Journal of the Korean Institute of Industrial Engineers
Vol. 30, No. 4, pp. 306-316, December 2004.

자동창고의 저장 및 불출요구의 대기시간에 관한 연구

조 면 식[1†] · Yavuz A. Bozer[2]

[1]경기대학교 산업공학과 / [2]미시간대학교

Expected Waiting Times for Storage and Retrieval Requests in Automated Storage and Retrieval Systems

Myeonsig Cho[1] · Yavuz A. Bozer[2]

[1]Department. of Industrial Engineering, Kyonggi University, Suwon, 442-760
[2]Department. of Industrial and Operations Engineering, The University of Michigan, Ann Arbor, MI, USA, 48109-2117

We present a closed form approximate analytical model to estimate the expected waiting times for the storage and retrieval requests of an automated storage/retrieval (AS/R) system, assuming that the storage/retrieval (S/R) machine idles either at the rack or at the input/output point. The expected waiting times (and the associated mean queue lengths) can play an important role to decide whether the performance of a stable AS/R system is actually acceptable, to determine buffer size (or length) of the input conveyor, and to compute the number of the rack openings which is required to hold the loads which are requested by processing machines but waiting in the rack to be retrieved by the S/R machine. This model can be effectively used in the early design stage of an AS/R system.

Keywords: automated storage/retrieval system, expected waiting time

1. Introduction

Automated storage/retrieval (AS/R) systems play a significant role in manufacturing and warehousing due to higher space utilization and accurate inventory control, among other benefits. The AS/R systems are not only used to store the raw materials and/or finished goods, but also used more often to store the work-in-process (WIP) in automated factories. Although the AS/R system in an automated factory is a supporting facility to store and retrieve WIP, if it is not properly designed, it would be a bottleneck to meet the manufacturing requirement. Furthermore, since the initial investment cost for AS/R system is high and reconfigurability of the system is limited, the performance of the proposed AS/R system should be thoroughly analyzed in the design stage of the system.

One of the common types of AS/R systems is the unit load AS/RS, where pallet loads are stored and subsequently retrieved, one at a time, by the S/R machine. In this paper, we derive a closed form analytical expression to estimate the expected waiting times for the storage and the retrieval requests, assuming that requests occur randomly and independently. The expected waiting times (and the associated mean queue lengths) can play an important role to decide whether the performance of a stable AS/R system is actually acceptable, to determine buffer size (or length) of the input conveyor, and to compute the number of the rack openings which is required to hold the loads which are requested by processing machines but waiting in the rack to be retrieved by the S/R machine.

An AS/R system consists of one or several aisles and each aisle consists of an input/output(I/O) point, a S/R machine, and storage racks on either side. Loads to be stored wait at the input point until the S/R machine is available and loads retrieved by the S/R machine are deposited at the output

† Corresponding author : Professor Myeonsig Cho, Department of Industrial Engineering, Kyonggi University, Suwon, 442-760, Korea,
Fax : +82-31-244-3534, E-mail : mscho@kyonggi.ac.kr
Received January 2004; revision received July 2004; accepted September 2004.

point. Typically the I/O point is located at the lower left-hand corner of the rack and a pair of short conveyors serves as the I/O point; one conveyor for input and one conveyor for output.

The storage and retrieval requests are served by the S/R machine which performs either single command (SC) or dual command (DC), assuming that each trip starts and finishes at the I/O point. There are two types of the SC: storage SC and retrieval SC. To perform a SC storage opeartion, the S/R machine picks up the load from the input conveyor, travels to an empty rack opening, deposits the load, and travels empty back to the I/O point. To perform a SC retrieval operation, the S/R machine travels empty from the I/O point to the appropriate rack opening, picks up the load, travels back to the I/O point and deposits the load on the output conveyor. When the S/R machine returning to the I/O point finds a storage and a retrieval request, then it performs a DC; i.e., it picks up the load to be stored, travels to an empty rack opening, deposits the load, then travels empty directly to the load to be retrieved, picks up the load, travels back to the I/O point, and deposits the load. When the S/R machine returning to the I/O point finds no requests to serve, it will become idle at the I/O point. In other words, the S/R machine will always idle at the I/O point.

In the above strategy, the S/R machine should return to the I/O point at the end of SC storage operation, even though there are no storage requests at the I/O point and/or there are retrieval requests to serve at the moment the load is deposited into the rack. An alternative strategy is to keep the idle S/R machine at the point of deposit (within the rack) following a storage, and at the I/O point following a retrieval (Bozer and White, 1984; Egbelu and Wu, 1993; Bozer and Cho, 1998). If the S/R machine is idle within the rack and the next request is a retrieval (storage), then the S/R machine travels directly to the retrieval point (I/O point) to pick up the load. If the S/R machine is idle at the I/O point and the next request is a retrieval, then a SC retrieval is performed as described above. If the next request is a storage, the S/R machine picks up the load and checks the two request queues after depositing the load in the rack. If the retrieval queue is non-empty, the S/R machine serves the retrieval request first, i.e., performs DC. If the retrieval queue is empty and the storage queue is non-empty, it travels to the I/O point and perform a storage operation. Otherwise, it idles within the rack until either storage or retrieval request occurs.

Bozer and Cho(1998) show empirically that the latter dwell point strategy, i.e., the S/R machine idles either at the rack or at the I/O point, is reasonable and performs well compared to the dwell point strategy where each trip starts and finishes at the I/O point. Therefore, in order to develop the analytical model to estimate the expected waiting times, we assume that the S/R machine can be idle either at the rack or at the I/O point.

2. Literature review

A number of papers concerned with AS/R system appear in the literature. Most of the research efforts focus on (1) estimating travel time of an S/R machine under various storage assignment policies to measure the performance of the system such as throughput (Bozer and White, 1990; Hwang and Lee, 1990; Foley and Frazelle, 1991; Chang *et al*., 1995); (2) sequencing of retrieval requests to reduce travel time of an S/R machine (Han *et al*., 1987; Bozer *et al*., 1990; Elsayed and Lee, 1996); (3) studying the dwell point strategies to minimize turnaround time (Egbelu, 1991; Egbelu and Wu, 1993; Hwang and Lim, 1993; Peters *et al*., 1996); (4) estimating the AS/R throughput and/or turnaround time of the requests in stochastic environment (Bozer and White, 1984; Lee, 1997; Lim *et al*., 2001; Bozer and Cho, 1998).

While studying the behavior of the AS/R system, most of the researchers tend to ignore the stochastic nature of an AS/RS, i.e., all the storage and retrieval requests are known and waiting to be processed, or employ simulation to analyze the stochastic system, which is costly and time consuming to develop.

Bozer and White(1990), Lee(1997), Lim *et al*.(2001), and Bozer and Cho(1998) present analytical results for AS/RS under stochastic storage and retrieval requests. Bozer and White(1990) study the performance of the miniload AS/RS with one picker as a two server closed queueing network, where the S/R machine and the picker are modeled as servers and the number of pick positions are modeled as the number of customers in the system.

