The optimal location and the size of the piezoelectric layers will be discussed in order to obtain better enhancement of the buckling load. The buckling capacity and the increase of the buckling capacity of the structure will also be studied versus the variations of the stiffness of the springs at the ends of the column strips.

2. Problem Description

Consider a host column with width b, thickness h, length L and a pair of piezoelectric layers bonded to the surfaces of the column whose length is and the thickness is as shown in Fig. 1. The beam is isotropic with Youngs modulus E. E_p is the equivalent elastic modulus of piezoelectric material; and $\bar{e}_{31}$ is the equivalent piezoelectric coefficients for one-dimensional problem. Let x denotes the coordinate along the length of the beam with its origin at the left end of the beam. The positive direction of the deflection of the beam, , is defined downward. The piezoelectric layers are located from to $x= L_2$. In order to obtain the buckling analysis of a general column strip with the pair of piezoelectric layers, we assume the column is subjected by a translational spring with stiffness constant α_1 and a rotational spring with constant β_1 at the left end, and the same kind of springs but with constants α_2 and β_2 at the right end separately.

As the length to the thickness ratio of the column is very large, the assumption for Euler-Bernoulli beam is adopted. Therefore, the shear deformation and rotary inertia effect in the structure are omitted. The displacement field of the column for a static analysis is written as follows:

$$u_x = -z\frac{dw(x)}{dx} \tag{1}$$

where $w(x)$ and u_x are the displacements in the transverse z-direction and longitudinal x-direction respectively.

The amplitude of the strain ε_x, stress σ_x, and the induced moment M_x in the column strip can thus be written as

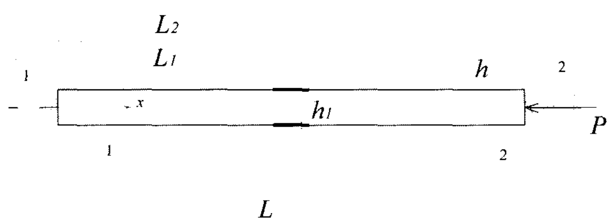

Fig. 1. A compressed column strip with a pair of piezoelectric layers.

$$\varepsilon_x = -z\frac{d^2w}{dx^2} \tag{2}$$

$$\sigma_x = -Ez\frac{d^2w}{dx^2} \tag{3}$$

$$M_x = \int_{\frac{h}{2}}^{\frac{h}{2}} z\sigma_x dz = -EI\frac{d^2w}{dx^2} \tag{4}$$

where

$$I = \frac{1}{12}bh^3 \tag{5}$$

According to electro-mechanical effects of the piezoelectric materials, the stress in the piezoelectric layer can be expressed as (Wang, 2002)

$$\bar{\sigma}_x = -E_p z\frac{d^2\bar{w}}{dx^2} - \bar{e}_{31}E_z \tag{6}$$

where the over-bar represents the variations in the piezoelectric layer.

Actually, the above equation is under the assumption that the piezoelectric layer is much thinner than the substrate metal.

It is noted from the above equation that the stress in the piezoelectric layer consists of two parts. The first term on the right hand side of Eq. (6) will lead to the bending moment applied to the column strip. This bending moment, $\bar{M}_x$ will be approximately expressed by:

$$\bar{M}_x = -E_P hbh_1\left(\frac{h}{2}+h_1\right)\frac{d^2w}{dx^2} \tag{7}$$

The second term of the right hand side of Eq. (6) will be modelled as a pure tensile force provided the pair of piezoelectric layers is placed symmetrically with poling direction oriented in the transverse direction of the column. When two equal voltages are applied to the upper and lower piezoelectric layers, the mechanical forces executed by the piezoelectric layer can lead to tensile stresses in the two piezoelectric layers. As pointed out by Crowley and Luis (1987), the shear forces acting at the interfaces of the piezoelectric layers and the host structure can be modelled as concentrated forces at the ends of the layers when the bonding layer is very thin. As such, the tensile forces excited by the two piezoelectric layers should be modelled as a pair of concentrated forces applied at the two ends of the piezoelectric layer, i.e. $x = L_1$ and, $x = L_2$,

$$F = 2bh_1\bar{\sigma}_x = -2bh_1E_p\bar{e}_{31} \tag{8}$$

As the electric field E_z in the piezoelectric layer is expressed by

$$E_z = \frac{V}{h_1} \tag{9}$$

The tensile force executed by the piezoelectric layers due to the voltages can thus be obtained from Eqs. (8) and (9),

$$F = -2b\bar{e}_{31}V \tag{10}$$

Such tensile force induced by the piezoelectric layer may be used to enhance the ability of a column structure against buckling.

Next, the buckling analysis will be conducted for this piezoelectric coupled column strip when a compression force P is subjected at the ends of the column. The mathematical model will be given first based on the above stress-strain analysis for the host column and the piezoelectric layer. An analytical solution for the mathematical model will be obtained hereafter.

3. Mathematical Model for the Buckling Analysis of the Column Strip

In order to obtain the governing equation for this piezoelectric coupled column, the column strip is analyzed by three parts: $0 \le x \le L_1$; $L_1 \le x \le L_2$; and $L_2 \le x \le L$, as there are different moment distributions for the different three parts. The displacement of the column strip for the above three parts will be obtained and expressed with $w_1(x)$, $w_2(x)$, and separately.

