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# MIMO OFDM 시스템을 위한 향상된 채널 추정 기법

## (An Enhaced Channel Estimation Technique for MIMO OFDM Systems)

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### 요약

MIMO OFDM 시스템에서 comb형태의 훈련심볼을 사용하는 기존의 채널 추정방식은 가장자리 반송파 영역에서 채널 추정의 MSE(Mean Squared Error)가 큰 단점이 있다. 본 논문에서는 이러한 문제점을 해결하기 위해 순환구조를 갖는 comb형태 훈련심볼을 제안한다. 제안된 순환구조에서는 모든 종류의 comb 훈련심볼이 각 안테나로부터 순환 전송되며, 수신 단에서는 각 comb 형태의 훈련심볼들을 이용해 채널을 추정한 후, MSE로부터 구해진 최적의 가중치를 곱함으로써 채널 주파수 응답을 추정한다. 모의실험을 통해 제안된 방식이 기존의 comb 형태의 훈련심볼에 비해 높은 성능을 나타냄을 확인하였다.

### Abstract

In MIMO-OFDM systems, conventional channel estimation techniques using comb type training symbols give relatively large mean squared errors(MSEs) at the edge subcarriers. To reduce the MSEs at these subcarriers, a cyclic comb type training structure is proposed. In the proposed cyclic training structure, all types of training symbols are transmitted cyclically at each antenna. At the receiver, the channel frequency responses that are estimated using each training symbol are averaged with weights obtained from the corresponding MSEs. Computer simulations showed that the proposed cyclic training structure gives more SNR gain than the conventional training structure.

**Keywords** : Channel estimation, cyclic training structure, MIMO-OFDM, training symbol

## I. Introduction

The orthogonal frequency division multiplexing (OFDM) technique has recently attracted considerable interest as an effective method for high rate communication systems[1]. Since the OFDM system can efficiently combat inter-symbol interference(ISI), it has been adopted as a standards for digital audio broadcasting, digital video broadcasting, and broad band indoor wireless systems. On the other hand, information theory indicates that a multi-input multi-output (MIMO) system is able to support enormous capacities[2][3], provided the multipath scattering of a wireless channel is exploited with appropriate space-time signal processing techniques. However, in a frequency selective broadband channel, the MIMO system requires a complicated channel equalization technique in order to eliminate the ISI. To alleviate this problem, the use of an OFDM technique for MIMO systems would be desirable, and recent studies have shown that combining the OFDM technique with a MIMO system can provide high performance transmission[4][5].

In a MIMO-OFDM system, the receiver should know the frequency responses of the spectral and spatial channels between the transmit and receive antennas to achieve coherent signal detection. For reliable

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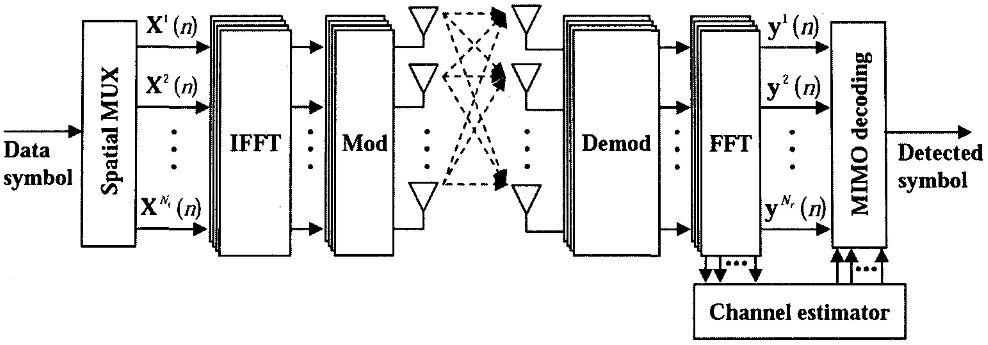

그림 1. 간략화된 MIMO OFDM 시스템
Fig. 1. Simplified MIMO-OFDM system.

channel estimation, the channel estimators exploit training symbols from each transmit antenna, which are known by the receivers, and these training symbols must be orthogonal with respect to each other[5][6]. The commonly used training symbol for a MIMO OFDM system is the "comb type training symbol" in which each transmitter uses a different subset of subcarriers for transmitting its training symbols[6]. In this method, the training symbols of the transmit antennas are orthogonal with respect to each other in the frequency domain thus permitting the channel frequency responses to be effectively estimated.