Lee(1997) presents an analytical stochastic model of an unit load AS/RS using a single server queueing model with two queues, one for storage and one for retrieval, and two different service modes, that is, SC and DC. It is assumed that two service times for the S/R machine, i.e., SC and DC times, follow the exponential distributions with the mean service times which can be easily obtained from Bozer and White(1984). (However, as shown in Bozer(1978), the variability of cycle times for both SC and DC in real system is far smaller than that of the exponential cycle times.) It is also assumed that the capacities of both the storage and the retrieval queues are finite and the requests denied to enter the queues are lost. Using the memoryless property of the exponential service times and Poisson arrival process, they model the operation of the unit load AS/R systems as a continuous time Markov chain.

Lim *et al.*(2001) develop an analytical model to compute the S/R machine utilization and mean waiting time using an M/G/1 queueing model with a single server and two queues. They assume that the S/R machine always idle at the I/O point, which is also used in Lee(1997). They first develop the waiting time model assuming that the SC and DC travel times follow the same probability distribution and then the distributions of SC and DC are approximated using the service amount required for the storage and retrieval requests.

Bozer and Cho(1998) develop a closed form analytical model to evaluate the throughput performance of an AS/R system under stochastic demand and general service time distributions for SC and DC, and determine whether it meets the required throughput or not. To develop the model, they assume that the S/R machine can idle either at the rack or at the I/O point. They show that the analytical model itself works well and they also show empirically that the dwell point strategy outperforms the dwell point strategy where the S/R machine is always idle at the I/O point, when S/R machine utilization level is moderate or high.

3. The waiting time model

In order to develop an analytical expected waiting time model for the storage and the retrieval requests, we assume that the arrival processes of these requests are Poisson. We also assume that the requests in each queue are served by First-Come-First-Served (FCFS) basis. However, the storage and retrieval requests as a whole are not necessarily served by FCFS, since the S/R machine can perform DC.

As discussed earlier, upon completion of servicing a storage request, if there is no other requests to serve, the S/R machine stays in the rack where the load is deposited. Also, upon completion of servicing the retrieval request, if there is no other requests to serve, the S/R machine stays at the I/O point where the load is delivered. Since the S/R machine resides either at the I/O point or in the rack when it becomes idle, we describe the operation of the S/R machine as a two nodes model as shown in <Figure 1>

The I/O point is denoted as node 1 and the rack is denoted as node 2 in <Figure 1>. Recall that under the dwell strategy used in this paper, whenever the S/R machine delivers a load, it checks if there is a load to pick up. The time taken by the S/R machine to pick up a load from the node i, travel to node j, $j \neq i$, and deposit it at node j, is assumed to be a random variable with mean α_{ij} and second moment $\alpha_{ij}^{(2)}$. The empty S/R machine travel time from node i to j, on the other hand, is a random variable with mean β_{ij} and second moment $\beta_{ij}^{(2)}$. When the S/R machine deposits a load into a storage location in the rack and there is at least one retrieval request to serve, the S/R machine has to travel empty to pick up the corresponding retrieval request. (That is, the S/R machine is performing a DC.) This empty travel time, which is known as the travel time between storage and retrieval points, is a random variable with mean β_{22} and second moment $\beta_{22}^{(2)}$. When the S/R machine deposits a load into the I/O point and finds at least one storage request, no empty travel time is required. Hence, $\beta_{11} = \beta_{11}^{(2)} = 0$.

In order to obtain the second moments of the travel times discussed above, we first represent the travel time model developed by Bozer and White(1984). To develop the mean travel times, they assume that (1) the rack is continuous; (2) the S/R machine travels simultaneously in the horizontal and vertical directions; (3) randomized storage is used; and (4) I/O point is located lower left corner of the rack. Let $t_h(t_v)$ denote the horizontal(vertical) travel time required to go to the farthest column(row) from the I/O point. Let $T = Max(t_h, t_v)$ and $b = Min(t_h/T, t_v/T)$, which is known as the shape factor. Let Z denote a random variable which represents the travel time from the I/O point to (x, y), which is the storage (or retrieval) point in time, $0 \leq x \leq 1$ and $0 \leq y \leq b$. (Without loss of generality, we assume that $T = t_h$.) They show that the probability density function, $g(z)$, is

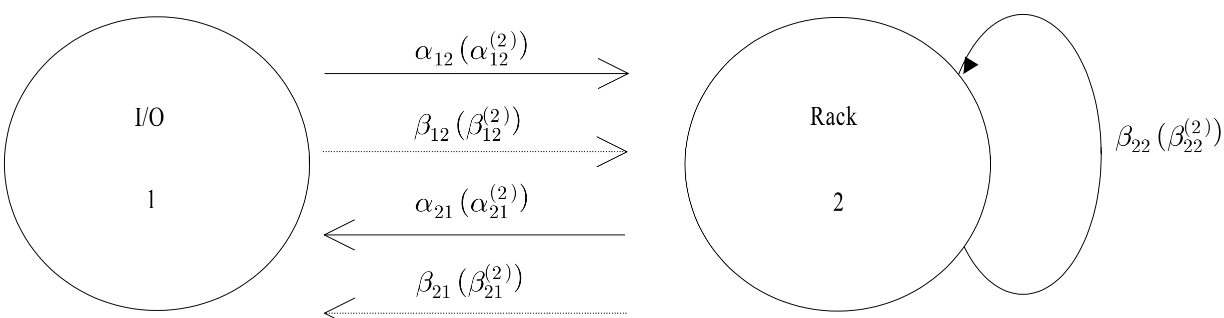

Figure 1. Two nodes model of an AS/RS.

$$g(z) = \begin{cases} 2z/b & \text{for } 0 \le z \le b \\ 1 & \text{for } b < z \le 1 \end{cases} \quad (1)$$

Using equation (1), they show that the normalized expected one way travel time, i.e., *E(Z),* excluding the pick up and deposit times, is

$$E(Z) = \frac{1}{6}b^2 + \frac{1}{2}. \quad (2)$$

The second moment of the normalized one way travel time can be easily obtained as follows.

$$\begin{aligned} E(Z^2) &= \int_0^1 z^2 g(z)dz \\ &= \frac{1}{6}b^3 + \frac{1}{3}. \end{aligned} \quad (3)$$

Let D denote a random variable which represents the actual, i.e., denormalized, one way travel time. Then,

$$D = \begin{cases} ZT & \text{for } \textit{empty travel} \\ ZT + K & \text{for } \textit{loaded travel} \end{cases} \quad (4)$$

where K is the sum of the load pick-up and deposit times. Therefore the first and second moments of the one way empty and loaded travel times are

$$\begin{aligned} \alpha_{12} &= \alpha_{21} = E(D) \\ &= E(ZT+K) = E(Z)T + K, \\ \beta_{12} &= \beta_{21} = E(D) \\ &= E(ZT) = E(Z)T, \\ \alpha_{12}^{(2)} &= \alpha_{21}^{(2)} = E(D^2) \\ &= E(Z^2)T^2 + 2E(Z)TK + K^2, \\ \beta_{12}^{(2)} &= \beta_{21}^{(2)} \\ &= E(D^2) = E(Z^2)T^2. \end{aligned} \quad (5)$$