Since a general column is used here for analysis, it will be easy to obtain the solution for all the other particular cases by the current general analysis. For example, if we want to study the buckling analysis of a pinned-pinned column, the four constants for the springs at the twos ends of the column can be assigned by $\alpha_1 = \alpha_2 = \infty$ and $\beta_1 = \beta_2 = \infty$ in the solutions for the general case. Similarly, the analyses for a fixed-free column and a fixed-pinned column can be conducted by substituting $\alpha_1 = \beta_2 = \infty$; $\alpha_2 = \beta_2 = 0$ and $\alpha_1 = \beta_1 = \alpha_1 = \infty$; $\beta_2 = 0$ respectively.

For the buckling analysis of the general column in Fig. 1, we assume possible variations of the displacements and rotations at the two ends of the column are Δ_1, Δ_2, θ_1, and θ_2 respectively.

When $0 \le x \le L_1$, a simple moment analysis yields

$$M_x = \alpha_1 \Delta_1 x - \beta_1 \theta_1 + P(w_1 - \Delta_1) \tag{11}$$

Substituting Eq. (11) into Eq. (4) yields,

$$EI\frac{d^2 w_1}{dx^2} + \alpha_1 \Delta_1 x - \beta_1 \theta_1 + P(w_1 - \Delta_1) = 0 \tag{12}$$

whose solution can be easily obtained as

$$w_1(x) = A_1 cos\lambda_1 x + B_1 sin\lambda_1 x + \left(\Delta_1 + \frac{\beta_1}{P}\theta_1\right) - \frac{\alpha_1 \Delta_1}{P} x \tag{13}$$

where $\lambda_1 = \sqrt{\frac{P}{EI}}$.

The coefficients A_1 and B_1 can be derived below by considering the boundary conditions

$$w_1\big|_{x=0} = \Delta_1 \text{ and } \frac{dw_1}{dx}\bigg|_{x=0} = \theta_1,$$

$$A_1 = -\frac{\beta_1}{P}\theta_1 \text{ and } B_1 = \frac{\alpha_1 \Delta_1}{P\lambda_1} + \frac{\theta_1}{\lambda_1} \tag{14}$$

Similarly, when $L_2 \le x \le L$, the governing equation is obtained by

$$EI\frac{d^2 w_3}{dx^2} + \alpha_2 \Delta_2 (L - x) - \beta_2 \theta_1 + P(w_3 - \Delta_2) = 0 \tag{15}$$

whose solution can be expressed as

$$w_3(x) = A_1 cos\lambda_1 x + B_3 sin\lambda_1 x + \left(\Delta_2 + \frac{\beta_2}{P}\theta_2\right) - \frac{\alpha_2 \Delta_2}{P}(L - x) \tag{16}$$

Again the coefficients A_3 and B_3 can be derived below by considering the boundary conditions $w_3\big|_{x=l} = \Delta_2$ and $\frac{dw_3}{dx}\Big|_{x=L} = \theta_2$,

$$A_3 = M_2 \Delta_2 + N_2 \theta_2 \text{ and } B_3 = M_3 \Delta_2 + N_3 \theta_2 \tag{17}$$

where $M_2 = \frac{\alpha_2 sin\lambda_1 L}{P\lambda_1}$, $N_2 = -\frac{sin\lambda_1 L}{\lambda_1} - \frac{\beta_2 cos\lambda_1 L}{P}$,

$M_3 = -\frac{\alpha_2 cos\lambda_1 L}{P\lambda_1}$, and $N_3 = \frac{cos\lambda_1 L}{\lambda_1} - \frac{\beta_2 sin\lambda_1 L}{P}$.

For the part $L_1 \le x \le L_2$, the moment due to the tensile force F and the bending moment due to the piezoelectric layer will be involved and the governing equation can thus be expressed by

$$M_x + \overline{M}_x = \alpha_1 \Delta_1 x - \beta_1 \theta_1 + P(w_2 - \Delta_1) - F(w_2 - w_1\big|_{x=L_1}) \tag{18}$$

Eq. (18) will change to the following form by considering Eqs. (4) and (7),

$$(EI)'\frac{d^2 w_2}{dx^2} + \alpha_1 \Delta_1 x - \beta_1 \theta_1 + P(w_2 - \Delta_1) - F(w_2 - w_1\big|_{x=L_1}) = 0 \tag{19}$$

where $(EI)' = EI + E_p hbh_1\left(\frac{h}{2} + h_1\right)$.

The analytical solution for w_2 can be written as

$$w_2 = A_2 cos\lambda_2 x + B_2 sin\lambda_2 x + \frac{P}{P-F}\Delta_1 - \frac{F}{P-F}w_1\big|_{x=L_1} + \frac{\beta_1 \theta_1}{P-F} - \frac{\alpha_1 \Delta_1}{P-F}x \quad \text{for } P \ge F \tag{20a}$$

$$w_2 = A_2 cosh\lambda_2 x + B_2 sinh\lambda_2 x + \frac{P}{P-F}\Delta_1 - \frac{F}{P-F}w_1\big|_{x=L_1} + \frac{\beta_1 \theta_1}{P-F} - \frac{\alpha_1 \Delta_1}{P-F}x \quad \text{for } P < F \tag{20b}$$

where $\lambda_2 = \sqrt{\frac{|P-F|}{(EI)'}}$.

For any given external voltage applied to the piezoelectric layers, the tensile force F can be obtained from Eq. (10). Thus the buckling capacity for this piezoelectric coupled column strip will be derived based on the above governing equations together with the continuity conditions for the deflection and rotation at $x = L_1$ and $x = L_2$. We will provide the detailed analyses in the next section.