However, the conventional comb type training symbol produces high mean squared errors(MSEs) at the edge subcarriers[7], so that the overall bit error rate is increased. To reduce the MSEs at such subcarriers, we propose a "cyclic comb type training structure" for the training symbols. In the proposed cyclic training structure, all types of training symbols are transmitted cyclically at each transmitter. At the receiver, the channel frequency responses are estimated using each training symbol and are then averaged with weights obtained from the corresponding MSEs.

## II. MIMO-OFDM system model

A simplified MIMO-OFDM system is shown in Fig. 1. If the time-varying and frequency-selective fading channel is considered, the received signal vector $\boldsymbol{y}^q(n)$ at the $q^{th}$ antenna can be expressed as [5]:

$$\boldsymbol{y}^q(n) = \sum_{p=1}^{N_t} \boldsymbol{X}^p(n)\boldsymbol{F}_{[1:L]}\boldsymbol{h}_t^{p,q}(n) + \boldsymbol{w}^q(n) \qquad (1)$$

where $N_t$ indicates the number of transmit antennas, $\boldsymbol{h}_t^{p,q}$ is the channel impulse response vector with $L \times 1$ dimension between the $p^{th}$ and $q^{th}$ antennas, and $\boldsymbol{w}^q(n)$ is a noise vector at the $q^{th}$ receive antenna with variance $\sigma^2$ per component. $\boldsymbol{X}^p(n)$ is diagonalized OFDM symbol matrix at the $q^{th}$ transmit antenna, and can be expressed as

$$\boldsymbol{X}^p(n) = \\ diag([0 \dots 0 \;\; x_1^p \;\dots\; x_{N_\alpha}^p \;\; 0 \;\; x_{N_\alpha+1}^p \;\dots\; x_{2N_\alpha}^p \;\; 0 \;\dots\; 0]) \qquad (2)$$

where $N_\alpha$ denotes the number of one sided available subcarriers so that the total number of subcarriers for data transmission is $2N_\alpha$, the subscript indicates the available subcarrier index, and $diag(\cdot)$ denotes diagonalize operator. In addition, $\boldsymbol{F}$ describes the $K \times K$ Fourier transform matrix and $\boldsymbol{F}_{[1:L]}$ indicates the first columns of $\boldsymbol{F}$. Assume that the virtual subcarriers at the edges of the spectrum are not used to avoid aliasing problems at receiver and the center subcarrier is not used to avoid intermodulation effects and difficulties in D/A and A/D conversion[8].

## III. Conventional channel estimation technique

In the MIMO-OFDM system, the comb type training symbol is a well-known training structure for channel estimation[6]. In the comb type training structure, it is assumed that each transmit antenna has the same number of training subcarriers, so that the total number of available subcarriers is

$$2N_\alpha = N_c N_t \tag{3}$$

where $N_c$ denotes the number of pilot subcarriers reserved for the training symbols of each transmit antenna and is assumed to be not smaller than the channel dispersion length $L$. Then, the matrix of comb type training symbols at the $p^{th}$ transmit antenna, $\boldsymbol{X}^p_{comb}$ has diagonal elements $x^p_i$ which satisfies

$$x^p_i = \begin{cases} c_i, & i = (m-1)N_t + p \\ 0, & otherwise \end{cases} \tag{4}$$

where $c_i$ denotes an arbitrary complex number with magnitude $\sqrt{N_t}$ and $m$ indicates an arbitrary positive integer which is not greater than $N_c$. In the comb type training structure, $\boldsymbol{X}^p_{comb}$ is repeatedly transmitted $N_t$ times as depicted in Fig. 2 for 4 transmit antenna system, for example.