Bozer and White(1984) also show that the probability density function, *f(z),* of travel time between the storage and retrieval points is

$$f(z) = \begin{cases} (2-2z)(2z/b - z^2/b^2) \\ \quad + (2z - z^2)(2/b - 2z/b^2) \\ \qquad\qquad \text{for } 0 \le z \le b \\ 2 - 2z \qquad \text{for } b < z \le 1 \end{cases} \quad (6)$$

From equation (6), they show that the normalized expected travel time between the storage and retrieval points, *E(TB)*, is

$$E(T_B) = \frac{1}{3} + \frac{1}{6}b^2 - \frac{1}{30}b^3 \quad (7)$$

One can also easily obtain the normalized second moment of travel time between the storage and retrieval points as

$$\begin{aligned} E(T_B^2) &= \int_0^1 z^2 f(z)dz \\ &= \frac{1}{6} + \frac{2}{15}b^3 - \frac{1}{30}b^4. \end{aligned} \quad (8)$$

Hence, the actual first and second moments of the empty travel time between the storage and retrieval points are

$$\beta_{22} = E(T_B)T, \text{ and}$$

$$\beta_{22}^{(2)} = E(T_B^2)T^2 \quad (9)$$

Let λ_1 and λ_2 denote external arrival rates of storage and retrieval requests, respectively. Let Λ_1 denote the rate at which loads are delivered at the I/O point by the S/R machine and let Λ_2 denote the rate at which loads are stored into the rack by the S/R machine. That is, $\lambda_1 = \Lambda_2$ and $\lambda_2 = \Lambda_1$. Let λ_T denote the total arrival rate of storage and retrieval requests, i.e., $\lambda_T = \lambda_1 + \lambda_2$. Let Δ_f denote the proportion of time that the S/R machine is traveling with a load, i.e., S/R machine utilization due to loaded travel which must be less than 1. The term Δ_f is easily computed as

$$\Delta_f = \lambda_1 \alpha_{12} + \lambda_2 \alpha_{21}. \quad (10)$$

Let q_i denote the probability that node *i* is empty at the instant the (loaded) S/R machine has just delivered a load at node *i* and let $\overline{q_i}$ denote its complement, i.e., $\overline{q_i} = 1 - q_i$. A closed form expression for q_i is presented in Bozer and Cho(1998) as follows. Assuming that the S/R machine completes service at the random points in time, they obtain the following expression for the S/R machine utilization, ρ:

$$\rho = 1 - q_1 q_2 \quad (11)$$

They show that q_1 and q_2 can be obtained as

$$q_1 = \Phi_1 \rho - \Phi_2 \text{ and}$$

$$q_2 = (\lambda_2/\lambda_1)(q_1 - 1) + 1, \quad (12)$$

where

$$\Phi_1 = \frac{1}{\lambda_2(2\beta_{12} - \beta_{22})} - \frac{\lambda_1}{\lambda_T} \text{ and}$$

$$\Phi_2 = \frac{2\lambda_1\beta_{12} + \lambda_2\beta_{22} + \lambda_T K}{\lambda_2(2\beta_{12} - \beta_{22})} - \frac{\lambda_1}{\lambda_T}.$$

Substituting q_1 and q_2 of equation (12) into equation (11) and solving the second degree equation, they obtain the value of ρ. Hence, the values of q_1 and q_2 can be obtained from equation (12).

With the above results, we proceed as follows to develop a closed form expression for the expected waiting times for the storage and the retrieval requests. Consider a tagged storage(retrieval) request which arrives at node *i, i*=1, 2. Let $P_i(n)$ denote the probability that the tagged request finds *n* requests already present at node *i*, and let $W_i(n)$ present the conditional expected waiting time for the tagged request, given that it finds *n* requests at node *i* upon arrival. Let W_i denote the expected waiting time for the tagged request arriving at node *i*. Then

$$W_i = \sum_{n=0}^{\infty} P_i(n)\, W_i(n). \tag{13}$$

If we can estimate the values of $P_i(n)$ and $W_i(n)$, we can determine W_i from the above equation. To estimate $W_i(n)$, we consider two cases : $n = 0$ and $n > 0$.

If the tagged request finds no requests at node *i* upon arrival, the S/R machine is either busy or it is idle at node *i, i* = 1, 2. Let π_i denote the probability that the S/R machine is busy when the tagged request arrives at node *i*. For this case, let C_i^B denote the expected time for the S/R machine to return to node *i*. On the other hand, if the tagged request finds the S/R machine idle, then the idle S/R machine is dispatched to node *i*. Let C_i^I denote the expected time for the S/R machine to arrive at node *i* from the idle state. Thus, for $n = 0$, we have

$$W_i(0) = \pi_i C_i^B + (1-\pi_i) C_i^I. \tag{14}$$

If the tagged request finds n>0 requests at node *i* upon arrival, we define the request at the head of this node as the Head-Of-Line (HOL) request. The expected waiting time for the tagged request is the sum of two quantities: (i) the expected time, C_i^H, starting from the time of its arrival until the time the S/R machine arrives at that node to pick up the HOL request, and (ii) the expected time for the S/R machine to pick up the n-1 remaining requests, followed by a visit to pick up the tagged request; that is, the expected time for the S/R machine to complete *n* successive cycles where the expected time of a cycle is denoted by C_i^S. Thus,

$$W_i(n) = C_i^H + nC_i^S, \qquad n>0. \tag{15}$$

Hence, from equations (13) ~ (15),

$$\begin{aligned} W_i &= P_i(0)[\pi_i C_i^B + (1-\pi_i)C_i^I] + \sum_{n=1}^{\infty} P_i(n)(C_i^H + nC_i^S) \\ &= P_i(0)[\pi_i C_i^B + (1-\pi_i)C_i^I] + C_i^H \sum_{n=1}^{\infty} P_i(n) + C_i^S \sum_{n=1}^{\infty} nP_i(n) \end{aligned}$$

Noting, from Littles formula, that $\sum_{n=1}^{\infty} nP_i(n) = \lambda_i W_i$, we have

$$W_i = \frac{P_i(0)[\pi_i C_i^B + (1-\pi_i)C_i^I] + [1-P_i(0)]C_i^H}{1-\lambda_i C_i^S} \tag{16}$$