4. Analytical Solution for the Buckling Capacity of the Piezoelectric Column

The continuity conditions for this sectional analysis of the column is listed below for deducing the buckling capacity.

$$w_1\big|_{x=L_1} = w_2\big|_{x=L_1} \tag{21}$$

$$w_1'\big|_{x=L_1} = w_2'\big|_{x=L_1} \tag{22}$$

$$w_2\big|_{x=L_2} = w_3\big|_{x=L_2} \tag{23}$$

$$w_2'\big|_{x=L_2} = w_3'\big|_{x=L_2} \tag{24}$$

Besides these continuity conditions, two equilibrium equations for the column strip are provided for the deduction. The balance of the force in flexural direction of the beam leads to

$$\alpha_1 \Delta_1 + \alpha_2 \Delta_2 = 0 \tag{25}$$

The balance of the moment of the column leads to

$$-\beta_1\theta_1 + \beta_2\theta_2 - \alpha_2 \Delta L - F(w_2|_{x=L_2} - w_2|_{x=L_1}) + P(\Delta_2 - \Delta_1) = 0 \tag{26}$$

From Eqs. (20a) and (20b), the derivation of relies on the expression of $w_1|_{x=L_1}$. According to Eq. (13), the solution of $w_1|_{x=L_1}$ is found to be,

$$w_1|_{x=L_1} = M_1\Delta_1 + N_1\theta_1 \tag{27}$$

where $M_1 = \frac{\alpha_1 sin\lambda_1 L_1}{P\lambda_1} + 1 - \frac{\alpha_1 L_1}{P}$, $N_1 = -\frac{\beta_1 cos\lambda_1 L_1}{P} + \frac{sin\lambda_1 L_1}{\lambda_1} + \frac{\beta_1}{P}$

Hence w_2 can be expressed as

$$w_2 = A_2 cos\lambda_2 x + B_2 sin\lambda_2 x + \frac{P}{P-F}\Delta_1 - \frac{F}{P-F}(M_1\Delta_1 + N_1\theta_1) + \frac{\beta_1\theta_1}{P-F} - \frac{\alpha_1\Delta_1}{P-F}x$$

for $P \geq F$ (28a)

$$w_2 = A_2 cosh\lambda_2 x + B_2 sinh\lambda_2 x + \frac{P}{P-F}\Delta_1 - \frac{F}{P-F}(M_1\Delta_1 + N_1\theta_1) + \frac{\beta_{1,}\theta_1}{P-F} - \frac{\alpha_1\Delta_1}{P-F}x$$

for $P < F$ (28b)

The solution of w_1, w_2, and w_3 from Eqs. (13), (16), and (28) will be substituted into six conditions in Eqs. (21)-(26), which will leads to six homogeneous equations. The requirement for the non-trivial solutions for the unknown variables, $\Delta_1, \Delta_2, \beta_1, \beta_2, A_2,$ and B_2 will lead to an eigenvalue problem from which the buckling capacity P can thus be obtained.

Eqs. (25)-(26) and Eqs. (21)-(24) can be re-arranged into the following matrix expressions respectively after substituting the solutions of w_1, w_2, and w_3 from Eqs. (13), (16), and (28) into them,

$$\begin{bmatrix} r_{11} & r_{12} & 0 & 0 & 0 & 0 \\ r_{21} & r_{22} & r_{23} & r_{24} & r_{25} & r_{26} \\ r_{31} & 0 & r_{33} & 0 & r_{35} & r_{36} \\ r_{41} & 0 & r_{43} & 0 & r_{45} & r_{46} \\ r_{51} & r_{52} & r_{53} & r_{54} & r_{55} & r_{56} \\ r_{61} & r_{62} & 0 & r_{64} & r_{65} & r_{66} \end{bmatrix} \begin{Bmatrix} \Delta_1 \\ \Delta_2 \\ \beta_1 \\ \beta_2 \\ A_2 \\ B_2 \end{Bmatrix} = [R] \begin{Bmatrix} \Delta_1 \\ \Delta_2 \\ \beta_1 \\ \beta_2 \\ A_2 \\ B_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

for $P \geq F$ (29a)

$$\begin{bmatrix} s_{11} & s_{12} & 0 & 0 & 0 & 0 \\ s_{21} & s_{22} & s_{23} & s_{24} & s_{25} & s_{26} \\ s_{31} & 0 & s_{33} & 0 & s_{35} & s_{36} \\ s_{41} & 0 & s_{43} & 0 & s_{45} & s_{46} \\ s_{51} & s_{52} & s_{53} & s_{54} & s_{55} & s_{56} \\ s_{61} & s_{62} & 0 & s_{64} & s_{65} & s_{66} \end{bmatrix} \begin{Bmatrix} \Delta_1 \\ \Delta_2 \\ \beta_1 \\ \beta_2 \\ A_2 \\ B_2 \end{Bmatrix} = [S] \begin{Bmatrix} \Delta_1 \\ \Delta_2 \\ \beta_1 \\ \beta_2 \\ A_2 \\ B_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

for $P < F$ (29b)

where $r_{11} = \alpha_1, r_{12} = \alpha_2, r_{21} = -P + \frac{F}{P-F}\alpha_1(L_2 - L_1)$, $r_{22} = -\alpha_1 L + P$, $r_{23} = s_{23} = -\beta_1$,

$r_{24} = \beta_2$, $r_{25} = -F(cos\lambda_2 L_2 - cos\lambda_2 L_1)$, $r_{26} = -F(sin\lambda_2 L_2 - sin\lambda_2 L_1)$

$r_{31} = \frac{P}{P-F} - M_1\left(1 + \frac{F}{P-F}\right) - \frac{\alpha_1 L_1}{P-F}$, $r_{33} = \frac{\beta_1}{P-F} - N_1\left(1 + \frac{F}{P-F}\right)$, $r_{35} = cos\lambda_2 L_1$,