In the comb type structure, the training symbol transmitted at the $p^{th}$ antenna can be separated from the received signal vector $\boldsymbol{y}^q(n)$, because the comb type training symbols are orthogonal in the frequency domain. The received training symbol vector of the $q^{th}$ receive antenna at the $n^{th}$ OFDM block can then be modified from (1) as

$$\boldsymbol{y}^{p,q}(n) = \boldsymbol{X}^p(n)\boldsymbol{F}_{[1:L]}\boldsymbol{h}^{p,q}_t(n) + \boldsymbol{w}^q(n) \tag{5}$$

where $\boldsymbol{X}^p_{comb}(n)$ does not hae full rank because of virtual subcarriers and DC subcarrier. The dimension of $\boldsymbol{X}^p_{comb}(n)$ can be reduced to an $N_c \times N_c$ matrix, $\dot{\boldsymbol{X}}^p_{comb}(n)$, by removing the null diagonal elements. In the same manner, $\boldsymbol{F}_{[1:L]}$ and $\boldsymbol{y}^{p,q}(n)$ can be reduced

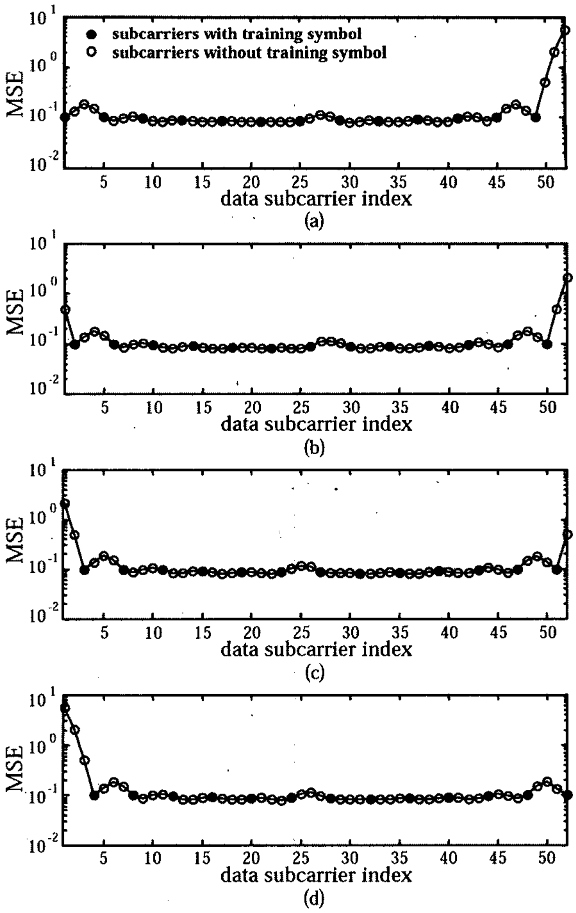

그림 3. 기존 훈련심볼을 사용한 경우 각 반송파의 MSE의 예( SNR 10dB ) (a) 첫째, (b) 둘째, (c) 셋째, (d) 넷째 송신 안테나

Fig. 3. An example of the MSE of each data subcarrier using the conventional training symbol at SNR 10dB (a) 1st Tx antenna, (b) 2nd Tx antenna, (c) 3rd Tx antenna and (d) 4th Tx antenna.

according to the dimension of $\dot{\boldsymbol{X}}^p_{comb}(n)$. Consequently, the reduced receive vector can be expressed as

$$\dot{\boldsymbol{y}}^{p,q}(n) = \dot{\boldsymbol{X}}^p(n)\dot{\boldsymbol{F}}_{[1:L]}\boldsymbol{h}^{p,q}_t(n) + \boldsymbol{w}^q(n) \tag{6}$$

where $\dot{\boldsymbol{F}}^p_{[1:L]}$ denotes the reduced Fourier transform matrix. From (6), the channel impulse response can be then estimated by the least square technique [9]:

$$\hat{\boldsymbol{h}}^{p,q}_t(n) = (\dot{\boldsymbol{X}}^p_{comb}(n)\dot{\boldsymbol{F}}^p_{[1:L]})^+ \tag{7}$$

where $(\cdot)^+$ indicates a pseudo-inverse operation. However, the channel frequency response is actually required for MIMO-OFDM channel equalization and can be estimated by