As assumed in Bozer and Cho(1998), we also assume that q_i represents the probability that the node *i* is empty at an arbitrary instant in time. Hence, $P_i(0)$ can be approximated by q_i. Therefore, to estimate W_i, the values of π_i, C_i^I, C_i^H, C_i^B, C_i^S need to be determined. Since the probability that the S/R machine is busy is ρ, the expression for π_i is derived by conditioning on the number of requests present at node *i* when the tagged request arrives as follows:

ρ = Probability that S/R machine is busy

= P(S/R machine busy | n=0) $P_i(0)$

$+ P$(S/R machine busy | n>0)(1 − $P_i(0)$)

$= \pi_i P_i(0) + 1 - P_i(0)$,

since the S/R machine cannot be idle when n>0. Hence,

$$\pi_i = 1 - \frac{1-\rho}{P_i(0)} \approx 1 - \frac{1-\rho}{q_i}. \tag{17}$$

The expected time for the S/R machine to arrive at node *i* from the idle state, C_i^I, is determined by assuming that the location of the idle S/R machine is proportional to the rate at which the S/R machine delivers loads at node *i*, that is

$$c_i^I = \sum_{j=1,2} (\Lambda_j/\lambda_T)\beta_{ji}. \tag{18}$$

Consider next, C_i^H, that is, the expected time required for the S/R machine to pick up the HOL request. In this case, the tagged request always finds the S/R machine busy, i.e., traveling either loaded or empty. Let l_{jk}^i (e_{jk}^i) denote the event that the S/R machine is traveling loaded (empty) from *j* to *k* at the time of arrival of the tagged request at node *i*. Let χ_i^H denote the time required for the S/R machine to pick up the HOL request at node *i*. Then

$$\begin{aligned} C_i^H &= E[\chi_i^H] \\ &= \sum_{j=1,2}\sum_{k=1,2}\big[E[\chi_i^H \mid l_{jk}^i]P(l_{jk}^i) \\ &\qquad + E[\chi_i^H \mid e_{jk}^i]P(e_{jk}^i)\big] \end{aligned} \tag{19}$$

Note that Δ_f/ρ is the probability that the S/R machine is traveling loaded since the tagged request always finds the S/R machine busy. Given that the S/R machine is traveling loaded, the proportion of time that it is traveling loaded from *j* to *k* is obtained as $\lambda_j\alpha_{jk}/\Delta_f$. Therefore, $P(l_{jk}^i)$ is given by

$$P(l_{jk}^i) \;=\; \frac{\Delta_f}{\rho}\frac{\lambda_j\alpha_{jk}}{\Delta_f} \;=\; \frac{\lambda_j\alpha_{jk}}{\rho}, \tag{20}$$

which is independent of node where the tagged request arrives.

The term $P(e_{jk}^i)$ is obtained in a similar manner. The tagged request finds the S/R machine traveling empty with probability $1-(\Delta_f/\rho)$. To determine the proportion of time that the tagged request finds the S/R machine traveling empty from *j* to *k*, we proceed as follows. Each time the S/R machine delivers a load at node 1(2) (which occurs at a rate of Λ_1 (Λ_2)), it checks the node and with probability q_1 (q_2), it finds the node empty. Consequently, an empty trip is initiated from 1(2) at a rate of $\Lambda_1 q_1$ ($\Lambda_2 q_2$) and the S/R machine next travels to node 2(1) to pick up a waiting request and the expected empty travel time to node 2(1) is β_{12} (β_{21}). On the other hand, it finds the node non-empty with probability $\overline{q_1}$ ($\overline{q_2}$) and the S/R machine next travels to node 1(2) to pick up a waiting request in there. The expected empty travel time to node 1(2) is β_{11} (β_{22}). (Recall that $\beta_{11}=0$ in our application.)

Since we are considering the case where the tagged request finds *n* > 0 requests at node *i*, if the tagged request finds the S/R machine traveling empty out of node *i* to node *j*, $i\neq j$, it implies that the *n* requests must have all arrived during the empty trip out of node *i*. We assume that the probability of this event is negligible. Therefore, given that the S/R machine is traveling empty, the probability, H_{jk}^i, that the tagged request arriving at node *i* finds the S/R machine traveling empty from *j* to *k* is

$$H_{11}^1 = H_{12}^1 \;=\; H_{11}^2 = \; H_{21}^2 \;=\; 0,$$

$$H_{21}^1 = \frac{\Lambda_2 q_2\beta_{21}}{\Lambda_2 q_2\beta_{21}+\Lambda_2\overline{q_2}\beta_{22}} = \frac{q_2\beta_{21}}{q_2\beta_{21}+\overline{q_2}\beta_{22}}$$

$$H_{22}^1 = \frac{\overline{q_2}\beta_{22}}{q_2\beta_{21}+\overline{q_2}\beta_{22}},$$

$$H_{12}^2 = \frac{\Lambda_1 q_1\beta_{12}}{\Lambda_1 q_1\beta_{12}+\Lambda_2\overline{q_2}\beta_{22}}, \quad \text{and} \tag{21}$$

$$H_{22}^2 = \frac{\Lambda_2\overline{q_2}\beta_{22}}{\Lambda_1 q_1\beta_{12}+\Lambda_2\overline{q_2}\beta_{22}}.$$

The term $P(e_{jk}^i)$ is now computed from

$$P(e_{jk}^i) \;=\; \frac{\rho-\Delta_f}{\rho}H_{jk}^i. \tag{22}$$

We next develop an expression for $E[\chi_i^H \mid l_{jk}^i]$. Let B_k^i denote the expected time for the S/R machine to first visit node *i* from the instant at which the loaded S/R machine arrives at node *k*. The term $E[\chi_i^H \mid l_{jk}^i]$ is then simply obtained as

$$E[\chi_i^H \mid l_{jk}^i] \;=\; \frac{\alpha_{jk}^{(2)}}{2\alpha_{jk}} + \; B_k^i; \quad j\neq k. \tag{23}$$

Equation (23) follows since the tagged request interrupts a loaded travel from *j* to *k*. Note that $B_1^1=0$ and $B_2^2=\beta_{22}$. The value of $B_1^2(B_2^1)$ is obtained by conditioning on the possible events that can occur when the (loaded) S/R machine delivers a request at node 1(2): (a) there is no request at node 1(2), in other words, HOL request is the next request to serve ; (b) there is a request at node 1(2). Noting that q_k is the conditional probability that the S/R machine finds no request at node *k*, given that it just delivered a request, we obtain $B_1^2 = q_1\beta_{12}+\overline{q_1}(\alpha_{12}+\beta_{22})$ and $B_2^1 = q_2\beta_{21}+\overline{q_2}(\beta_{22}+\alpha_{21})$.

The expression for $E[\chi_i^H \mid e_{jk}^i]$ is obtained in a similar manner. Let F_k^i denote the expected time for the S/R machine to first visit node *i* from the instant at which the S/R machine arrives empty at node *k* and picks up a request waiting there. Since the tagged request interrupts an empty trip from node *j* to node *k*, we have

$$E[\chi_i^H \mid e_{jk}^i] \;=\; \frac{\beta_{jk}^{(2)}}{2\beta_{jk}} + \; F_k^i, \tag{24}$$

where $F_1^1 = F_2^2 = 0$, $F_1^2 = \alpha_{12}+\beta_{22}$, and $F_2^1 = \alpha_{21}$. Therefore, we can easily obtain the value of C_i^H, given in (19), from equations (20) ~ (24).