$r_{36} = sin\lambda_2 L_1$, $r_{41} = \frac{\alpha_1 cos\lambda_1 L_1}{P} - \frac{\alpha_1}{P} + \frac{\alpha_1}{P-F}$, $r_{43} = \frac{\lambda_1\beta_1 sin\lambda_1 L_1}{P} + cos\lambda_1 L_1$,

$r_{45} = \lambda_2 sin\lambda_2 L_1$, $r_{46} = -\lambda_2 cos\lambda_2 L_1$, $r_{51} = \frac{P - FM_1 - \alpha_1 L_2}{P-F}$,

$r_{52} = -M_2 cos\lambda_1 L_2 - M_3 sin\lambda_1 L_2 - 1 + \frac{\alpha_2(L-L_2)}{P}$, $r_{53} = \frac{FN_1 - \beta_l}{P-F}$,

$r_{54} = -N_2 cos\lambda_1 L_2 - N_3 sin\lambda_1 L_2 - \frac{\beta_2}{P}$, $r_{55} = cos\lambda_2 L_2$, $r_{56} = sin\lambda_2 L_2$, $r_{61} = \frac{\alpha_l}{P-F}$,

$r_{62} = -\lambda_1 M_2 sin\lambda_1 L_2 + \lambda_1 M_3 cos\lambda_1 L_2 + \frac{\alpha_2}{P}$,

$r_{64} = -\lambda_1 N_2 sin\lambda_1 L_2 + \lambda_1 N_3 cos\lambda_1 L_2$,

$r_{65} = \lambda_2 sin\lambda_2 L_2$, $r_{66} = -\lambda_2 cos\lambda_2 L_2$;

$s_{ij} = r_{ij} i = 1, 2, \ldots, 6$; $j = 1, 2, \ldots. 4$.

$s_{25} = -F(cosh\lambda_2 L_1 - cosh\lambda_2 L_1)$,

$s_{26} = -F(sinh\lambda_2 L_2 - sinh\lambda_2 L_1)$, $s_{35} = cosh\lambda_2 L_2$,

$s_{36} = sinh\lambda_2 L_1$, $s_{45} = -\lambda_2 sinh\lambda_2 L_1$, $s_{46} = -\lambda_2 cosh\lambda_2 L_2$,

$s_{56} = sinh\lambda_2 L_2$,

$s_{65} = -\lambda_2 sinh\lambda_2 L_2$, $s_{66} = -\lambda_2 cosh\lambda_2 L_2$.

The condition for non-trivial solution for the vector of $((\Delta_1, \Delta_2, \theta_1, \theta_2, A_2, B_2)^T)$ is thus obtained as

$$\det[R] = 0 \text{ for } P \geq F \qquad (30a)$$
$$\det[S] = 0 \text{ for } P < F \qquad (30b)$$

Hence the buckling capacity for this column strip with piezoelectric layers will be determined through Eqs. (30a) and (30b) given the geometry of the host column and the piezoelectric layers as well as the size of the layer and the voltage applied to the layer.

In the next section, the enhancement of the buckling capacity of the column strip will be studied computationally.

5. Buckling Results and Discussions

In this section, buckling results for a column strip with a pair of piezoelectric patches that are bonded on its two surfaces (see Fig. 1) are presented. The adopted values for material and geometrical parameters of the column strip and the piezoelectric layers are given in Table 1. The non-dimensional buckling capacity will be taken as $\bar{P} = P/P_{cr}$ for all the following studies, where $P_{cr} = \frac{\pi^2 EI}{L^2}$ is the Euler buckling load (Timoshenko and Gere, 1961). First, a validation is conducted by comparing the results of the buckling capacities for the columns with standard pinned-pinned (P-P), fixed-free (Fi-F), and fixed-pinned (Fi-P) ends by the current study and the results in Timoshenko and Gere (1961). The computational solutions by the current study are obtained from Eqs. (30a) and (30b) based on the mathematical models discussed in the paper when voltages and the thickness of the layers are taken zero. The results shown in Table 2 reveals that the error percentage is within 0.1%.

Next the buckling capacities for the (P-P) column versus the voltages applied to the piezoelectric layers are plotted in Figure 2(a). The parameters for the locations of the layers are chosen as $L_1 = 0m$; $L_2 = 0.3m$ and $L_2 = 0.4m$ respectively. It is shown that the buckling capacities of the column strip are increased up to 20% for the case $L_2 = 0.4m$ when voltage $V = 26.03v$. As is also noted at $v=0$, there is slight increase for the buckling capacity which is due to the composite effects of the piezoelectric layers. However, such increase is incomparable with that by the electro- mechanical effects of the piezoelectric actuator. Figure 2(b) and Figure 2(c) show the buckling capacities of the (Fi-F) and (Fi-P) column when $L_1 = 0.5m$ and $L_2 = 0.8m$; $L_2 = 0.9m$ respectively. The maximum increases of the buckling capacity for (Fi-F) and (Fi-P) are 86% and 13% respectively when $V = 26.03v$. From the results obtained above, it can be seen that the piezoelectric layer has more effects on the enhancement of the buckling capacity for "softer" structures, as is found that the enhancement is 86% for Fixed-free column. This observation is reasonable because the critical buckling load for

Table 1. Geometrical and Material Properties of Column Strip and Piezoelectric Patch

	Column Strip	Piezoelectric Patch
Length (m)	1.	-
Width (m)	0.02	0.02
Height (m)	0.002	0.0001
Youngs modulus (N/m^2)	210.0×10^9	78.6×10^9
$\bar{e}_{31}$ (C/m^2)	-	- 9.29