$$\hat{\boldsymbol{h}}_f^{p,q}(n)=\dot{\boldsymbol{F}}_{[1:L]}\hat{\boldsymbol{h}}_t^{p,q}(n)=\hat{\boldsymbol{h}}_f^{p,q}(n)+e^{p,q}(n) \tag{8}$$

where $\dot{\boldsymbol{F}}_{[1:L]}$ is the reduced Fourier transform matrix of $2N_\alpha \times L$ dimension with rows corresponding to data subcarriers and $e^{p,q}(n)$ denotes the estimation error vector. The mean squared error(MSE) vector can then be obtained as

$$\sigma^2_{p,q}(n)=\Psi(\boldsymbol{E}\{e^{p,q}(n)e^{p,q}(n)^H\}) \tag{9}$$

where $\Psi(\cdot)$ denotes the de-diagonalizaton operator which creates a vector from a matrix by taking the diagonal term of the matrix, and $\boldsymbol{E}\{\cdot\}$ indicates mean operation.

In the comb type training symbol, the channel frequency responses between the transmit and receive antennas are sampled by sparse training subcarriers and the sampled channel frequency responses are interpolated to estimate the channel coefficient of all subcarriers. In this case, there are no training symbols to obtain channel information at the edge subcarriers, so that the channel estimation errors are inevitably increased in such subcarriers[7]. Fig. 3 shows a typical example of the MSE of each subcarrier when 4 transmit antennas are used in a MIMO-OFDM system with 64 subcarriers. It clearly shows that the MSEs of the edge subcarriers are relatively large.

In the comb type training structure, the replica of $\dot{\boldsymbol{X}}^p_{comb}$ is transmitted $N_t$ times, and the estimated channel frequency responses are averaged under the assumption that the channel is invariant during the training phase. The averaged channel frequency response can be expressed as

$$\begin{aligned}\bar{\boldsymbol{h}}_f^{p,q}&=\frac{1}{N_t}\sum_{n=1}^{N_t}\hat{\boldsymbol{h}}_f^{p,q}(n)\\&=\boldsymbol{h}_f^{p,q}+\frac{1}{N_t}\sum_{n=1}^{N_t}\boldsymbol{e}^{p,q}(n)\end{aligned} \tag{10}$$

In this case, since $\boldsymbol{e}^{p,q}(n)$ is identical with respect to $n$, the averaged MSE vector, $\bar{\boldsymbol{\sigma}}^2_{p,q}$ can be expressed as

$$\bar{\boldsymbol{\sigma}}^2_{p,q}=\frac{1}{N_t}\boldsymbol{\sigma}^2_{p,q}(n) \tag{11}$$

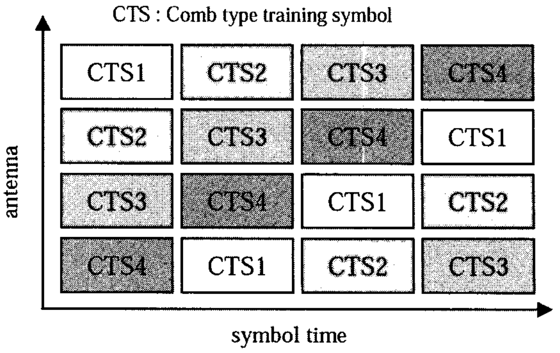

그림 4. 제안된 훈련심볼 구조
Fig. 4. Proposed trainig symbol structure.

Furthermore, the overall normalized MSE(NMSE), which is defined by the sum of all MSEs normalized by the number of antennas and data subcarriers, can be expressed as

$$NMSE_{conv}=\frac{\sum_{p=1}^{N_t}\sum_{q=1}^{N_r}trace(diag(\bar{\boldsymbol{\sigma}}^2_{p,q}))}{2N_\alpha N_t N_r} \tag{12}$$

## Ⅳ. Proposed channel estimation technique

As discussed above, in the conventional training structure, the estimated channel frequency responses have estimation error variances of equal patterns, so that the averaged MSEs are relatively large at the edge subcarriers. Therefore, an approach in which the same training symbols are sent repeatedly to each transmitter and the estimated channel frequency res-ponses are averaged with equal weight is not desirable.