The term C_i^B is obtained, analogous to C_i^H, as

$$\begin{aligned} C_i^B &= E[\chi_i^B] \\ &= \sum_{j=1,2}\sum_{k=1,2}\big[E[\chi_i^B \mid l_{jk}^i]P(l_{jk}^i) \\ &\qquad + E[\chi_i^B \mid e_{jk}^i]P(e_{jk}^i)\big] \end{aligned} \tag{25}$$

where χ_i^B is the time required for the busy S/R machine to pick up the tagged request and $P(l_{jk}^i)$, $E[\chi_i^B \mid l_{jk}^i]$, $E[\chi_i^B \mid e_{jk}^i]$ can be obtained by (23), (24), and (20), respectively

In equation (25), the term $P(e_{jk}^i)$ is obtained using similar arguments as were used to derive (21) and (22). When the tagged request arrives at node i, no empty trip toward this node is in progress, since the tagged request sees no requests waiting at node i. Hence, the term H_{jk}^i is obtained as

$$H_{11}^1 = H_{21}^1 = H_{11}^2 = H_{12}^2 = H_{22}^2 = 0,\ H_{21}^2 = 1,$$

$$H_{12}^1 = \frac{\Lambda_1 q_1 \beta_{12}}{\Lambda_1 q_1 \beta_{12} + \Lambda_2 q_2 \beta_{22}}, \quad \text{and} \qquad (26)$$
$$H_{22}^1 = \frac{\Lambda_2 q_2 \beta_{22}}{\Lambda_1 q_1 \beta_{12} + \Lambda_2 q_2 \beta_{22}}.$$

Recall that $\sum_j \sum_k H_{jk}^i = 1$ for a given i. Therefore, the term $P(e_{jk}^i)$ is now obtained as

$$P(e_{jk}^i) = \frac{\rho - \Delta_f}{\rho} H_{jk}^i. \qquad (27)$$

Therefore, we can evaluate C_i^B from equations (20), (23), (24), (26), and (27).

Finally, C_i^S is obtained as follows. Since the S/R machine picks up a request from node i, it travels loaded to node j, $j \neq i$, following which it takes a time B_j^i to next return to i. Hence,

$$C_i^S = \alpha_{ij} + B_j^i \qquad (28)$$

Thus far, we developed equations to determine the values of π_i, C_i^I, C_i^H, C_i^B, C_i^S. Therefore, it is straightforward to obtain the expected waiting times for the storage and the retrieval requests, given in equation (16), using equations (17), (18), (19), (25), (28). The expected turnaround time, i.e., sum of the service time and waiting time, for both requests can be simply obtained by adding one way expected loaded travel time of the S/R machine to equation (16). It is also straightforward to obtain the mean queue lengths for the storage and retrieval requests due to Little's formula.

Numerical Example : Suppose $T = 1.0$ *min,* $b = 1.0$, $\lambda_1 = 0.5$ requests/min, $\lambda_2 = 0.25$ requests/min, and K is negligible. From equations (11) and (12), we obtain $\rho =$ 0.8184, $q_1 = 0.2830$, and $q_2 = 0.6415$. Also from equations (5) and (9), it is straightforward to obtain $\alpha_{12} = \alpha_{21} = \beta_{12} = \beta_{21} = 0.6667$ min, $\alpha_{21}^{(2)} = \beta_{12}^{(2)} = \beta_{21}^{(2)} = 0.5$ min^2, $\beta_{22} = 0.4667$ min, and $\beta_{22}^{(2)} = 0.2667$ min^2. Therefore, from equations (17), (18), (19), (25), and (28), we obtain $\pi_1 = 0.3585$, $\pi_2 = 0.7170$, $C_1^I = 0.4444$ min, $C_2^I = 0.5333$ min, $C_1^H = 0.7778$ min, $C_2^H = 0.7468$ min, $C_1^B = 0.9518$ min, $C_2^B = 1.2099$ min, $C_1^S = 1.5006$ min, and $C_2^S = 1.6679$ min. Substituting these values into equation (16), we finally obtain $W_1 = 2.94345$ min and $W_2 = 1.57976$ min.

4. Numerical results

In order to test the performance of the analytical model, we simulated AS/R systems with three different values of shape factors, i.e., $b = 1.0, 0.7, 0.3$. For each shape factor, we simulated AS/R systems under different values and ratios of arrival rates of storage requests and retrieval requests, i.e., $\lambda_1 = \lambda_2$, $\lambda_1 = 2\lambda_2$, $2\lambda_1 = \lambda_2$. We assume that the value of $T = Max\{t_h, t_v\}$ is one and the sum of pick up and deposit times, K, is negligible in the simulation experiment. (Lee(1997) also assumes T=1 and K=0 in their experiment.) That is, the influence of the pick up and deposit times, which is usually constant and known, on the performance of the analytical model is eliminated.

In order to obtain steady state statistics, we first make a single simulation run starting with empty storage and retrieval queues and the S/R machine idling at the I/O point. For warm-up purpose, appropriate statistics are cleared when 5,000 time units are passed. After the warm-up period, ten replications are recorded. Each replication is based on 10,000 storage and retrieval requests served by the S/R machine.

<Table 1> shows the simulated and analytical results for the expected waiting times both in the storage and the retrieval queues, and simulated S/R machine utilizations under different interarrival times of the storage and retrieval requests for b=1. (The interarrival times of storage and retrieval requests are $t_1 = 1/\lambda_1$ and $t_2 = 1/\lambda_2$, respectively.) This table also shows 95% confidence intervals for the simulated waiting times in the storage and retrieval queues. <Table 2> and <Table 3> show the same information, but for b=0.7 and b=0.3, respectively. In order to test the performance of the analytical model extensively, we obtained waiting times under wide range of the S/R machine utilization. For example, the minimum and the maximum values of S/R machine utilization reported in <Table 1>, <Table 2> and <Table 3> are 0.262 and 0.974, respectively.

The simulation results indicate that the analytical model performs reasonably well for different values of the shape factors and interarrival times of the storage and retrieval requests. However, the performance of the analytical model becomes worse when the S/R machine is highly utilized. The worst percentage differences in estimating the storage and retrieval waiting times are 22.2% (b = 0.7, t_1 = 1.4, and t_2 = 2.8) and 26.2% (b = 0.7, t_1 = 2.8, and t_2 = 1.4), respectively. In these cases, the simulated S/R machine utilizations are 0.973 and 0.974, respectively. Excluding the cases where the S/R machine is highly utilized, i.e, 0.9 or more, the worst percentage differences in estimating the storage and retrieval waiting times are -10.1% (b = 0.3, t_1 = 1.75, and t_2 = 1.75) and -8.4% (b = 0.7, t_1 = 2.0, and t_2 = 2.0). In these cases, the simulated S/R machine utilizations are 0.881 and 0.877, respectively. (As mentioned earlier, since AS/R system in a supporting facility to store and retrieve WIP, it is unlikely to have a system whose S/R machine utilization is very high.)