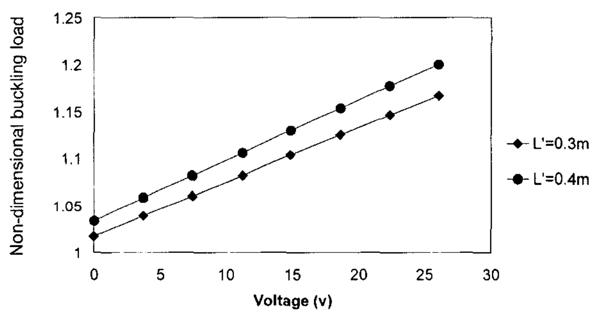

Fig. 2(a). Buckling capacity of the pined-pined column versus the voltage on the piezoelectirc layer

Table 2. Validation of the Non-dimensional value of the buckling load

	Pinned-pinned	Fixed-free	Fixed-pinned
Theoretical solution (Timoshemko, 1961)	1.000	0.250	2.047
Current finite difference method	1.001	0.251	2.046

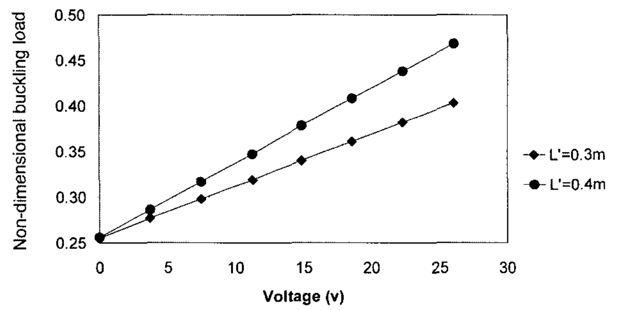

Fig. 2(b). Buckling capacity of the fixed-free column versus the voltage on the piezoelectirc layer

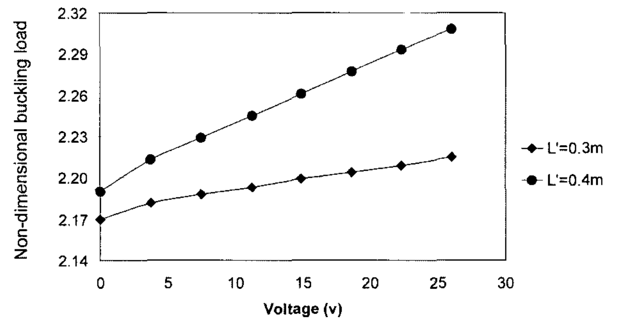

Fig. 2(c). Buckling capacity of the fixed-pined column versus the voltage on the piezoelectirc layer

a fixed-free column is only $0.25P_{cr}$. It is with no doubt the same amount of tensile force executed by the piezoelectric layer will produce more enhancement of buckling capacity for such "softer" structure than for other "harder" structures, such as (P-P) and (Fi-P) columns.

The buckling capacities of the (P-P), (Fi-P), and (Fi-P) columns versus the sizes of the piezoelectric layer are studies in Figures 3(a), (3b), and (3c) respectively when voltages are chosen as $V = 26.03v$. In all the cases, the left end of the piezoelectric layer is put at the left end of the column. The enhancement of the buckling capacities in the three cases are up to 47%, 152%, and 30% when the piezoelectric layers are in full size of the host column.

The studies of the placements of the piezoelectric layer for obtaining the best enhancement of the buckling capacities of the column stripes are studied in Figures 4(a)-4(c) based on the analyses of (P-P), (Fi-F), and (Fi-P) columns where the length of the piezoelectric layer is 0.3 m. It is found that the optimal placements of the piezoelectric layer are at the two ends of (P-P) column, and right ends of the (Fi-F) and (Fi-P) columns. It is concluded that the best locations for the placement of the piezoelectric layer in enhancement of the buckling capacity of the column strip is at the point with least curvature of the strip.

All the above conclusions are all based on the sim-

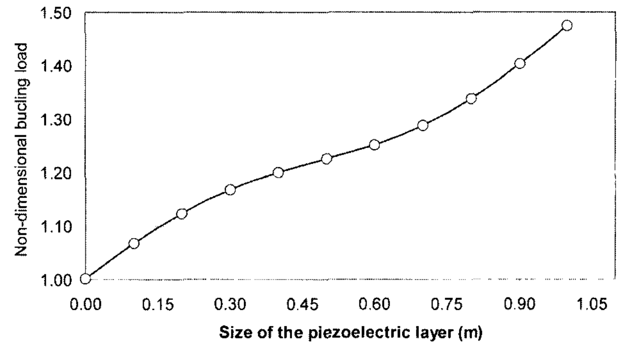

Fig. 3(a). Buckling capacity of the pined-pined column versus the size of the piezoelectirc layer

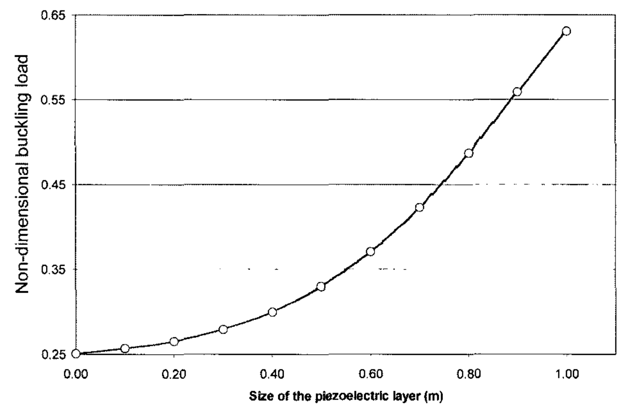

Fig. 3(b). Buckling capacity of the fixed-free column versus the size on the piezoelectirc layer