To alleviate this problem, we propose "the cyclic comb type training structure". In the proposed structure, a comb type training symbol, transmitted through the 1st antenna, is transmitted through the 2nd antenna, and will be transmitted through the 3rd antenna the next time, and so on. Using this cyclical scheme, all types of training symbols are transmitted at each antenna. Fig. 4 describes the proposed structure in the case of a 4 transmit antenna system.

In the proposed structure, the estimated channel frequency responses $\hat{\boldsymbol{h}}_f^{p,q}(n)$ should not be averaged using (10), because $\boldsymbol{\sigma}^2_{p,q}(n)$ has different patterns

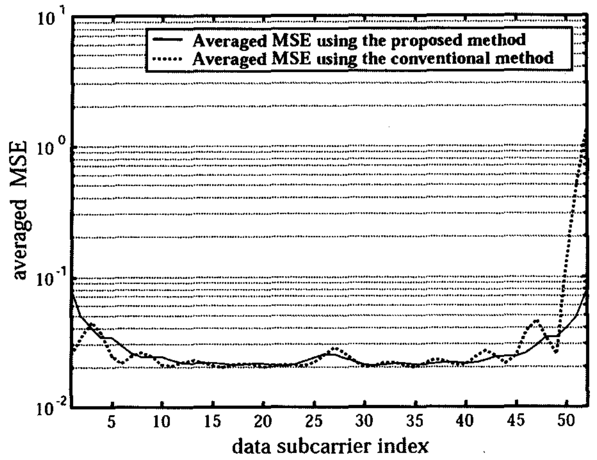

그림 5. 기존 훈련심볼과 제안된 훈련심볼의 성능비교 ( SNR 10dB)

Fig. 5. An example of averaged MSEs of channel estimation using the conventional training symbol and the proposed cyclic training symbol at SNR 10dB.

according to $n$. Therefore, it would be better to weight $\hat{h}_f^{p,q}(n)$ according to the estimation error in the averaging process, so as to ensure that the averaged channel frequency response $\bar{h}_f^{p,q}$ has a minimum error variance:

$$\begin{aligned}\bar{h}_f^{p,q} &= \Big(\sum_{n=1}^{N_t} C^{p,q}(n)\Big)^{-1} \sum_{n=1}^{N_t} C^{p,q}(n)\hat{h}_f^{p,q}(n) \\ &= h_f^{p,q} + \left\{\sum_{n=1}^{N_t} C^{p,q}(n)\right\}^{-1} \sum_{n=1}^{N_t} C^{p,q}(n) e^{p,q}(n) \\ &= h_f^{p,q} + \Omega^{p,q}\end{aligned} \tag{13}$$

where $C^{p,q}(n)$ is a diagonal weight matrix to minimize the MSEs, and $\Omega^{p,q}$ denotes the channel estimation error. $C^{p,q}(n)$ can be obtained by solving the following minimization problem [9]:

$$\min_{C^{p,q}} E\{\Omega^{p,q}(\Omega^{p,q})^H\} \tag{14}$$

The weight matrix satisfying (14) is then given by

$$C^{p,q}(n) = (diag(\sigma^2_{p,q}(n)))^{-1} \tag{15}$$

The corresponding minimum MSE of each subcarrier is expressed as

$$\bar{\sigma}^2_{p,q} = \Psi\Big(\Big(\sum_{n=1}^{N_t}(diag(\sigma^2_{p,q}(n)))^{-1}\Big)^{-1}\Big) \tag{16}$$

In this case, the normalized MSE of the proposed cyclic training structure ($NMSE_{cydic}$) can also be obtained using (12).

Fig. 5 gives an example of typical MSEs of channel estimation for the conventional and the proposed structures at 10dB SNR. This clearly shows that the proposed structure is capable of reducing MSEs at the edge subcarriers.