<Table 4>, <Table 5> and <Table 6> show expected turnaround times of the storage and retrieval requests for different values of shape factors. As can be noted from these tables, the analytical model performs better in estimating the turnaround time than estimating the waiting time in the queue, since the turnaround time is simply sum of the waiting time in the queue and the one way loaded travel time, which can be accurately obtained from Bozer and White(1990).

Table 1. Expected waiting times in the storage and retrieval queues, b=1.0

Interarrival time		simul	Expected waiting time in storage queue					Expected waiting time in retrieval queue				
Storage	Retrieval	S/R Util	95% CI		Simul model	Analytical model	% diff	95% CI		Simul model	Analytical model	% diff
6	6	0.367	0.660	0.670	0.665	0.634	-4.6%	0.911	0.929	0.920	0.926	0.6%
5	5	0.439	0.763	0.791	0.777	0.727	-6.5%	1.014	1.035	1.024	1.031	0.7%
4	4	0.542	0.928	0.963	0.945	0.901	-4.6%	1.206	1.240	1.223	1.226	0.2%
3	3	0.706	1.41	1.48	1.45	1.359	-6.3%	1.72	1.77	1.75	1.718	-1.8%
2.3	2.3	0.877	2.89	3.11	3.00	2.728	-9.1%	3.15	3.57	3.36	3.121	-7.1%
2	2	0.959	6.14	6.94	6.54	1.641	-6.1%	5.94	7.71	6.82	6.533	-4.2%
5	10	0.339	0.763	0.778	0.771	0.759	-1.6%	0.804	0.821	0.813	0.812	-0.1%
3.75	7.5	0.448	0.949	0.973	0.961	0.939	-2.3%	0.935	0.957	0.946	0.933	-1.4%
3	6	0.556	1.20	1.23	1.21	1.198	-1.0%	1.07	1.10	1.08	1.075	-0.5%
2	4	0.809	2.94	3.19	3.07	2.943	-4.1%	1.61	1.66	1.64	1.580	-3.7%
1.7	3.4	0.928	7.30	8.74	8.02	8.580	7.0%	1.99	2.10	2.04	2.011	-1.4%
10	5	0.339	0.480	0.496	0.488	0.457	-6.4%	0.941	0.950	0.945	0.961	1.7%
7.5	3.75	0.450	0.594	0.614	0.604	0.563	-6.8%	1.12	1.14	1.130	1.160	2.6%
6	3	0.559	0.722	0.751	0.737	0.690	-6.3%	1.41	1.45	1.43	1.439	0.6%
4	2	0.816	1.23	1.28	1.25	1.167	-6.6%	3.29	3.43	3.36	3.241	-3.5%
3.4	1.7	0.934	1.61	1.76	1.68	1.599	-4.8%	8.59	10.2	9.38	8.912	-5.0%

Table 2. Expected waiting times in the storage and retrieval queues, b=0.7

Interarrival time		simul	Expected waiting time in storage queue					Expected waiting time in retrieval queue				
Storage	Retrieval	S/R Util	95% CI		Simul model	Analytical model	% diff	95% CI		Simul model	Analytical model	% diff
6	6	0.321	0.530	0.543	0.537	0.512	-4.7%	0.744	0.753	0.749	0.757	1.0%
5	5	0.383	0.601	0.622	0.612	0.575	-6.1%	0.811	0.834	0.823	0.830	0.8%
4	4	0.477	0.728	0.751	0.739	0.689	-6.7%	0.950	0.968	0.959	0.959	0.0%
3	3	0.623	0.994	1.040	1.02	0.959	-6.0%	1.25	1.29	1.27	1.254	-1.3%
2.3	2.3	0.787	1.58	1.70	1.64	1.570	-4.3%	1.90	2.04	1.97	1.896	-3.8%
2	2	0.877	2.54	2.79	2.66	2.417	-9.1%	2.83	3.20	3.01	2.757	-8.4%
1.75	1.75	0.956	5.07	5.79	5.43	5.176	-4.7%	5.41	6.25	5.83	5.515	-5.4%
5	10	0.296	0.624	0.634	0.629	0.617	-1.9%	0.668	0.681	0.675	0.672	-0.5%
3.75	7.5	0.392	0.744	0.764	0.754	0.737	-2.3%	0.758	0.776	0.767	0.759	-1.1%
3	6	0.488	0.914	0.937	0.926	0.897	-3.1%	0.862	0.882	0.872	0.858	-1.6%
2	4	0.716	1.71	1.80	1.76	1.700	-3.4%	1.20	1.27	1.23	1.184	-3.7%
1.7	3.4	0.828	2.93	3.16	3.05	2.877	-5.7%	1.44	1.49	1.46	1.430	-2.1%
1.4	2.8	0.973	16.5	23.4	19.90	24.311	22.2%	1.94	2.02	1.98	1.960	-1.0%
10	5	0.297	0.393	0.408	0.400	0.370	-7.6%	0.762	0.779	0.771	0.786	2.0%
7.5	3.75	0.393	0.478	0.487	0.482	0.446	-7.6%	0.896	0.911	0.904	0.920	1.7%
6	3	0.489	0.572	0.588	0.580	0.533	-8.0%	1.07	1.09	1.08	1.094	1.3%
4	2	0.718	0.877	0.915	0.896	0.834	-6.9%	1.92	2.05	1.99	1.938	-2.6%
3	1.5	0.926	1.40	1.52	1.46	1.369	-6.3%	6.81	7.96	7.39	6.857	-7.2%
2.8	1.4	0.974	1.68	1.76	1.72	1.612	-6.3%	13.6	25.4	19.5	24.611	26.2%