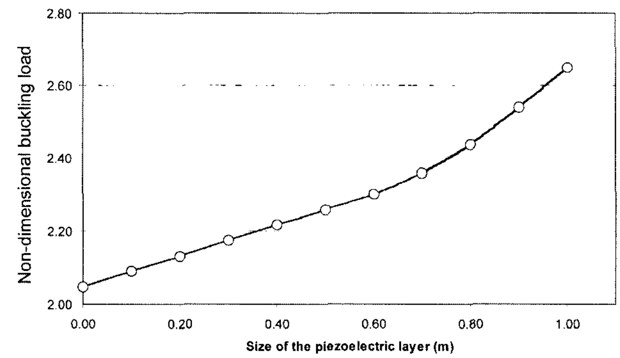

Fig. 3(c). Buckling capacity of the fixed-pined column versus the size of the piezoelectirc layer

ulations for the column strips with standard boundary conditions. Next, further investigations on the enhancement of the buckling capacities of the columns with general boundary constraints are studied. The non-dimensional spring constants in the following simulations are chosen as $\bar{\alpha}_1 = \alpha_1/\alpha$, $\bar{\alpha}_2 = \alpha_2/\alpha$, $\bar{\beta}_1 = \beta_1/\beta$, and $\bar{\beta}_2 = \beta_2/\beta$, where $\alpha = EI/L^3$ and $\beta = EI/L$.

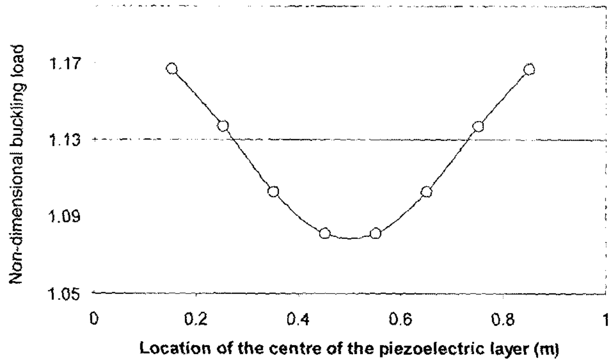

Fig. 4(a). Buckling capacity of the pined-pined column versus the location of the piezoelectirc layer.

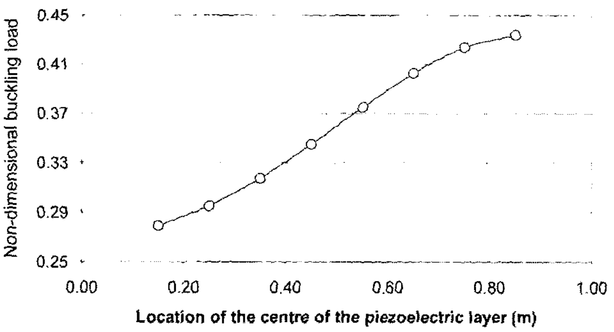

Fig. 4(b). Buckling capacity of the fixed-free column versus the location of the piezoelectirc layer.

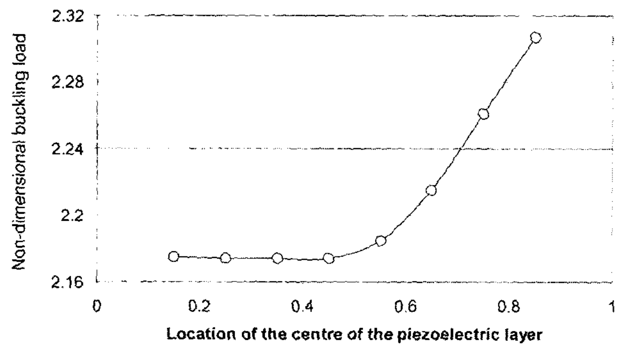

Fig. 4(c). Buckling capacity of the fixed-pined column versus the location of the piezoelectirc layer

Assume $\bar{P}_0$ represents the buckling capacity of column with general boundaries without piezoelectric layer bonded on it. The variations of the buckling capacity, $\bar{P}$, and the increase of the buckling capacity of the strips in percentage, $(\bar{P}-\bar{P}_0)/\bar{P}_0$, versus the variation of $\bar{\alpha}_1$ and $\bar{\alpha}_2$ when $\bar{\beta}_1 = \bar{\beta}_2 = 0$ are studied in Figure 5(a) and 5(b). In this simulations, voltage applied to the layers are taken as V = 26.03v, and the positions of the ends of the layer are assumed to be L_1=0.3*m* and L_2=0.7*m*. From Figure 5(a), it