In addition, the performance of the proposed structure can be evaluated by the NMSE ratio (NMSER) defined as :

$$NMSER = \frac{NMSE_{conv}}{NMSE_{cydic}} \tag{17}$$

Fig.6 shows the NMSER with respect to the number of transmit antennas. This indicates that the NMSE of the proposed structure is always less than that of the conventional structure, when the number of transmit antennas is greater than one. It also indicates that the NMSER tends to be increased, with increasing the number of transmit antennas. From this result, we can infer that the proposed structure becomes increasingly useful as the number of antennas increases.

## V. Simulation results

It was assumed that each transmit antenna of the target MIMO-OFDM system has an individual OFDM block, which is based on the IEEE 802.11a system[8]. Although the IEEE 802.11a system supports variable data rates, only the 12 Mbps rate case which uses a QPSK symbol set and 1/2 coding rate was considered. As a MIMO configuration, a 4X4 system is considered. For channel estimation, the training symbols were transmitted 4 times in a preamble to 16$\mu$s. At the receive end, the V-BLAST algorithm[3]was used to detect spatially multiplexed signals.

In order to evaluate the performance of the channel estimation in the MIMO-OFDM system, the channel dispersion length was assumed to be 12 samples. It was assumed that the power delay profile of the

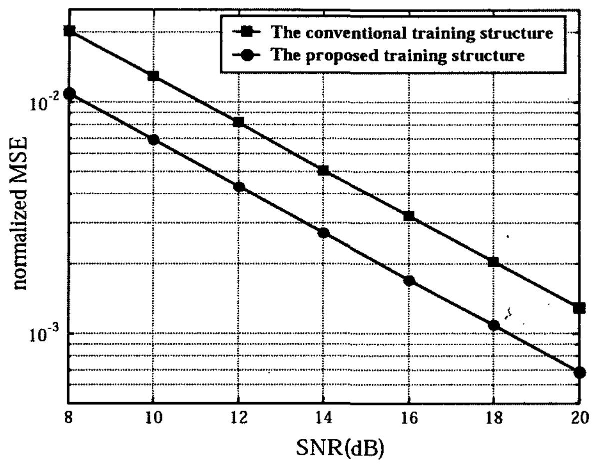

그림 7. 기존 훈련심볼과 제안된 훈련심볼의 NMSE
Fig. 7. NMSE of the conventional training symbol and the proposed training symbol.

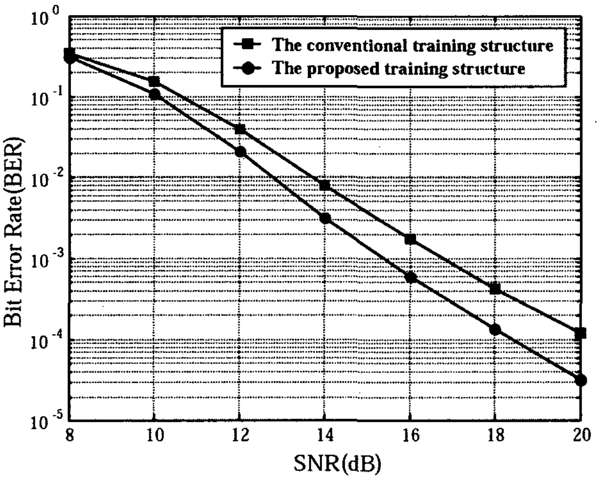

그림 8. 기존 훈련심볼과 제안된 훈련심볼의 BER
Fig. 8. BER of the conventional training symbol and the proposed training symbol.

channel decays exponentially. It was also assumed that the channel state is static during the transmission of the OFDM symbol block.

Computer simulations were performed to verify the effects of the proposed cyclic training structure. Fig. 7 shows the NMSE of channel estimation for a 4X4 system. It can be seen that the proposed structure has a 2.5dB SNR gain over the conventional structure.

Fig. 8 compares bit error rate(BER) performance and indicates that the cyclic structure has about a 1.3dB SNR gain at BER=$10^{-3}$ over the conventional structure.

## VI. Conclusions

A cyclic comb type training symbol structure for training symbols is proposed to enhance the channel estimation performance of MIMO-OFDM systems. Computer simulations indicate that the proposed structure gives a higher SNR gain than the conventional structure. Therefore, the proposed method effectively enhances the channel estimation performance of MIMO-OFDM systems.

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