Table 3. Expected waiting times in the storage and retrieval queues, b=0.3

Interarrival time		Simul	Expected waiting time in storage queue				Expected waiting time in retrieval queue			
Storage	Retrieval	S/R Util	95% CI	Simul model	Analytical model	% diff	95% CI	Simul model	Analytical model	% diff
6	6	0.284	0.446 0.463	0.455	0.434	-4.6%	0.632 0.645	0.639	0.640	0.2%
5	5	0.340	0.507 0.525	0.516	0.482	-6.6%	0.684 0.696	0.690	0.695	0.8%
4	4	0.421	0.597 0.610	0.604	0.565	-6.4%	0.775 0.804	0.790	0.790	0.0%
3	3	0.553	0.780 0.805	0.792	0.749	-5.5%	0.995 1.030	1.01	0.993	-1.7%
2	2	0.793	1.58 1.68	1.63	1.495	-8.3%	1.81 1.99	1.90	1.777	-6.5%
1.75	1.75	0.881	2.42 2.67	2.54	2.282	-10.1%	2.61 2.92	2.77	2.576	-7.0%
1.5	1.5	0.971	6.12 8.00	7.06	6.170	-12.6%	6.27 9.67	7.97	6.455	-19.0%
5	10	0.262	0.523 0.538	0.531	0.525	-1.1%	0.573 0.586	0.579	0.571	-1.3%
3	6	0.430	0.738 0.760	0.749	0.727	-3.0%	0.709 0.731	0.720	0.715	-0.7%
2	4	0.636	1.23 1.27	1.25	1.205	-3.6%	0.957 0.987	0.972	0.950	-2.3%
1.7	3.4	0.737	1.71 1.88	1.79	1.724	-3.7%	1.13 1.18	1.15	1.114	-3.2%
1.4	2.8	0.873	3.54 4.24	3.89	3.824	-1.7%	1.43 1.53	1.48	1.427	-3.6%
1.3	2.6	0.928	5.88 7.49	6.69	7.432	11.1%	1.60 1.68	1.64	1.611	-1.8%
10	5	0.262	0.329 0.345	0.337	0.315	-6.6%	0.655 0.666	0.661	0.667	1.0%
7.5	3.75	0.348	0.397 0.416	0.406	0.374	-7.9%	0.751 0.767	0.759	0.766	1.0%
4	2	0.639	0.709 0.735	0.722	0.656	-9.2%	1.41 1.48	1.44	1.398	-2.9%
3	1.5	0.830	1.07 1.11	1.09	0.987	-9.4%	2.80 3.25	3.03	2.879	-5.0%
2.8	1.4	0.881	1.18 1.25	1.22	1.120	-8.2%	3.88 4.72	4.30	4.063	-5.5%
2.6	1.3	0.936	1.37 1.41	1.39	1.307	-5.9%	7.29 9.10	8.19	7.683	-6.2%

Table 4. Expected turnaround times for storage and retrieval requests, b=1.0

Interarrival time		Storage request			Retrieval request			Weighted turnaround time		
Storage	Retrieval	Simul model	Analytical model	% Diff	Simul model	Analytical model	% Diff	Simul model	Analytical model	% Diff
6	6	1.33	1.30	-2.2%	1.59	1.59	0.1%	1.46	1.45	-0.9%
5	5	1.44	1.39	-3.2%	1.69	1.70	0.5%	1.57	1.55	-1.6%
4	4	1.61	1.57	-2.6%	1.89	1.89	0.1%	1.75	1.73	-1.1%
3	3	2.11	2.03	-4.0%	2.41	2.38	-1.0%	2.26	2.21	-2.4%
2.3	2.3	3.67	3.39	-7.5%	4.03	3.79	-6.0%	3.85	3.59	-6.7%
2	2	7.21	6.81	-5.6%	7.49	7.20	-3.9%	7.35	7.00	-4.7%
5	10	1.44	1.43	-1.0%	1.48	1.48	-0.1%	1.45	1.44	-0.5%
3.75	7.5	1.63	1.61	-1.5%	1.61	1.60	-0.6%	1.62	1.60	-1.0%
3	6	1.88	1.87	-0.8%	1.75	1.74	-0.5%	1.84	1.82	-0.9%
2	4	3.74	3.61	-3.5%	2.31	2.25	-2.8%	3.26	3.16	-3.2%
1.7	3.4	8.68	9.25	6.5%	2.71	2.68	-1.2%	6.69	7.06	5.5%
10	5	1.15	1.12	-2.3%	1.61	1.63	1.1%	1.46	1.46	0.0%
7.5	3.75	1.27	1.23	-3.2%	1.80	1.83	1.5%	1.62	1.63	0.5%
6	3	1.40	1.36	-3.1%	2.09	2.11	0.8%	1.86	1.86	-0.2%
4	2	1.92	1.83	-4.5%	4.03	3.91	-3.0%	3.33	3.22	-3.4%
3.4	1.7	2.35	2.27	-3.6%	10.0	9.58	-4.2%	7.48	7.14	-4.5%

Table 5. Expected turnaround times for storage and retrieval requests, b=0.7

Interarrival time		Storage request			Retrieval request			Weighted turnaround time		
Storage	Retrieval	Simul model	Analytical model	% Diff	Simul model	Analytical model	% Diff	Simul model	Analytical model	% Diff
6	6	1.12	1.09	-2.4%	1.33	1.34	0.6%	1.22	1.22	-0.3%
5	5	1.19	1.16	-2.8%	1.40	1.41	0.8%	1.30	1.28	-1.2%
4	4	1.32	1.27	-3.7%	1.54	1.54	0.0%	1.43	1.41	-1.7%
3	3	1.60	1.54	-3.7%	1.85	1.84	-0.8%	1.73	1.69	-2.4%
2.3	2.3	2.22	2.15	-3.1%	2.55	2.48	-2.8%	2.39	2.31	-3.2%
2	2	3.25	3.00	-7.7%	3.59	3.34	-7.0%	3.42	3.17	-7.3%
1.75	1.75	6.01	5.76	-4.2%	6.41	6.10	-4.9%	6.21	5.93	-4.6%
5	10	1.21	1.20	-0.9%	1.26	1.25	-0.5%	1.23	1.22	-1.1%
3.75	7.5	1.34	1.32	-1.6%	1.35	1.34	-0.7%	1.34	1.33	-1.0%
3	6	1.51	1.48	-2.1%	1.45	1.44	-0.7%	1.49	1.47	-1.6%
2	4	2.34	2.28	-2.5%	1.82	1.77	-3.0%	2.17	2.11	-2.8%
1.7	3.4	3.63	3.46	-4.7%	2.04	2.01	-1.4%	3.10	2.98	-4.0%
1.4	2.8	20.50	24.89	21.4%	2.56	2.54	-0.7%	14.60	17.44	19.5%
10	5	0.98	0.95	-3.1%	1.35	1.37	1.3%	1.23	1.23	-0.1%
7.5	3.75	1.06	1.03	-3.1%	1.49	1.50	0.8%	1.35	1.34	-0.5%
6	3	1.16	1.12	-3.9%	1.66	1.68	1.0%	1.49	1.49	-0.1%
4	2	1.48	1.42	-4.3%	2.57	2.52	-2.0%	2.20	2.15	-2.2%
3	1.5	2.04	1.95	-4.4%	7.97	7.44	-6.7%	6.00	5.61	-6.5%
2.8	1.4	2.30	2.19	-4.6%	20.10	25.19	25.3%	14.10	17.53	24.3%