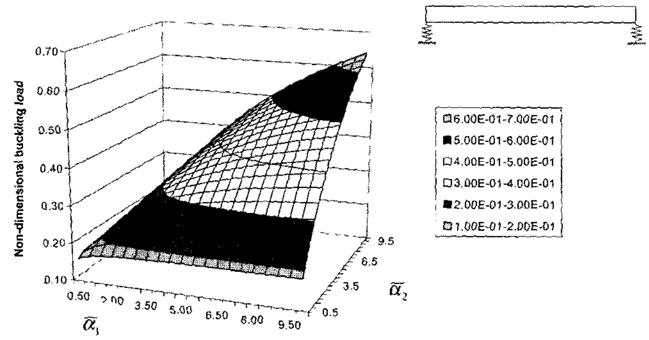

Fig. 5(a). Variation of the buckling load versus the stiffness of the springs

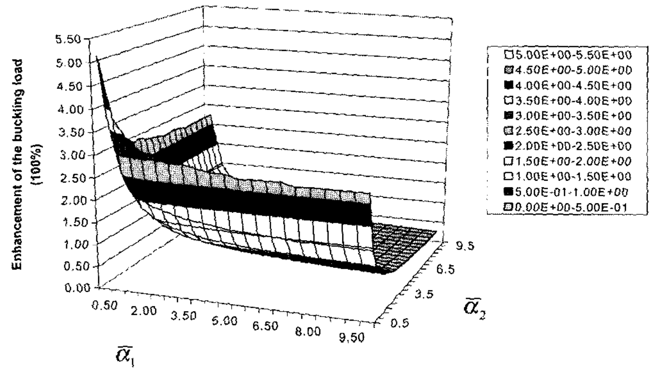

Fig. 5(b). Enhancement of the buckling capacity versus the stiffness of the springs

shows that for the columns with stiffer springs at the ends, the higher buckling capacities of the column will be obtained. This is an obvious conclusion as a higher buckling capacity is always obtained for a stiffer structure. A more interesting observation is found, in Figure 5(b), that the highest value for the increase of the buckling capacity is obtained at the lowest values of $\bar{\alpha}_1$ and $\bar{\alpha}_2$. Thus it is concluded that it is more effective to enhance the buckling capacities of column with "softer" boundary constraints by use of the piezoelectric layers.

The above observations are further validated through the following similar simulations:

(1). Figures 6(a) and (b) for the variations of the buckling capacity and the increase of the buckling capacity of the strips in percentage versus the variation of $\bar{\alpha}_1$ and $\bar{\beta}_1$ when $\bar{\alpha}_1 = \bar{\beta}_2 = 0$;

(2). Figures 7(a) and (b) for the variations of the buckling capacity and the increase of the buckling capacity of the strips in percentage versus the variation of $\bar{\alpha}_1$ and $\bar{\alpha}_2$ when $\bar{\beta}_1 = \infty$, $\bar{\beta}_2 = 0$;

(3). Figures 8(a) and (b) for the variations of the buck-

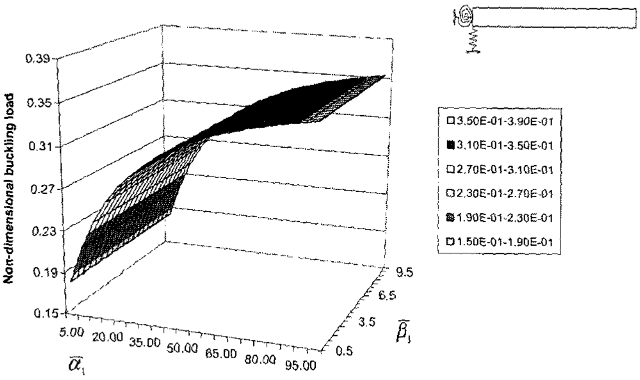

Fig. 6(a). Variation of the buckling capacity versus the stiffness of the springs.

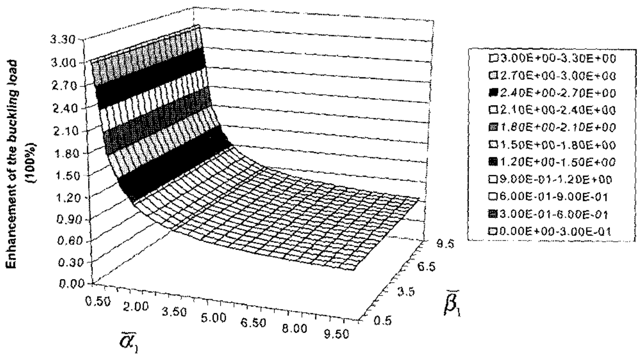

Fig. 6(b). Enhancement of the buckling capacity versus the stiffness of the springs.

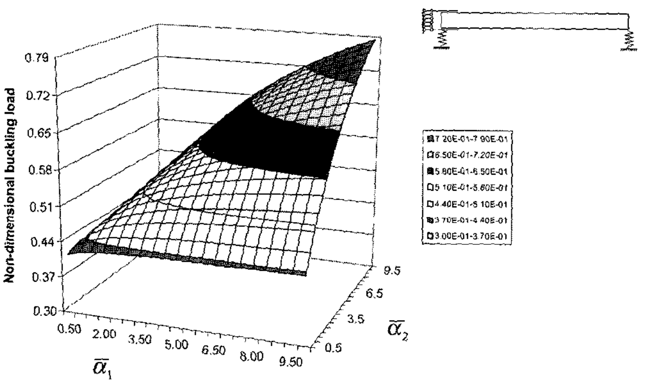

Fig. 7(a). Variation of the buckling load versus the stiffness of the springs.

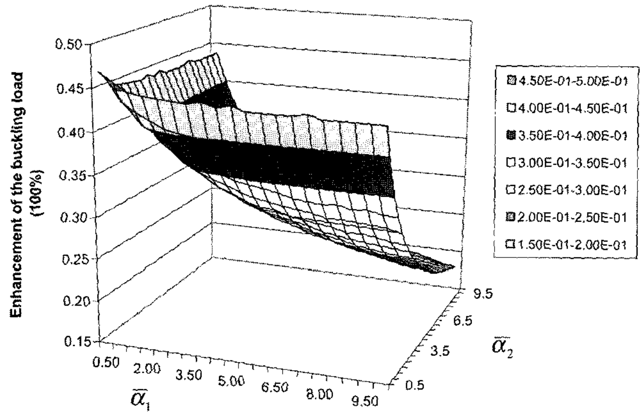

Fig. 7(b). Enhancement of the buckling capacity versus the stiffness of the springs.

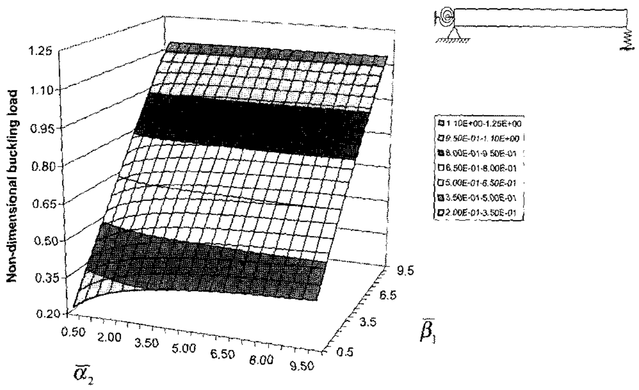

Fig. 8(a). Variation of the buckling capacity versus the stiffness of the springs.

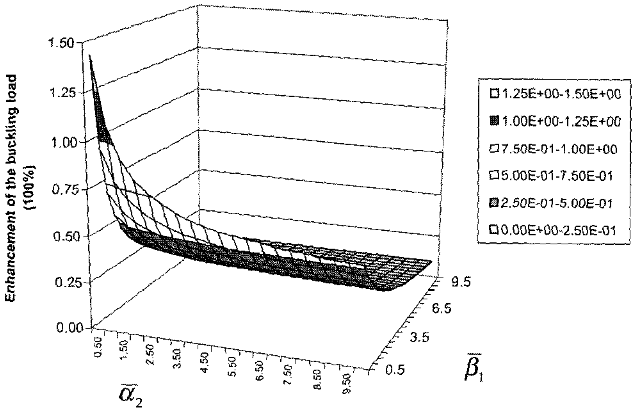

Fig. 8(b). Enhancement of the buckling capacity versus the stiffness of the springs.

ling capacity and the increase of the buckling capacity of the strips in percentage versus the variation of $\bar{\alpha}_2$ and $\bar{\beta}_1$ when $\bar{\alpha}_1 = \infty$, $\bar{\beta}_2 = 0$;

(4). Figures 9(a) and (b) for the variations of the buckling capacity and the increase of the buckling capacity of the strips in percentage versus the variation of $\bar{\alpha}_1$ and $\bar{\beta}_1$ when $\bar{\alpha}_2 = \infty$, $\bar{\beta}_2 = 0$.

Similar to the observations in Figures 5(a) and (b), all the higher buckling capacities of the column strips are obtained for the columns with stiffer springs on the ends of the strips. However, all the higher increase of the buckling

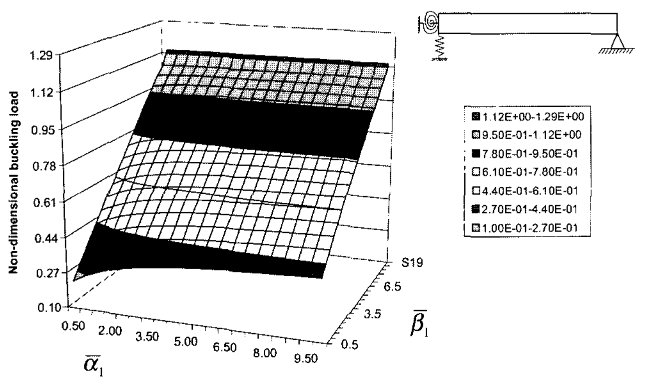

Fig. 9(a). Variation of the buckling capacity versus the stiffness of the springs.

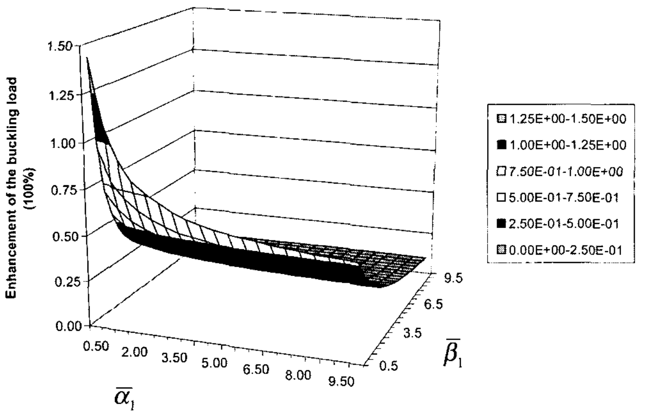

Fig. 9(b). Enhancement of the buckling capacity versus the stiffness of the springs.

capacities of the strips are obtained for strips with softer springs on the ends of the column.

6. Concluding Remarks

The enhancement of the buckling capacity of general column strips by use of a pair of piezoelectric layers is discussed in the paper. The analytical model is provided for the column strip with general boundary conditions by assuming two types of springs applied at the ends of the column.

The numerical simulations are conducted to show that the piezoelectric layer has great potential in enhancing the buckling capacity of the column strip. In addition, the results of the variation of such enhancement for the standard Pinned-pinned, fixed-free, and fixed pinned columns reveal that the optimal location of the piezoelectric layer is around the position of the column with minimum curvature. Further investigation on the buckling capacity of the column strip with general boundaries also indicates that the buckling capacity is higher with stiffer springs at the ends of the strip. In addition, the results also show that for columns with softer springs, the increase of the buckling capacity of the strip is more obvious.

This research may provide a benchmark for the estimate of the buckling capacity of a column strip by use of piezoelectric layer. Further research is under way to discuss the effects of multi-piezoelectric layer in the enhancement of the buckling capacities of column structures.

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