Table 6. Expected turnaround times for storage and retrieval requests, b=0.3

Interarrival time		Storage request			Retrieval request			Weighted turnaround time		
Storage	Retrieval	Simul model	Analytical model	% Diff	Simul model	Analytical model	% Diff	Simul model	Analytical model	% Diff
6	6	0.97	0.95	-2.1%	1.15	1.16	0.5%	1.06	1.05	-0.7%
5	5	1.03	1.00	-3.2%	1.20	1.21	0.9%	1.12	1.10	-1.5%
4	4	1.12	1.08	-3.5%	1.30	1.31	0.4%	1.21	1.19	-1.4%
3	3	1.31	1.26	-3.5%	1.53	1.51	-1.4%	1.42	1.39	-2.4%
2	2	2.14	2.01	-6.1%	2.41	2.29	-4.9%	2.28	2.15	-5.7%
1.75	1.75	3.06	2.80	-8.6%	3.28	3.09	-5.8%	3.17	2.94	-7.1%
1.5	1.5	7.57	6.68	-11.7%	8.48	6.97	-17.8%	8.03	6.83	-15.0%
5	10	1.05	1.04	-0.9%	1.09	1.09	-0.3%	1.06	1.06	-0.4%
3	6	1.26	1.24	-1.4%	1.24	1.23	-0.8%	1.25	1.24	-1.0%
2	4	1.77	1.72	-2.8%	1.49	1.47	-1.7%	1.67	1.64	-2.1%
1.7	3.4	2.31	2.24	-3.1%	1.67	1.63	-2.5%	2.09	2.04	-2.6%
1.4	2.8	4.41	4.34	-1.6%	1.99	1.94	-2.4%	3.60	3.54	-1.7%
1.3	2.6	7.20	7.95	10.4%	2.16	2.13	-1.6%	5.52	6.01	8.8%
10	5	0.85	0.83	-2.4%	1.18	1.18	0.2%	1.07	1.06	-0.5%
7.5	3.75	0.92	0.89	-3.6%	1.27	1.28	0.9%	1.16	1.15	-0.8%
4	2	1.24	1.17	-5.6%	1.96	1.91	-2.4%	1.72	1.67	-3.2%
3	1.5	1.61	1.50	-6.7%	3.54	3.39	-4.1%	2.90	2.76	-4.7%
2.8	1.4	1.73	1.64	-5.5%	4.81	4.58	-4.8%	3.79	3.60	-5.1%
2.6	1.3	1.90	1.82	-4.1%	8.70	8.2	-5.8%	6.43	6.07	-5.6%

Excluding the cases where the S/R machine is highly utilized, i.e, 0.9 or more, the worst percentage differences in estimating the turnaround times of the storage and retrieval requests, and weighted turnaround time are -8.6% ($b = 0.3$, $t_1 = 1.75$, and $t_2 = 1.75$), -7.0% ($b = 0.7$, $t_1 = 2.0$, and $t_2 = 2.0$), and -7.3% ($b = 0.7$, $t_1 = 2.0$, and $t_2 = 2.0$), respectively.

5. Conclusions

In this paper, we developed a closed form approximate analytical model to estimate the expected waiting times for the storage and retrieval requests under stochastic demand and general service time distributions for SC and DC. From this model, it is straightforward to obtain the turnaround times and the associated mean queue lengths for the storage and retrieval requests. To develop the analytical model, we assume that the S/R machine idles either at the rack or at the I/O point, which is reasonable and performs well compared to the dwell point strategy where each trip starts and finishes at the I/O point, as discussed in Bozer and Cho(1998).

The analytical waiting time model presented here can be used to evaluate the performance of stable systems by examining the expected waiting times for the storage and retrieval requests. In fact, even if the system is said to be stable, the expected waiting times (and the corresponding mean queue lengths) can be unacceptably long, due to non-linear relationship between the expected S/R utilization and waiting times of the requests. Furthermore, using the expected storage and retrieval queue lengths, we can determine the buffer size (or length) of the input conveyor and compute the number of the rack openings which is required to hold the loads which are requested by processing machines but waiting in the rack to be retrieved by the S/R machine.

References

Bozer, Y.A. (1978), A minimum cost design for an automated warehouse, Masters Thesis, Georgia Institute of Technology, Atlanta, GA.

Bozer, Y.A. and Cho, M.S. (1998), Throughput Performance of Automated Storage/Retrieval Systems under Stochastic Demand, Working paper, The University of Michigan, Ann Arbor, MI, 48109-2117, To appear in *IIE Transactions on Design and Manufacturing.*

Bozer, Y.A., Schorn, E.C., and Sharp, G.P. (1990), Geometric approaches to solve the Chebyshev traveling salesman problem, *IIE Transactions*, **22**(3), 238-254.

Bozer, Y.A. and White, J.A. (1984), Travel-time models for automated storage/retrieval systems, *IIE Transactions*, **16**(4), 329-338.

Bozer, Y.A. and White, J.A. (1990), Design and performance models for end-of-aisle order picking systems, *Management Science*, **36**(7), 852-866.

Chang, D.T., Wen, U.P., and Lin, J.T. (1995), The impact of acceleration/deceleration on travel-time models for automated storage/retrieval systems, *IIE Transactions*, **27**(1), 108-111.

Egbelu, P.J. (1991), Framework for dynamic positioning of storage/retrieval machines in an automated storage/retrieval system, *International Journal of Production Research*, **29**(1), 17-37.

Egbelu, P.J. and Wu, C.T. (1993), A comparison of dwell point rules in an automated storage/retrieval system, *International Journal of Production Research*, **31**(11), 2515-2530.

Elsayed, E.A. and Lee, M.K. (1996), Order processing in automated storage/retrieval systems with due dates, *IIE Transactions*, **28**(7), 567-577.

Foley, R.D. and Frazelle, E.H. (1991), Analytical results for miniload throughput and the distribution of dual command travel time, *IIE Transactions*, **23**(3), 273-280.

Han, M.H., McGinnis, L.F., Shieh, J.S., and White, J.A. (1987), On sequencing retrievals in an automated storage/retrieval system, *IIE Transactions*, **19**(3), 56-66.

Hwang, H. and Lee, S.B. (1990), Travel-time models considering the operating characteristics of the storage and retrieval machine, *International Journal of Production Research*, **28**(10), 1779-1789.

Hwang, H. and Lim, J.M. (1993), Deriving an optimal dwell point of the storage/retrieval machine in an automated storage/retrieval system, *International Journal of Production Research*, **31**(11), 2591-2602.

Lee, H. F. (1997), Performance analysis for automated storage and retrieval systems, *IIE Transactions*, **29**(1), 15-28.

Lim, S. Y., Hur, S., Lee, M. H., Lee, and Y. H. (2001), M/G/1 queueing model for the performance estimation of AS/RS, *Journal of the Korean Institute of Industrial Engineers,* **27**(1), 111-117.

Peters, B.A., Smith, J.S., and Hale, T.S. (1996). Closed form models for determining the optimal dwell point location in automated storage and retrieval systems, *International Journal of Production Research*, **34**(6), 1757-1771.